# Fin design for silent PC

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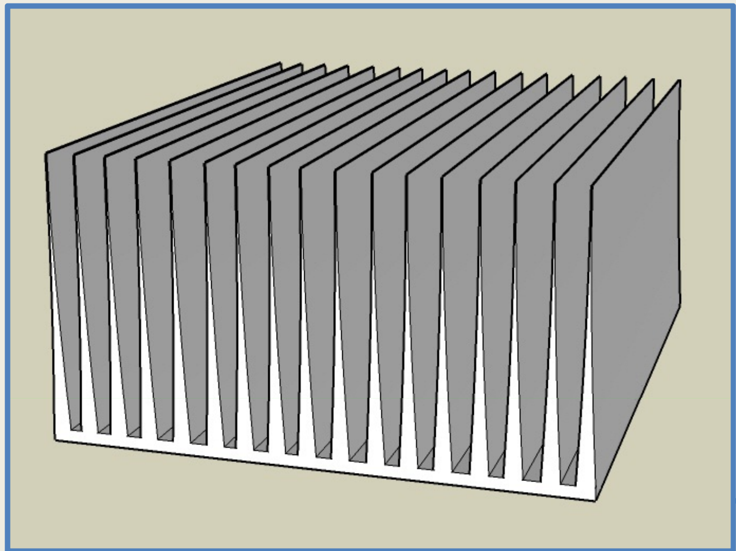
# Problem Statement

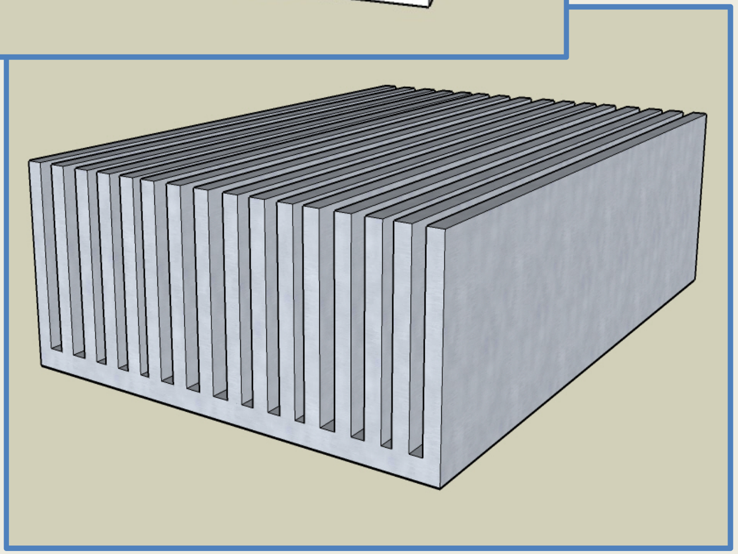
One day, a guy bought the brand new powerful high-end CPU for movie. He got a CPU at the retail store and brought it to his home. He installed his new CPU on the motherboard with CPU cooler. Sometimes later, he was annoyed by the noise from the CPU cooler. So he decided to get rid of the cooler from his motherboard. And then he recognized that he need more powerful cooling system consisted of only fins. Power of his new CPU at throttling is 65W, and critical temperature of motherboard is 60ºC. Size of heat sink is 75mm x 75mm x 1mm. He didn’t have enough money, so he had to minimize the consumption of the material for heat sink. If you were he, how did you design it?

# Design Object



**Advanced Model**

- Geometrical : Concave Parabolic
- Convection : Natural Convection
  Yovanovich, Culham(1994)
- Objective of design
  Minimize mass
  Power consumption of CPU(65W)



**Previous Model**

**2004, "Fin design" - 김민태**

- Geometrical : Rectangular
- Convection : Natural Convection
  Bar-Cohen, Rohsenow(1984)
- Objective of design
  Maximize fin effectiveness
  Minimize volume

# Assumptions
## *The Murray-Gardner assumptions*

- Steady state
- 1-Dimesional heat transfer
- Homogeneous and uniform characteristics of materials.
- Temp. of surrounding the fin is uniform
- Temp. at the base of the fin is uniform
- No heat source within the fin itself
- Neglect radiation effect

# Data and Design Variables

**Properties of Air**

- Temp of air is 60°C
- Prandtl number, Pr=0.7202
- Kinematic viscosity, $\nu = 1.896 \times 10^{-5}\ \mathrm{m}^2 / s$
- Thermal expansion coefficient

**Fin material**

- Pure aluminum
- Density, $\rho = 2702\ \mathrm{kg/m^3}$
- Thermal conductivity, $k = 237\ \mathrm{W/m \bullet} K$

# Data and Design Variables

**Intermediate Variable**

- H (Height of a heat sink) = $\delta_p + b$ [mm]

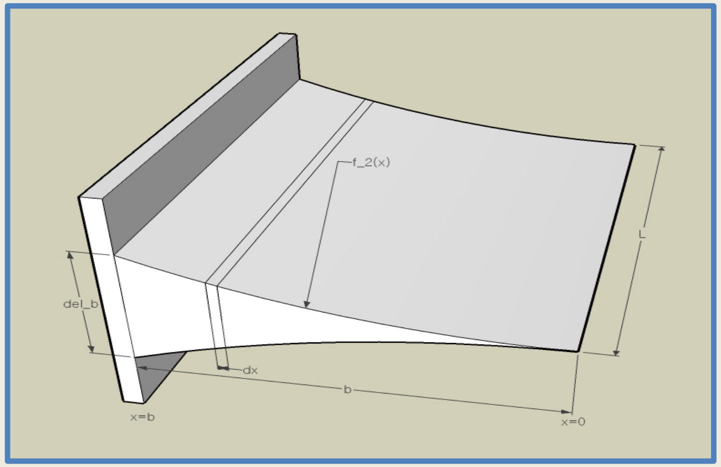
**Design Variables**

- $\delta_b$ (Thickness of a fin) [mm]
- b (height of Fin) [mm]
- n (Number of fins) [EA]

**Parameters**

- Q (Power Consumption of CPU) = 65 W
- L (Length of a Heat sink) = 75 mm
- W (Width of a heat sink) = 75 mm
- $T_a$ (Temp. of air) = 35°C
- $T_b$ (Temp. of base) = 60°C
- $\delta_p$ (Thickness of plate) = 1 mm

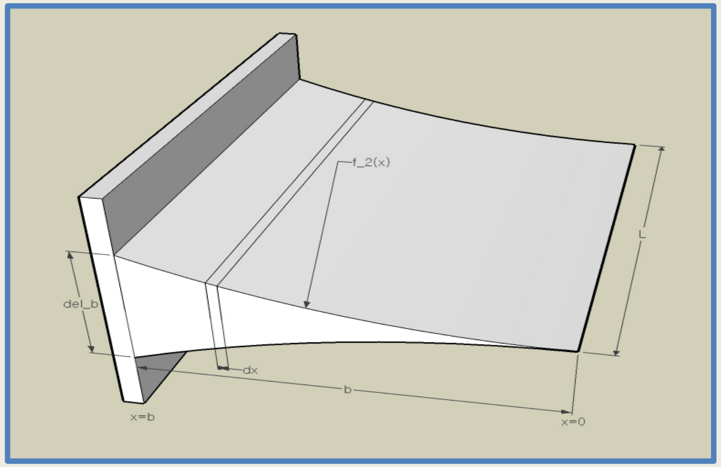
# Objective Function




**Minimize mass of the heat sink(fins).**

$$f(n,\delta_b,b)=\frac{n\delta_b bL}{3}\rho$$

# Formulation - Heat transfer




- Generalized Differential Equation

$$2Lf_2(x)\frac{d^2\theta}{dx^2}+\frac{2Ldf_2(x)}{dx}\frac{d\theta}{dx}-\frac{2h}{k}\theta=0$$

- Geometry function

$$f_2(x)=\frac{\delta_b}{2}\left(\frac{x}{b}\right)^2$$

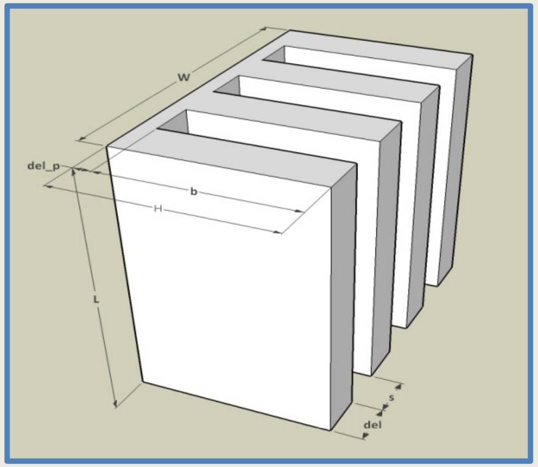
- Solution of temp. distribution

$$\theta(b,h,\delta_b,x)=\theta_b\left(\frac{x}{b}\right)^\alpha,\ \text{where } \alpha=-\frac{1}{2}+\frac{1}{2}(1+4m^2b^2)^{1/2},\ m=\left(\frac{2h}{k\delta_b}\right)^{1/2}$$

- Heat transfer through base of fin

$$\boxed{\dot{Q}(b,\delta_b)=\frac{k\delta_b L\theta_b}{2b}\left(-1+\sqrt{1+(2mb)^2}\right)}$$

# Formulation - Natural Convection *Yovanovich et al (1994)*




- Nusselt numbers, $\boxed{\begin{aligned} \mathrm{Nu}_{\sqrt{A}} &\equiv \frac{h\sqrt{A}}{k} \\ &= \mathrm{Nu}_{\sqrt{A}}^{\infty} + f(\mathrm{Pr})\mathrm{G}_{\sqrt{A}}\mathrm{Ra}_{\sqrt{A}}^{0.25} \end{aligned}}$

- Rayleigh numbers, $\mathrm{Ra}_{\sqrt{A}} \equiv \dfrac{g\beta \mathrm{Pr}\Delta T(\sqrt{A})^3}{\nu^2}$

- Universal Prandtl number, $f(\mathrm{Pr}) = \dfrac{0.67}{[1+(0.5/\mathrm{Pr})^{9/16}]^{4/9}}$

- Diffusive limit, $\mathrm{Nu}_{\sqrt{A}}^{\infty} = \dfrac{3.192 + 1.868(H/L)^{0.76}}{\sqrt{1+1.189(H/L)}}$

- Body gravity function

$$G_{\sqrt{A}} = 1.0904\left[\frac{L\Lambda^2}{(n\delta_b b + \delta_b W + L\Lambda)^{3/2}}\right]^{1/4}, \text{ where } \Lambda = nb + \delta_p + W$$

# Constraints

**h1. Heat transfer**

$$\dot{Q} - \dot{Q}_{total} = 65 - \left[ n \times \frac{k\delta_b L\theta_b}{2b}\left(-1 + \sqrt{1 + (2mb)^2}\right) + \left(W - n\delta_b\right) Lh\theta_b \right] = 0$$

**g1. 1-Dimensional heat transfer**

$$\frac{h\delta}{k} \leq 0.2,\ (\delta \leq \delta_b) \rightarrow\ \delta_b - \frac{h}{k} \leq 0$$

**g2. Fin efficiency**

$$\frac{2}{1 + \sqrt{1 + (2mb)^2}} \geq 0.6\ \rightarrow\ \frac{b^2 h}{0.96k} - \delta_b \leq 0$$

# Constraints

## g3. Laminar flow

$$Ra_{\sqrt{A}} - 10^9 \leq 0$$

## g4~9. Geometry

$$-\delta_b \leq 0 \qquad 0.001 - \delta_b \leq 0$$

$$-b \leq 0 \qquad b - 0.1 \leq 0$$

$$1 - n \leq 0 \qquad n\delta_b - W \leq 0$$

# Changes compared with PRJT 1.

- **Number of the design variables.**

  2 -> 3 (added b(height of the fin))

- **More specific linear and nonlinear constraints**

  1-Dimensional heat transfer

  Fin efficiency

  Laminar flow condition

- **More complicated situation**

  Caused by natural convection

# Results (by Matlab : fmincon)

- Initial Condition

  δ_b (Thickness of a fin) =1[mm]

  b (height of Fin) =70[mm]

  n (Number of fins) =75[EA]

- Number of iterations : 5
- Number of function count : 20
- Optimum values

  δ_b (Thickness of a fin) =1[mm]

  b (height of Fin) =55.5[mm]

  n (Number of fins) =75[EA]

  mass of the fins = 0.2789[kg]

  Fin efficiency = 91.12%

# Results (by Matlab : fmincon)

- Initial Condition

  $\delta\_b$ (Thickness of a fin) = 1.9035[mm]

  b (height of Fin) = 75[mm]

  n (Number of fins) =38[EA]

- Number of iterations : 12
- Number of function count : 64
- Optimum values

  $\delta\_b$ (Thickness of a fin) =1.5[mm]

  b (height of Fin) =92.8[mm]

  n (Number of fins) =47[EA]

  mass of the fins = 0.4494[kg]

  Fin efficiency = 87.08%

# Results (by Matlab : pattern search,1)

- Initial Condition

  δ_b (Thickness of a fin) =1[mm]

  b (height of Fin) =70[mm]

  n (Number of fins) =75[EA]

- Number of iterations : 4
- Number of function count : 919
- Search method : MADPositvieBasis2N
- Optimum values

  δ_b (Thickness of a fin) =1[mm]

  b (height of Fin) =72.9[mm]

  n (Number of fins) =59[EA]

  mass of the fins = 0.3022[kg](Min)

  Fin efficiency = 87.42%

# Results (by Matlab : pattern search,1)

- Initial Condition

  δ_b (Thickness of a fin) = 1.9035[mm]

  b (height of Fin) = 75[mm]

  n (Number of fins) =38[EA]

- Number of iterations : 5
- Number of function count : 1701
- Search method : MADPositvieBasis
- Optimum values

  δ_b (Thickness of a fin) =1[mm]

  b (height of Fin) =90.6[mm]

  n (Number of fins) =50[EA]

  mass of the fins = 0.3098 [kg](Min)

  Fin efficiency = 86.12%

# Results (by Matlab : pattern search,2)

- Initial Condition

  δ_b (Thickness of a fin) =1[mm]

  b (height of Fin) =70[mm]

  n (Number of fins) =75[EA]

- Number of iterations : 4
- Number of function count : 759
- Search method : MADPositvieBasisNp1
- Optimum values

  δ_b (Thickness of a fin) =1[mm]

  b (height of Fin) =72.9[mm]

  n (Number of fins) =59[EA]

  mass of the fins = 0.3022[kg](Min)

  Fin efficiency = 88.63%

# Results (by Matlab : pattern search,2)

- Initial Condition

  δ_b (Thickness of a fin) = 1.9035[mm]

  b (height of Fin) = 75[mm]

  n (Number of fins) =38[EA]

- Number of iterations : 5
- Number of function count : 1263
- Search method : MADPositvieBasisNp1
- Optimum values

  δ_b (Thickness of a fin) =1.2[mm]

  b (height of Fin) =98.4[mm]

  n (Number of fins) =46[EA]

  mass of the fins = 0.3707[kg] (Min)

  Fin efficiency = 85.66%

# Results (by Excel Solver)

- Initial Condition

  δ_b (Thickness of a fin) =1[mm]

  b (height of Fin) =70[mm]

  n (Number of fins) =75[EA]

- Number of iterations : 25
- Optimum values

  δ_b (Thickness of a fin) =1[mm]

  b (height of Fin) =55.5[mm]

  n (Number of fins) =75[EA]

  mass of the fins = 0.2789[kg]

  Fin efficiency = 91.12%

# Results (by Excel Solver)

- Initial Condition

  δ_b (Thickness of a fin) = 1.9035[mm]

  b (height of Fin) = 75[mm]

  n (Number of fins) =38[EA]

- Number of iterations : 71
- Optimum values

  δ_b (Thickness of a fin) =1[mm]

  b (height of Fin) =55.5[mm]

  n (Number of fins) =75[EA]

  mass of the fins = 0.2789[kg]

  Fin efficiency = 91.12%

# Compare the results

| | Function count | Thickness of a fin[mm] | Length of a fin[mm] | Number of fins[EA] | Mass of the fin[kg] |
|---|---|---|---|---|---|
| **fmincon 1.1** | **20** | **1** | **55.5** | **75** | **0.2789** |
| fmincon 1.2 | 64 | 1.5 | 92.8 | 47 | 0.4494 |
| Pattern Search 1.1 | 919 | 1 | 72.9 | 59 | 0.3022 |
| Pattern Search 1.2 | 1701 | 1 | 90.6 | 50 | 0.3098 |
| Pattern Search 2.1 | 759 | 1 | 72.9 | 59 | 0.3022 |
| Pattern Search 2.2 | 1263 | 1.2 | 98.4 | 46 | 0.3707 |
| **Excel 1.1** | **Unknown** | **1** | **55.5** | **75** | **0.2789** |
| **Excel 1.2** | **Unknown** | **1** | **55.5** | **75** | **0.2789** |

# Compared with the previous design

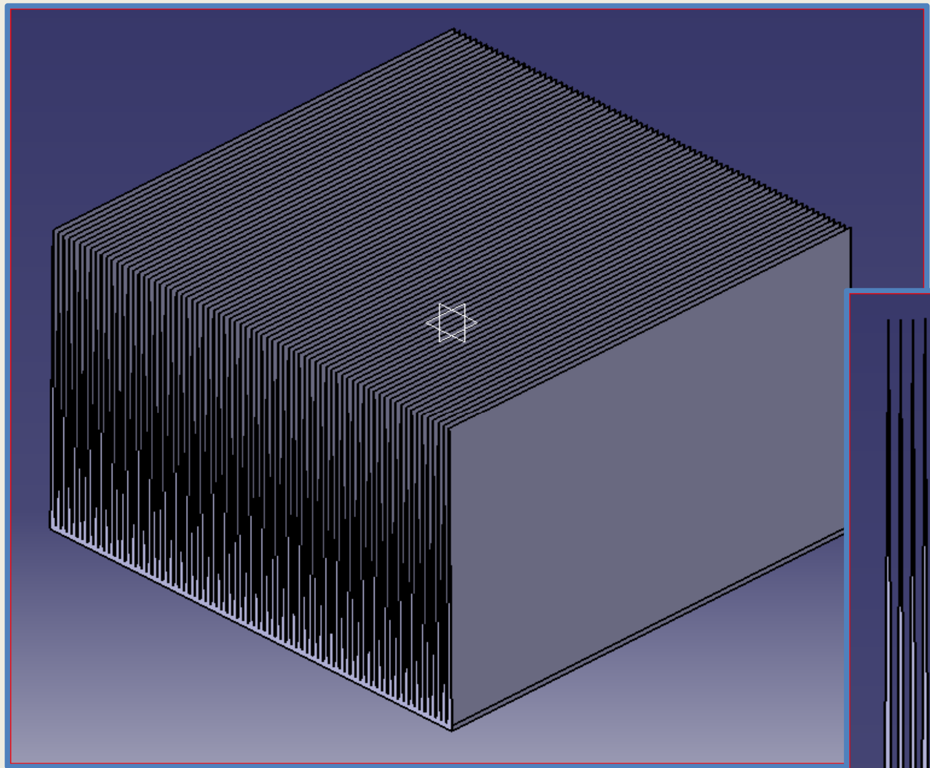
## Result of the design

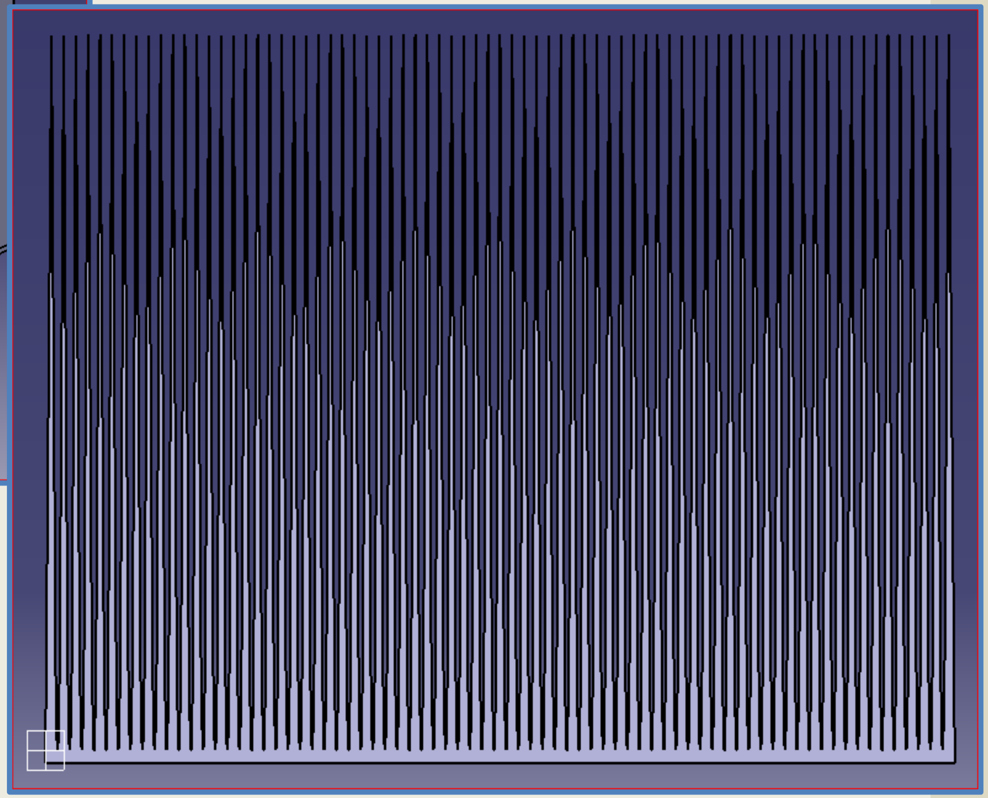
| | Thickness of a fin[mm] | Length of a fin[mm] | Number of fins[EA] | Mass of the fin[kg] | Fin efficiency[%] |
|---|---|---|---|---|---|
| **Final design** | **1** | **55.0** | **75** | **0.2789** | **91.12** |
| Interim | 1.9 | 75.0 | 38 | 0.3664 | 90.14 |
| Previous | 0.56 | 83 | 40 | 0.3783 | 19.58 |

## Improvements

| | Previous Design | Interim Design | Optimum Design | % |
|---|---|---|---|---|
| Mass | 0.3783 kg | 0.3664 kg | 0.2789 kg | -26.28% |
| Fin Efficiecncy | 19.58 % | 90.14 % | 91.12 % | 365.4% |

# Modeling of the heat sink





# Conclusion & discussion

**- Conclusion**

More improvements than our predict.

Each algorithm have a difference character.

Important of to decide the initial point.

**- Discussion**

Processing limit

Radiation effect

# References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Yunus A. Cengel
2004, McGraw-Hill

**Extended Surface Heat Transfer**
Allan D. Kraus, Abdul Aziz, James Welty
2001, Willy Inter-Science