



사각 단면 프로펠러의 형상 결정

OP. Course

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1. Contents

- 중간발표 요약
- 기존 문제의 수정
- 받음각(θ)의 영향
- 초기값의 영향 검토
- 알고리즘별 결과 비교
- 결과 및 고찰

2. Problem statement

- 프로펠러 전, 후의 공기의 운동량 차이만큼 추력(Trust)이 요구된다.
- 추력은 동일한 조건(엔진출력, 고도, 비행 속도 등)에서 프로펠러의 형상에 의해 그 크기가 결정된다.
- 사각 단면을 고려하여 최소의 크기 (경제성)를 고려한 최적의 형상을 결정하는 것에 이 프로젝트 수행의 목적을 둔다.



2. Problem statement

■ Physical Equations.

- Lift force : $\frac{1}{2} C_L \rho_f \int_0^L (\omega r)^2 D \cdot dr \times N = \frac{1}{2} C_L \rho_f \omega^2 D L^3$
- Drag force : $\frac{1}{2} C_D \rho_f \int_0^L (\omega r)^2 D \cdot dr \times N = \frac{1}{2} C_D \rho_f \omega^2 D L^3$
- Thrust : $L.F \times \cos \theta - D.F \times \sin \theta = \frac{1}{2} \rho_f \omega^2 D L^3 (C_L \cos \theta - C_D \sin \theta)$
- Solidity : $\frac{N \times \int_A dA}{\pi L^2} = \frac{N \times D L}{\pi L^2} = \frac{N D}{\pi L}$
- Resistance force : $\rho Q (V_{out} - V_{in}) = 2 \rho Q \left(\frac{Q}{\pi L^2} - V_{in} \right)$
- Total length : $L_{total} = L + \frac{D / 2}{\sin(\pi / N)}$

■ Parameters

Density	1.225 kg/m ³	Speed of Wind	400 km/h
Volume Flow (Q)	500 m ³ /s	Ang. Velocity	2000 rpm

3. Modified problem

- Thrust

$$T = \frac{1}{2} \rho_f \omega^2 D L^3 (C_L \cos \theta - C_D \sin \theta)$$

- Resistance Force

$$F = \rho Q (V_{out} - V_{in}) = 2 \rho Q \left(\frac{Q}{\pi L^2} - V_{in} \right)$$

- Solidity

$$\sigma = \frac{N \times \int_A dA}{\pi L^2} = \frac{ND}{\pi L}$$

Cost Function : Area (L*D)

Design Variable : L, D

θ의 영향은?

- Constraint 1

- Thrust $\geq$ Resistance Force

- Constraint 2

- Solidity ≥ 0.1

- Constraint 3

- $L + D/2 * \sin(\pi/N) \leq 1.5$

Cost Function : Area (L*D)

Design Variable : L, D, θ

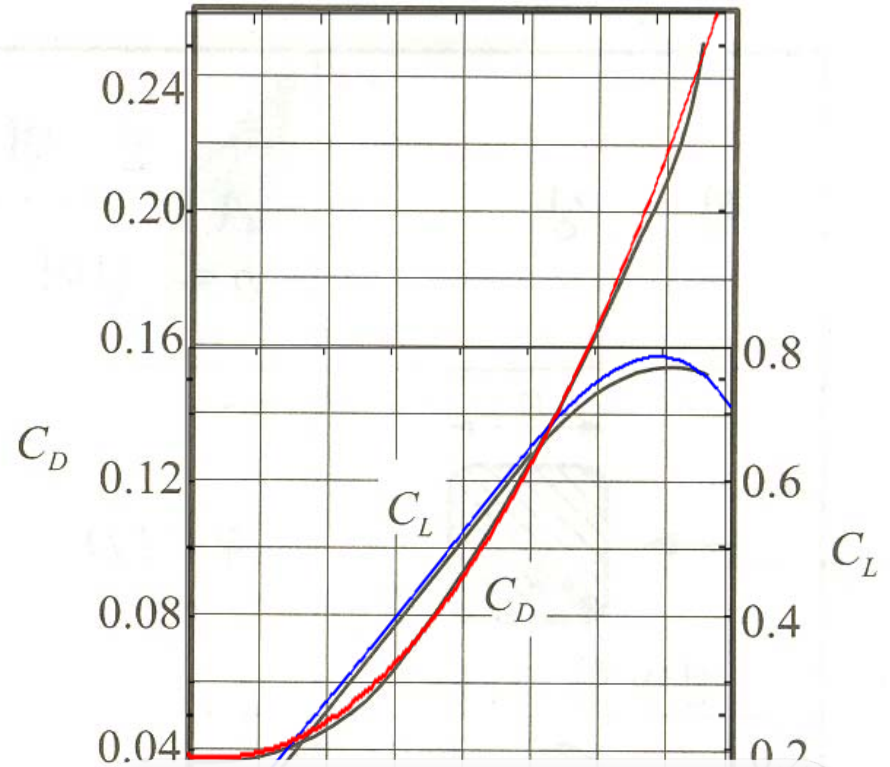
4. Angle of Attack

- C_L , C_D 의 결정

- 추력의 결정

$$T = \frac{1}{2} \rho_f \omega^2 D L^3 (C_L \cos \theta - C_D \sin \theta)$$

θ 변경



- Constraint 4

7k

Lift Coeff.	$-3.79 \cdot 10^{-5} \theta^4 + 8.96 \cdot 10^{-4} \theta^3 - 0.0071 \theta^2 + 0.0841 \theta - 0.0035$
Drag Coeff.	$0.001 \theta^2 - 0.0017 \theta + 0.0375$

5. Excel solver

기존의 결과

L (m)	D (m)	θ (deg)	Area (m ²)
1.180082	0.123578	12.841471	0.437496

변수 θ 추가

L (m)	D (m)	θ (deg)	Area (m ²)
1.180012	0.123571	13.364088	0.437444

예상과 달리 적은 변화
초기값의 영향은?

6. Change Initial Point

fmincon

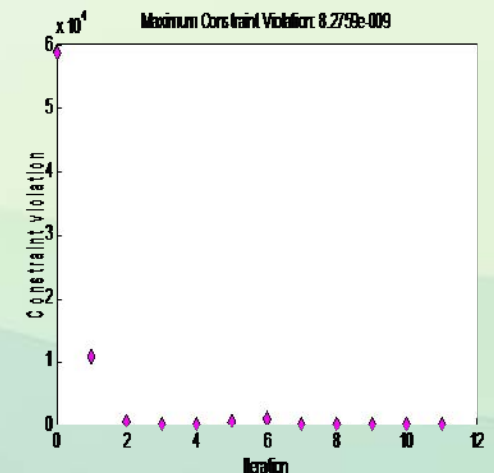
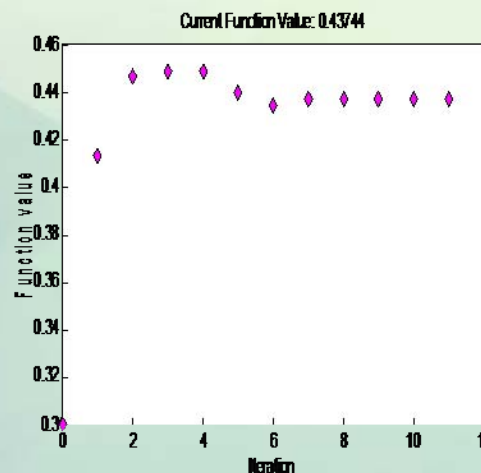
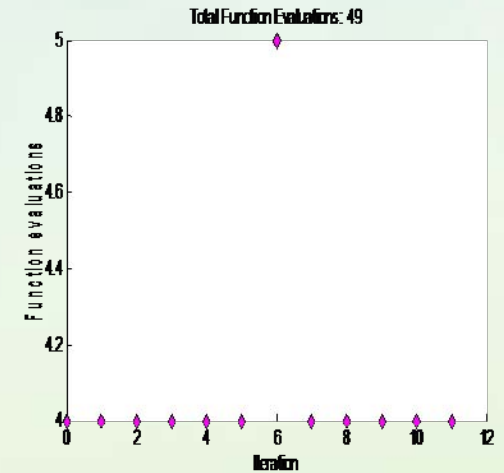
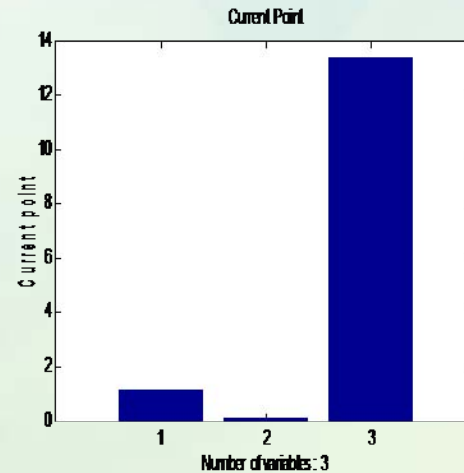
Infeasible (L, D, θ) = (1, 0.1, 1)

x_optimum
L=1.1800
D=0.1236
 θ =13.3641

F_value
0.4374

Exitflag
1

Lagrange Multiplier
2.96e-06 (Thrust)
4.26 (Solidity)
0
0
0
0
0



6. Change Initial Point

fmincon

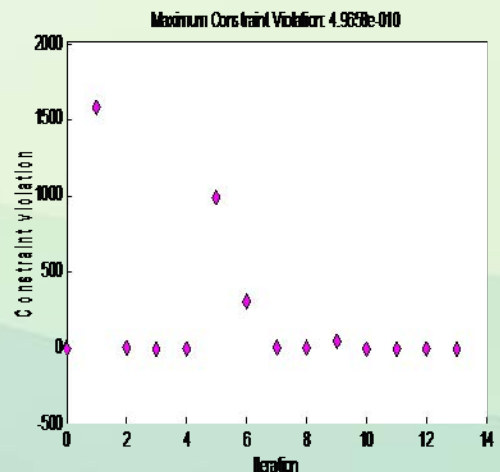
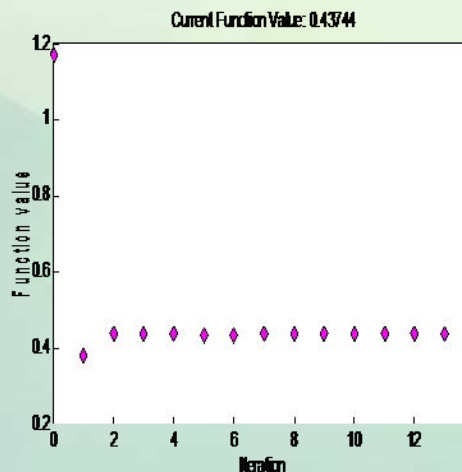
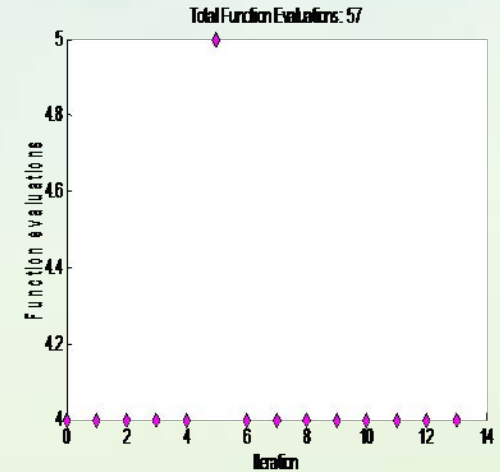
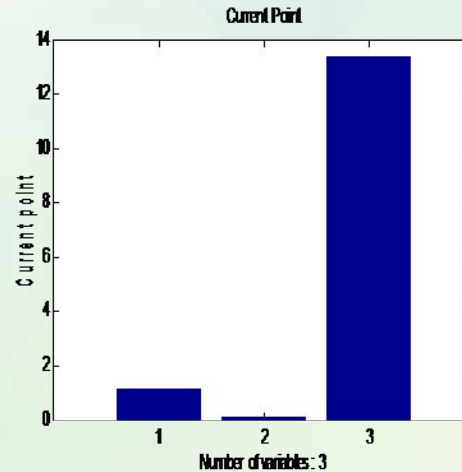
Feasible (L, D, θ) = (1.3, 0.3, 9)

x_optimum
L=1.1800
D=0.1236
 θ =13.3641

F_value
0.4374

Exitflag
1

Lagrange Multiplier
2.96e-06 (Thrust)
4.26 (Solidity)
0
0
0
0
0



6. Change Initial Point

Excel Solver	
Infeasible (L, D, θ) = (1, 0.1, 1)	
L (m)	1.18001
D (m)	0.12357
θ (deg)	13.36409
Area (m ²)	0.43744
Feasible (L, D, θ) = (1.3, 0.3, 9)	
L (m)	1.18001
D (m)	0.12357
θ (deg)	13.36400
Area (m ²)	0.43744

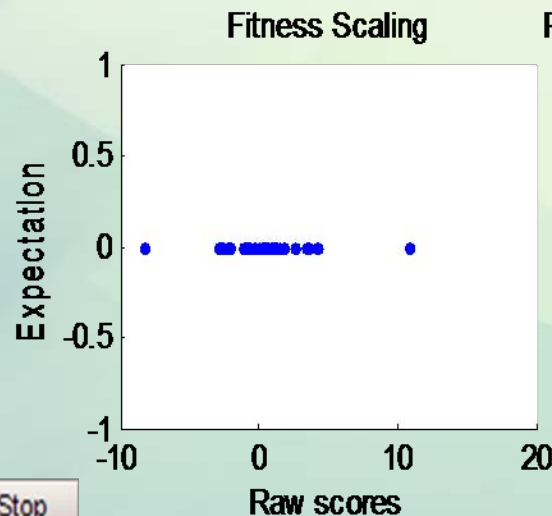
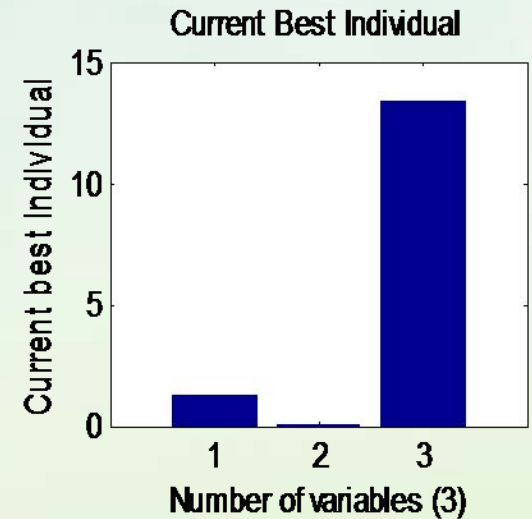
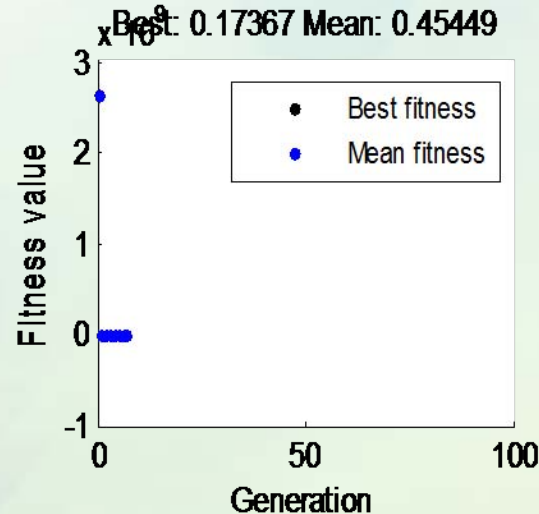
MATLAB fmincon	
Infeasible (L, D, θ) = (1, 0.1, 1)	
L (m)	1.18001
D (m)	0.12357
θ (deg)	13.36407
Area (m ²)	0.43744
Feasible (L, D, θ) = (1.3, 0.3, 9)	
L (m)	1.18001
D (m)	0.12357
θ (deg)	13.36407
Area (m ²)	0.43744

변수 θ 의 변화에 따른 목적함수의 영향은 무시할 수 있을 정도

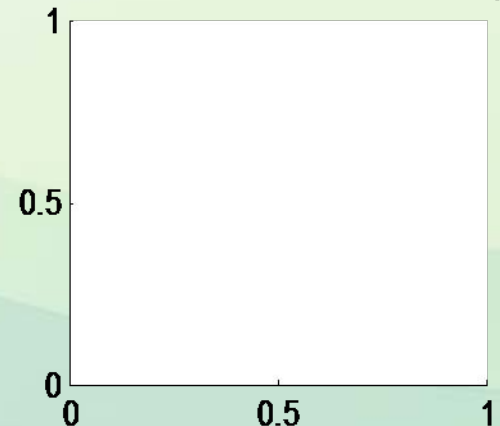
7. Comparing Algorithm

Genetic Algorithm

x_optimum	L=1.1852 D=0.1255 $\theta=13.3644$
F_value	0.4461
Exitflag	4



Plot not available for nonlinear constrained problems



Stop

7. Comparing Algorithm

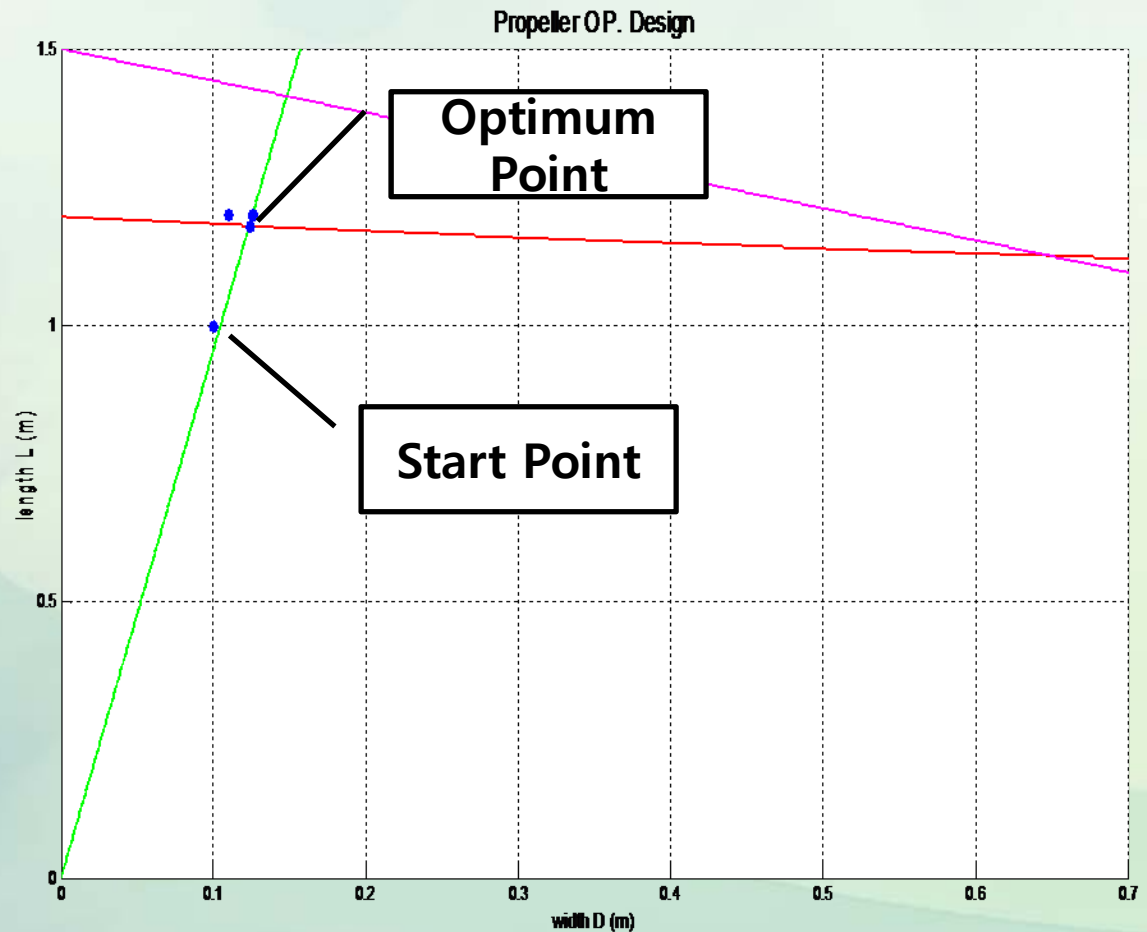
Excel Solver

Infeasible (L, D) = (1, 0.1)

x_optimum
L=1.1800
D=0.1236
 $\theta=13.3641$

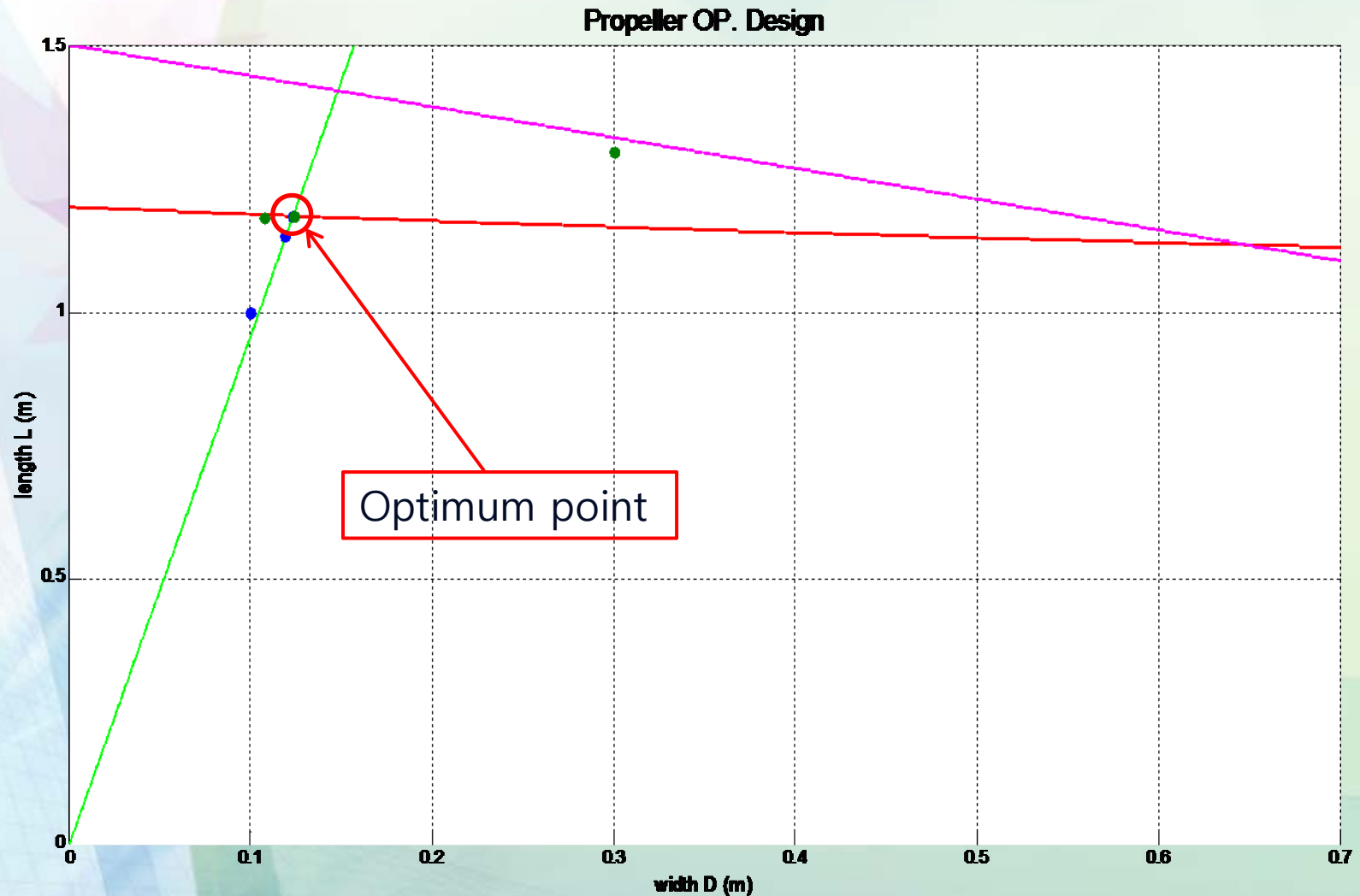
F_value
0.4374

Lagrange
Multiplier
2.96E-06 (Thrust)
4.26 (Solidity)
0
0
0
0
0

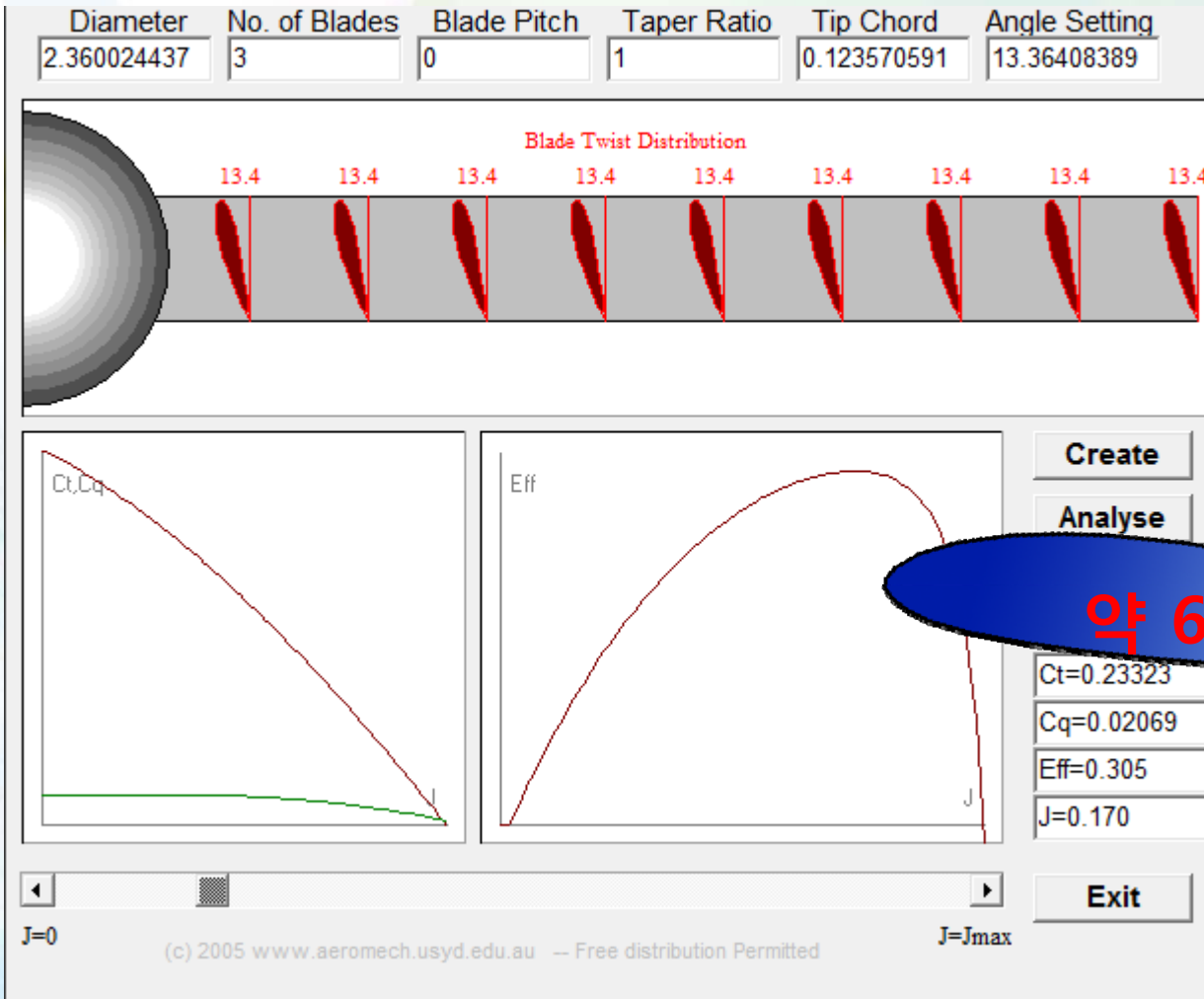


7. Comparing Algorithm

fmincon with 2 variables



8. Conclusion



Rectangle

Trust (N)	3906.68
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Airfoil

Trust (N)	6441.27
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약 64% 우수한 결과

9. Reference

- **Wind turbine engineering design** / Eggleston, David M / Van Nostrand Reinhold / 1987
- **Glauert Blade Element Theory** / www.aerodynamics4students.com
- **Propeller Design & Analysis Program** / www.aerodynamics4students.com
- **An Assessment of Propeller Aircraft Noise Reduction Technology** / NASA Contractor Report 198237
- **유체역학** / 맹주성/ 竝進 / 2001



Question??