2009 Optimum Design
- Design of Flywheel -

[Olleh_e+]

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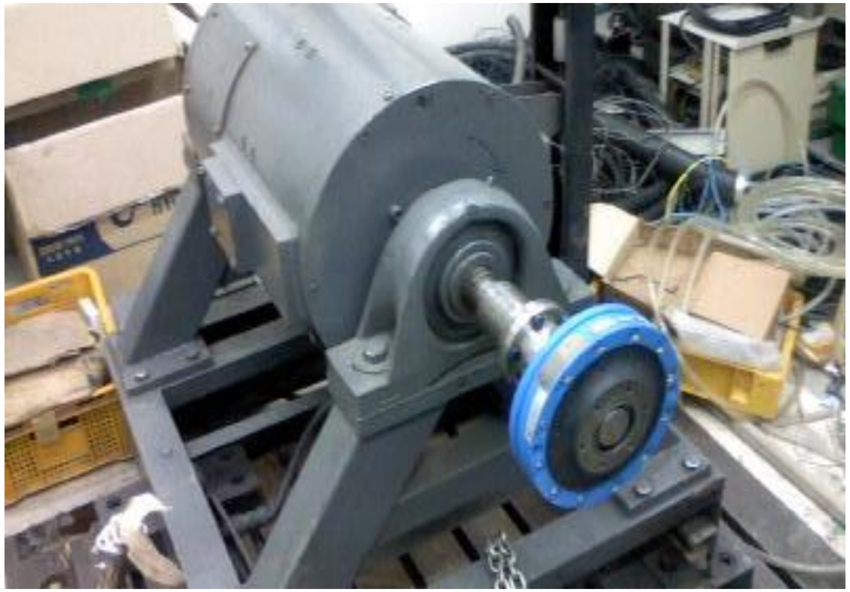
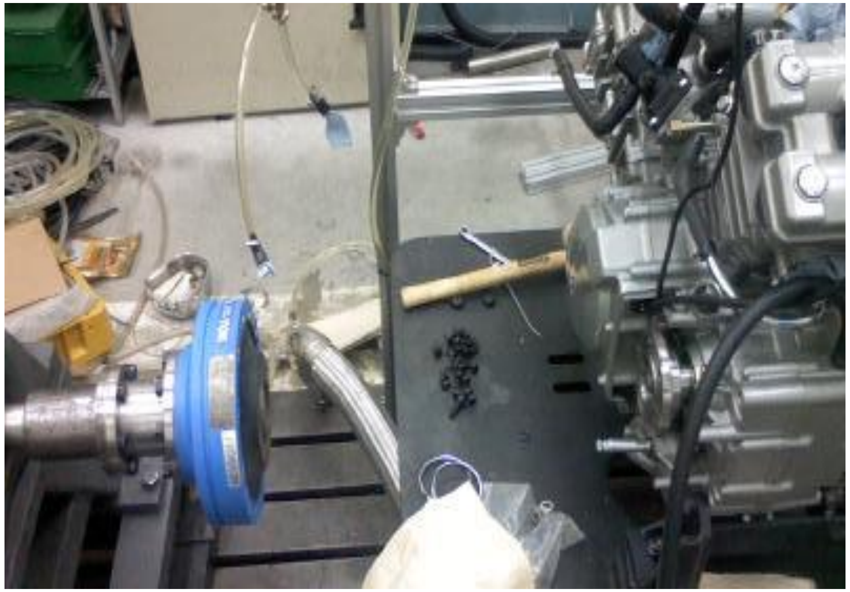
1. Introduction

◆ 플라이휠의 사용 목적

- 동력 전달
- 회전 진동 감소
- 동적 충격 완화





2. Problem Formulation Process

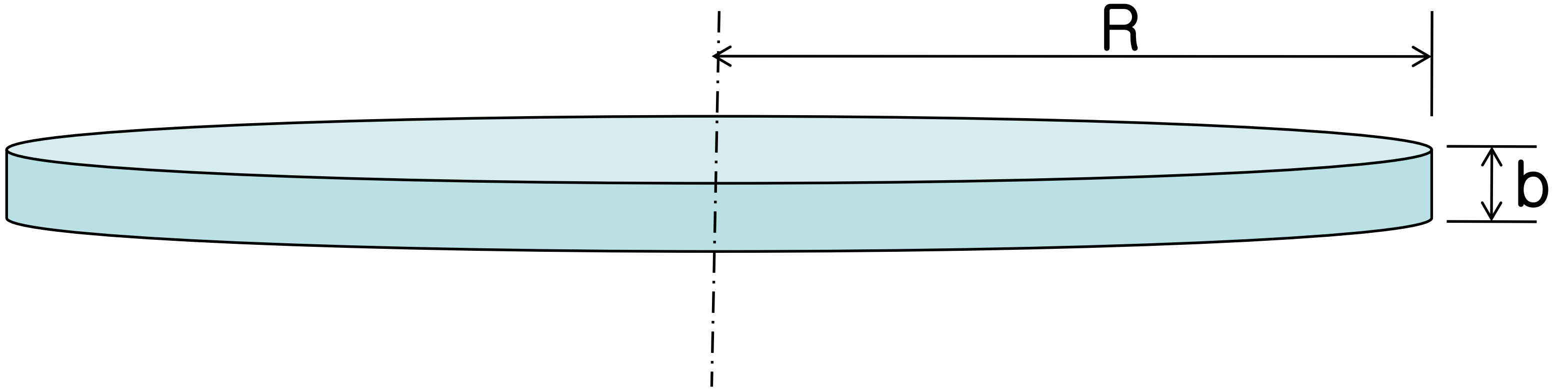
I. Project statement

a. Assumptions

- Disk 형태의 solid 원판
- Bolt, flange 무시
- 회전으로 인한 응력, damping 효과 무시

b. Objective

- 같은 질량관성모멘트 (J) 로 최소의 무게를 도출



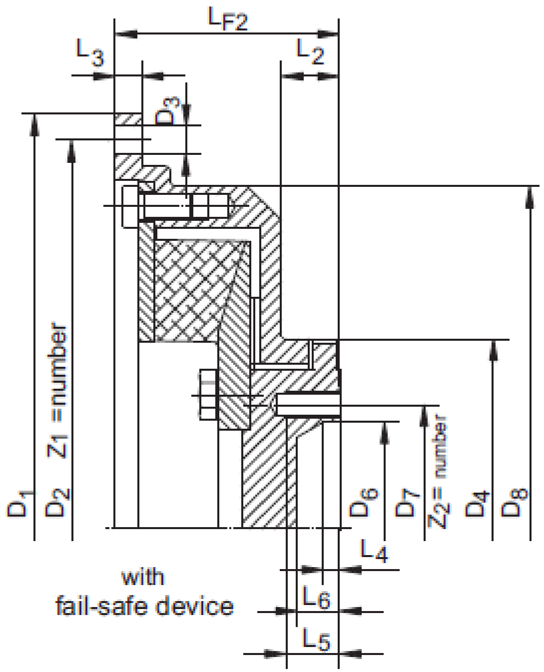
2. Problem Formulation Process

I. Project sta

c. Requirer

- Rad / wi

- $J \geq$



AC-VSK coupling size	Flywheel connection dimensions					D_4	D_8	L_2	L_3	L_{F1}	L_{F2}
	SAE size	D_1	D_2	D_3	Z_1						
-15.[1].F2	8	263.5	244.5	10.5	6	140	215	18	8	57	68
	10	314.3	295.3	10.5	8						
-25.[1].F2	10	314.3	295.3	10.5	8	144	260	22	10	74	85
	11.5	352.4	333.4	10.5							
-35.[1].F2	10	314.3	295.3	10.5	8	180	279	28	16	78	90
	11.5	352.4	333.4	10.5					8		
-45.[1].F2	11.5	352.4	333.4	10.5	8	180	314	25	26	89	100
	14	466.7	438.2	13.0					10		
-50.[1].F2	14	466.7	438.2	13.0	8	210	352	36	12	103	120
-55.[1].F2	14	466.7	438.2	13.0	8	285	417	35	28	115	130
										122	147
										134	155

AC-VSK coupling size	Flywheel connection dimensions					D_4	D_8	L_3	L_F	MECHANICS cardan shaft connecting dimensions							J_1 [kgm²]	J_2 [kgm²]	Total weight [kg]
	SAE size	D_1	D_2	D_3	Z_1					Size	D_6 H_7	L_4	L_5	L_6	L_7	M			
-15.[1].F2	8	263.5	244.5	10.5	6	140	215	8	83	4C	107.92	3.8	36.5	9.5	87.3	5/16"-24	0.055	0.010	8.8
	10	314.3	295.3	10.5	8					5C	115.06	5.1	42.9	14.26	88.9	3/8"- 24	0.084		10.2

2. Problem Formulation Process

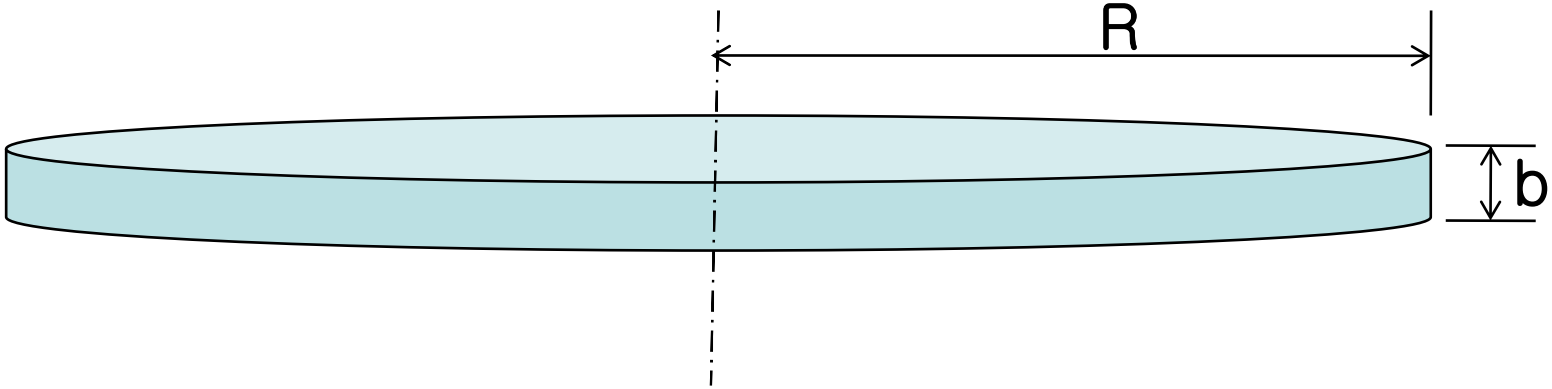
II. Data and Information Collection

- 기존 플라이휠 spec.
  - ✓ Mass: 8.8 kg

2. Problem Formulation Process

III. Design Variables

- R [radius]: Disk 의 반지름 [m]
- b [width]: Disk 의 폭 [m]



IV. Cost function

$$f_{\text{mass}} = \rho \cdot \pi R^2 \cdot b$$

2. Problem Formulation Process

V. Constraints

$$g_1 : -b \le 0$$

$$g_2 : -R \le 0$$

$$g_3 : R - 4b \le 0$$

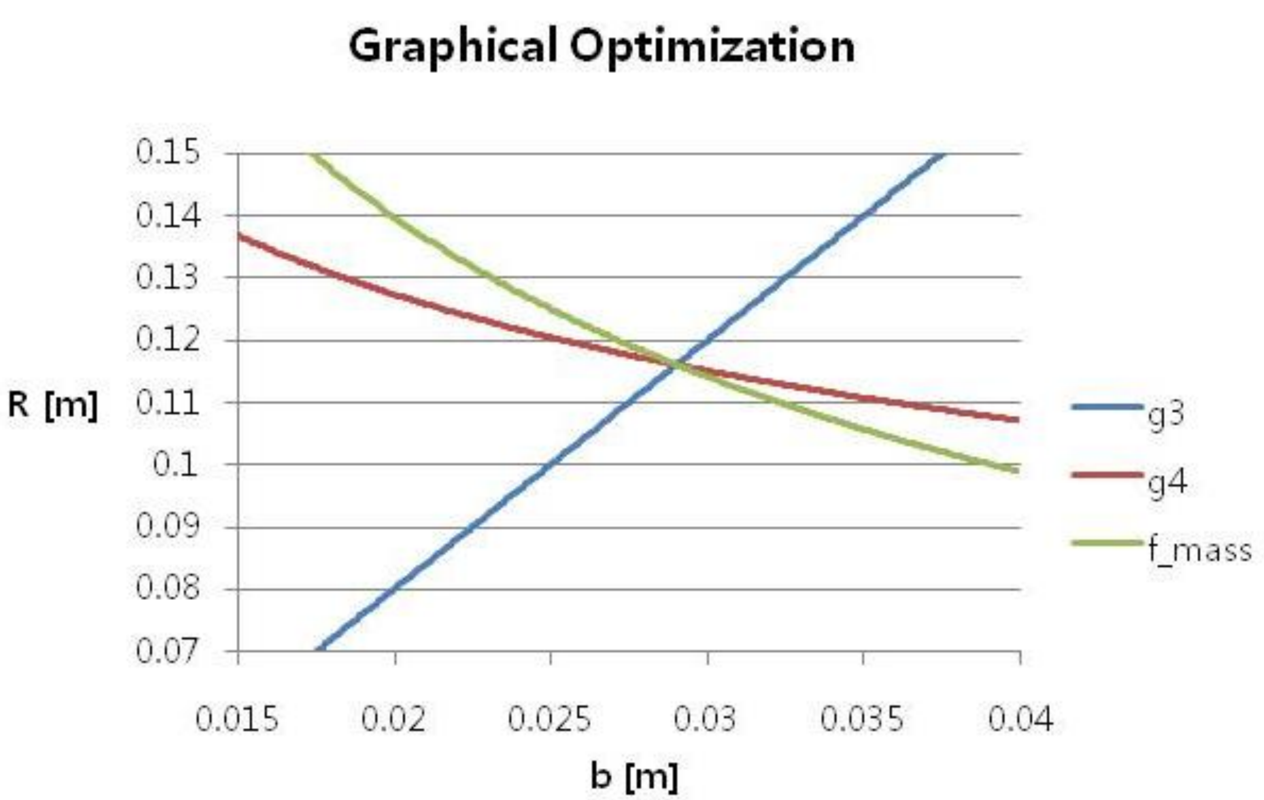
$$g_4 : J - \frac{1}{2}\rho\pi R^2 \cdot bR^2 \le 0 \qquad \left(J = 0.065\ \mathrm{kg} \cdot \mathrm{m}^2\right)$$

2. Problem Formulation Process

VI. Graphical Optimization

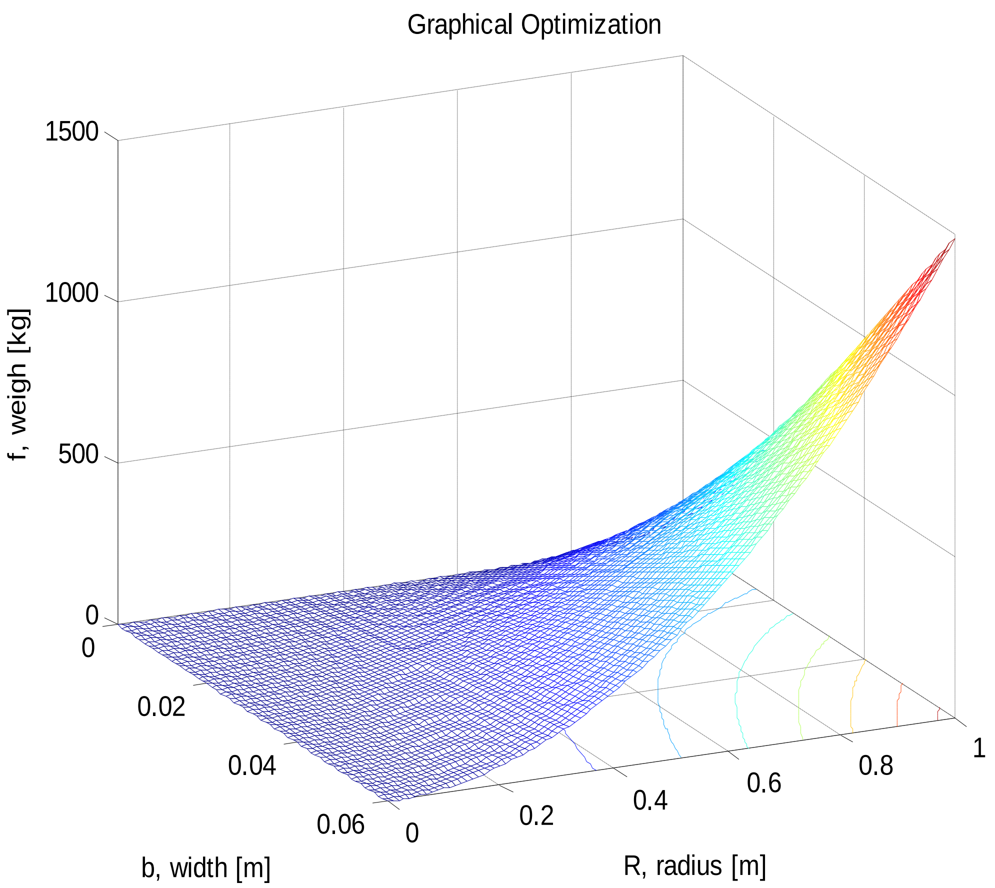
	A	B	C	D	E
1					
2					
3					
4	ρ	7870	kg/m^3		
5	J	0.065	kg·m^2		
6					96
7	b	g3	g4	f_mass	9.6
291	0.0284	0.1136	0.116647	0.117292	
292	0.0285	0.114	0.116545	0.117086	
293	0.0286	0.1144	0.116443	0.116881	
294	0.0287	0.1148	0.116341	0.116677	
295	0.0288	0.1152	0.11624	0.116474	
296	0.0289	0.1156	0.11614	0.116273	
297	**0.029**	**0.116**	**0.11604**	**0.11607**	
298	0.0291	0.1164	0.11594	0.115872	
299	0.0292	0.1168	0.11584	0.115674	

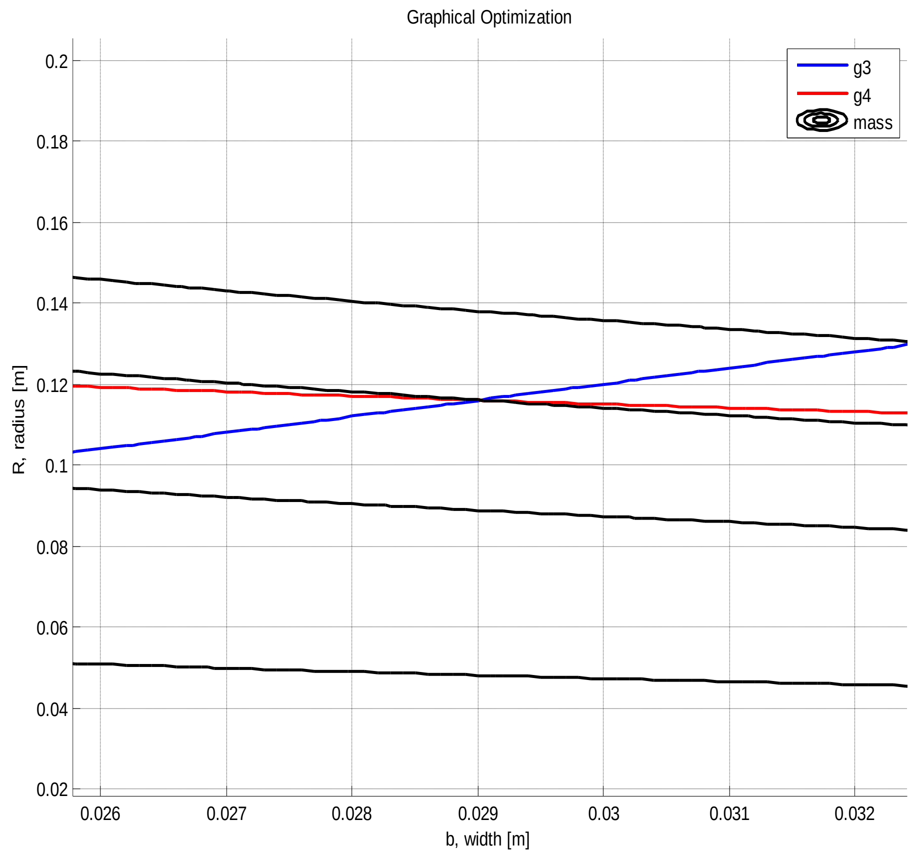
Project



2. Problem Formulation Process

VI. Graphical Optimization





3. Check for Convexity

- $g_1 : -b \le 0$

  $g_2 : -R \le 0$

  $g_3 : R - 4b \le 0$

  $g_4 : J - \frac{1}{2}\rho\pi R^2 \cdot bR^2 \le 0 \qquad \left(J = 0.065\ \mathrm{kg}\cdot\mathrm{m}^2\right)$

- g_1, g_2, g_3 **: linear inequality $\rightarrow$ convex**

- $g_4 : 5.26\times10^{-6} - bR^4 \le 0 \quad \rightarrow \quad \frac{5.26\times10^{-6}}{bR^4} - 1 \le 0$

$$\nabla g_4 = -5.26\times10^{-6}\begin{bmatrix} \frac{1}{b^2R^4} \\ \frac{4}{bR^5} \end{bmatrix}$$

$$\nabla^2 g_4 = 5.26\times10^{-6}\begin{bmatrix} \frac{2}{b^3R^4} & \frac{4}{b^2R^5} \\ \frac{4}{b^2R^5} & \frac{20}{bR^6} \end{bmatrix} = \frac{5.26\times10^{-6}}{b^3R^6}\begin{bmatrix} 2R^2 & 4bR \\ 4bR & 20b^2 \end{bmatrix}$$

$M_1 = 2R^2 > 0$

$M_2 = 40R^2b^2 - 16R^2b^2 > 0$

$\therefore$ **Positive definite**

$\rightarrow$ **Convex**

3. Check for Convexity

- $$f_{\text{mass}} = \rho \cdot \pi R^2 \cdot b$$

$$\nabla f = \begin{bmatrix} R^2 \\ 2bR \end{bmatrix}, \quad \nabla^2 f = \begin{bmatrix} 0 & 2R \\ 2R & 2b \end{bmatrix}$$

$$-4R^2 < 0$$

$\therefore$ **indefinite → Non-convex**

4. KKT necessary conditions

- **Lagrange function**

$$L = bR^2 + u_1\left(-b + s_1^2\right) + u_2\left(-R + s_2^2\right) + u_3\left(R - 4b + s_3^2\right) + u_4\left(5.26\times10^{-6} - bR^4 + s_4^2\right)$$

$$\frac{\partial L}{\partial b} = R^2 - u_1 - 4u_3 - R^4 u_4 = 0$$

$$\frac{\partial L}{\partial R} = 2Rb - u_2 + u_3 - 4bR^3 u_4 = 0$$

$$u_i s_i = 0,\ \ g_i + s_i^2 = 0,\ \ s_i^2 \ge 0,\ \ u_i \ge 0 \qquad \left(i = 1 \text{ to } 4\right)$$

✓ **16 switching conditions -> 4 switching conditions**

4. KKT necessary conditions

- **Case 1** $u_1 = u_2 = 0, \quad u_3 = 0, \; u_4 = 0$

  $R = 0$

  ✓ **not acceptable**

- **Case 2** $u_1 = u_2 = 0, \quad s_3 = 0, \; u_4 = 0$

  $$\frac{\partial L}{\partial R} = 8b^2 + u_3 = 0$$

  $$u_3 = -8b^2 < 0$$

  ✓ **not acceptable**

- **Case 3** $u_1 = u_2 = 0, \quad u_3 = 0, \; s_4 = 0$

  $$\frac{\partial L}{\partial R} = \frac{2 \times 5.26 \times 10^{-6}}{R^3} - \frac{4 \times 5.26 \times 10^{-6}}{R} u_4 = 0$$

  $$\therefore u_4 = \frac{1}{2R^2} \qquad \therefore \frac{\partial L}{\partial b} = R^2 - \frac{1}{2} R^2 = \frac{1}{2} R^2 \neq 0$$

  ✓ **not acceptable**

4. KKT necessary conditions

- **Case 4** $u_1 = u_2 = 0, \quad s_3 = 0, \; s_4 = 0$

$$5.26 \times 10^{-6} - 256 b^5 = 0$$

$$\therefore b = 0.02901, \; R = 0.11604$$

✓ **KKT 식의 조건을 만족 여부 확인**

$$s_1^2 > 0, \; s_2^2 > 0$$

$$\therefore u_3 = 0.001347, \qquad u_4 = 44.559$$

$$\therefore u_3 \,\&\, u_4 > 0$$

5. Sensitivity Analysis

- $\delta f^* = -u_3 e_3 - u_4 e_4$

$$u_3 = 0.001347$$

$$u_4 = 44.559$$

$$\delta f^* = -u_3 \cdot 0 - 44.559 \cdot \left(-1.5976 \times 10^{-7}\right) = 7.1187 \times 10^{-6}$$

$$\delta W = \rho \pi \delta f^* = 7870\pi \cdot \left(7.1187 \times 10^{-6}\right) = 0.176 \text{ kg}$$

- **cost function → g_3 보다 g_4 에 훨씬 더 민감**
- **J[kg·m²]: 3% 증가 → m[kg]: 1.76% 증가**

6. Conclusion & Comment

- 도해적으로 얻은 해와 Lagrange function을 사용한 해 거의 일치

	R [mm]	b [mm]	Mass [kg]
Origin Value	131.75	57	8.8
Optimum Value	116.04	29.01	9.66