

최적설계 프로젝트 최종보고
'09. 12. 9 (화)



2009 Optimum Design

Design of Main Wing for weight

2003007839

김 윤 성

2005007051

김 창 섭

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1 Introduction to Last design

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3 Analysis and Solution

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Intro. To Last Design

- 중간발표 참고 모델

수송기 C-295의 축소 모델 (Scale 1:15)



Intro. To Last Design

- 설계변수 선정
 - Thickness of Spar : t_s
 - Number of Ribs : N_r
- 목적함수 : 무게 최소화

$$f = \rho V = \rho(A_r t_r N_r + 2L w_s t_s)$$

Area of rib : A_r

Thickness of Rib : t_r

Wing span : L

Height of spar : w_s

Density of Material : ρ

Intro. To Last Design

- 구속조건

Rib개수 구속조건

$$\sigma = E\varepsilon \leq \sigma_Y$$

$$\varepsilon = \sqrt{1 + \frac{2ab}{a^2 + b^2} \sin \theta} - 1$$

$$E = 250 \text{ MPa}$$

$$\sigma_Y = 20 \text{ MPa}$$

$$\phi_{wing} \leq 4 \text{ deg}$$

Spar두께 구속조건

$$\begin{cases} \sigma \leq \sigma_{max} \\ \tau \leq \tau_{max} \end{cases}$$

$$\tau_{max} = \frac{3V}{2A} = \frac{3V_{max}}{2 \times 2ht_s}$$

$$\sigma_{max} = \frac{My}{I} = \frac{M_{max} \frac{h}{2}}{2 \times \frac{1}{12} t_s h^3}$$

Intro. To Last Design

- 최적해 도출 결과

$$\begin{array}{ll} \text{Spar의 두께} & t_s = 1.1024cm \\ \text{Rib의 개수} & N_r = 11 \end{array}$$

- 개선요소

- 구조를 구성하는 재료의 강성은 높는데 반면 매우 가벼움
- Fluent를 통하여 구한 압력 데이터의 신뢰성 검증 필요
- 모델 기체의 성능 및 제원이 알려져 있지 않음



실제 기체로의 모델 전환 필요

Problem Re-Formulation

STEP 1

Problem Statement

- Rib와 Spar의 개수가 많을 수록 강성이 증가하지만 무게가 무거워지므로 높은 양력이 필요

**Rib와 Spar에 따른 날개 구조의 하중과
Chord Length와 Aspect Ratio에 의해
결정되는 날개 크기에 의한 하중을 최소화
하도록 설계방향 설정**

- 필요한 양력을 만족시킬 수 있는 날개의 형상에 대한 최소한의 Chord Length와 Aspect Ratio를 결정

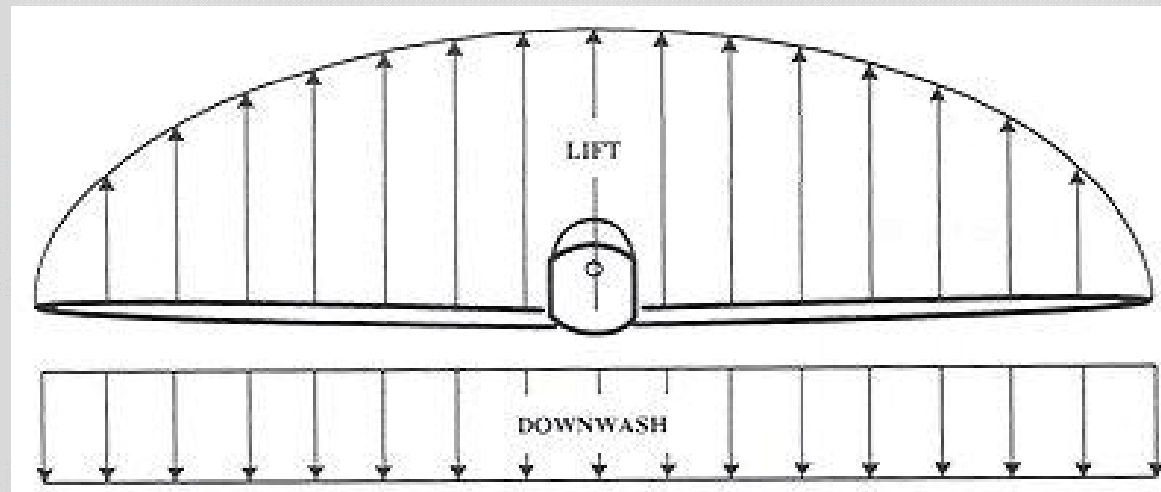
Problem Re-Formulation

STEP 2 / Data and Information Collection

- 주의 설계 파라미터

- Aspect Ratio : 날개의 길이(b : wing span)와 시위(c : chord)의 비
- Taper Ratio : 날개 끝 선의 길이와 시위의 비

Taper Ratio & 양력분포



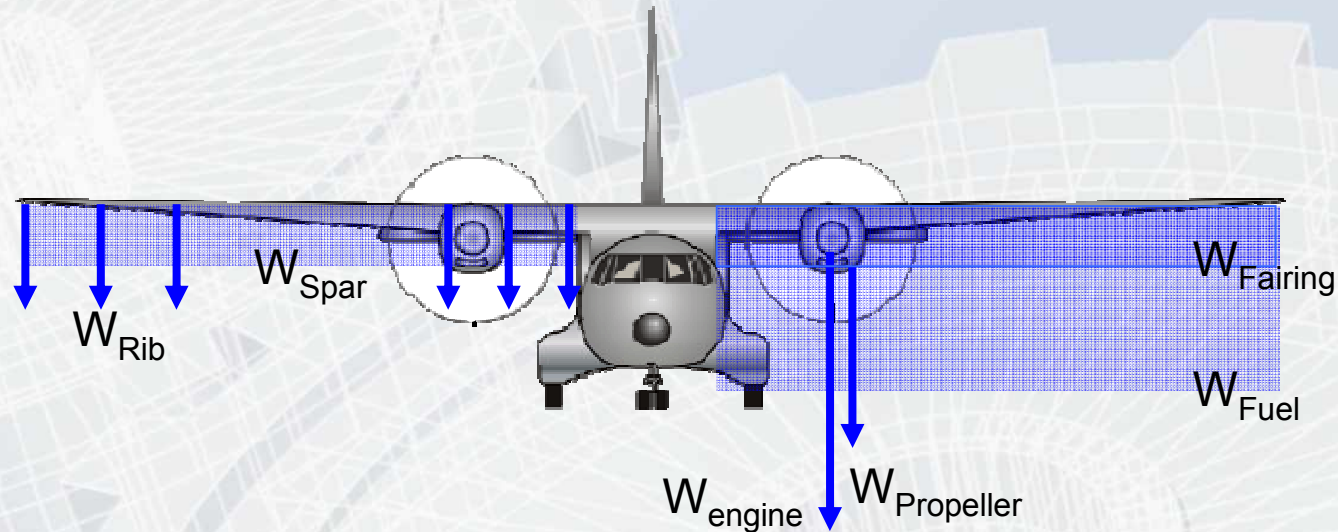
$\lambda \approx 0.25$ 출처 : Aircraft Design : A Conceptual Approach (AIAA Education series)

Problem Re-Formulation

STEP 2

Data and Information Collection

- 기체의 주익에 작용하는 힘 분석(계류중)



Property	Rating	Unit
W_{engine}	521.64	Kg
$W_{propeller}$	163.7	Kg
$W_{rib} , W_{spar} , W_{fuel} , W_{fairing}$	Depend on Design Variables	

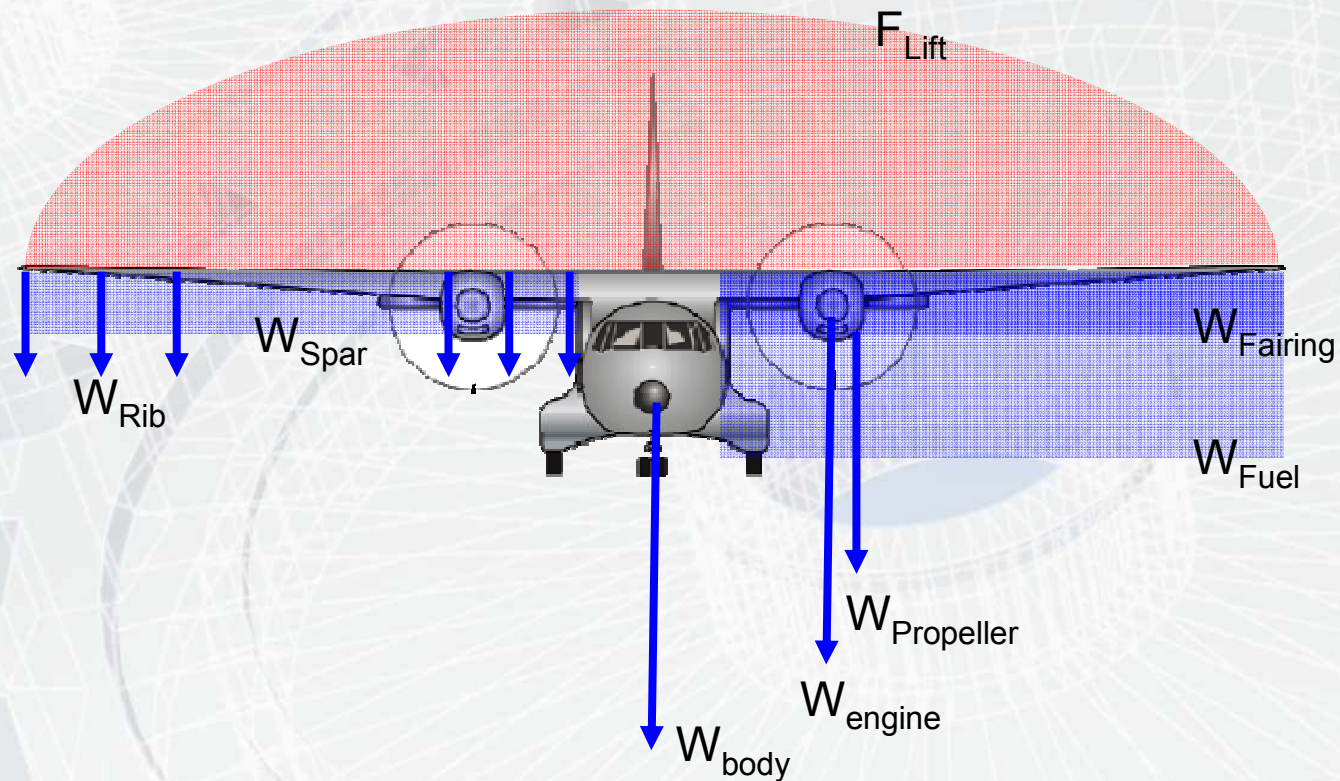
출처 : Pratt&Whitney corp.(Engine)
Hamilton Standard corp.(Propeller)

Problem Re-Formulation

STEP 2

Data and Information Collection

- 기체의 주익에 작용하는 힘 분석(운항중)

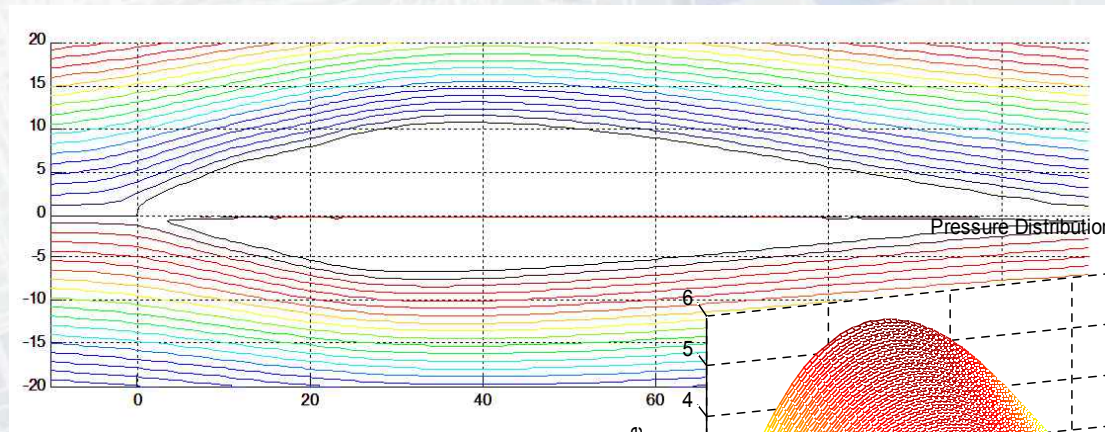


Problem Re-Formulation

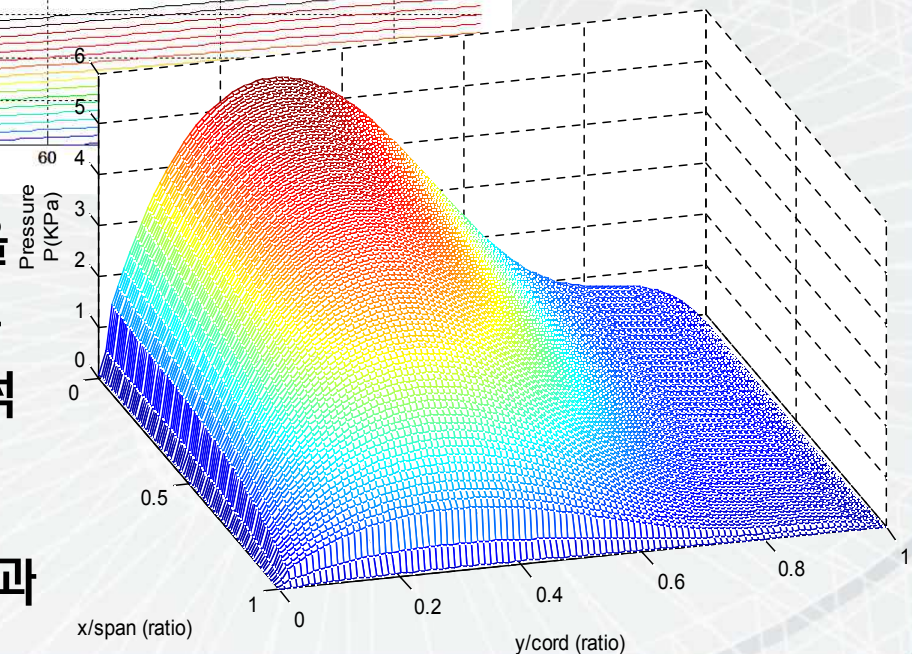
STEP 2

Data and Information Collection

- 날개에 작용하는 압력분포 계산
 - 익형은 NACA 66(2)-415로 가정



- Potential Flow 해석 방법을 통해 날개 형상을 구현 후 날개 주위의 유동장을 해석
- 기체의 순항 속도는 $V=100\text{m/s}$ 로 설정한 결과



Problem Re-Formulation

STEP 2 / Data and Information Collection

- 설계 재료 선정
 - 구조 재료 : 2024-T3(AlCuMg2)

Property	Rating	Unit
Young's Modulus	70000	MPa
Shear Modulus	27500	MPa
Tensile Strength	215	MPa
Elongation	13	%
Yield Strength	140	MPa
Density	2770	kg/m ³

출처 : www.matbase.com

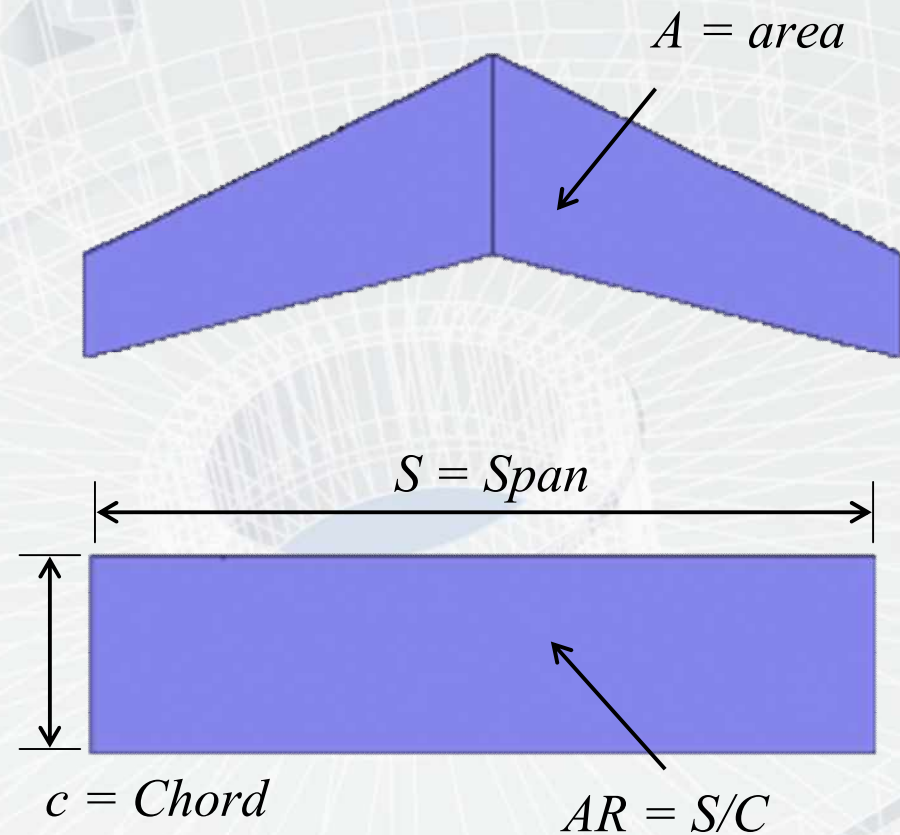
- 연료 : Kerosene계열(Jet A-1)

Property	Rating	Unit
Density	807.5	kg/m ³

Problem Re-Formulation

STEP 3 / Design Variables

- 최적화의 목적에 의해서 변수 결정
- Design Variables
 - Aspect Ratio : AR
 - Chord Length : c
 - Spar Thickness : t_s
 - Num. of Ribs : N_r



Problem Re-Formulation

STEP 4 / Cost function

- 주익의 무게 최소화

$$\begin{aligned} f &= f(r_{AR}, c, t_s, N_r) \\ &= W_{rib} + W_{fairing} + W_{spar} + W_{fuel} \\ &= \rho_{al}(V_{rib} + V_{fairing} + V_{spar}) + \rho_{fuel}V_{fuel} \end{aligned}$$

$$\text{Where, } V_{rib} = A_r - \sum_{i=1}^{N_r} \frac{i}{N_r} t_r A_r (1 - \lambda) = t_r A_r (N_r \lambda - (N_r + 1)(1 - \lambda) / 2)$$

$$V_{spar} = 0.1 \times L_c^2 r_{AR} t_s$$

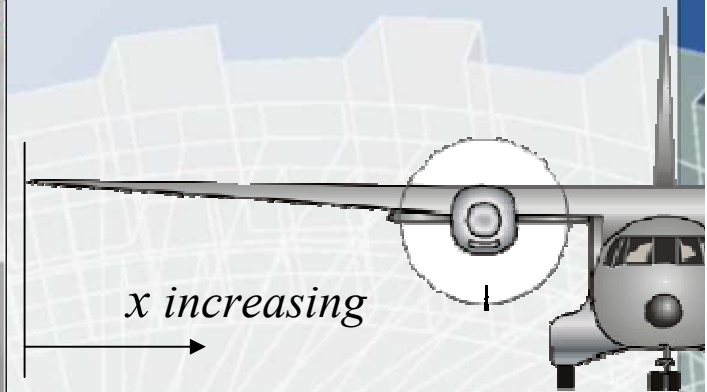
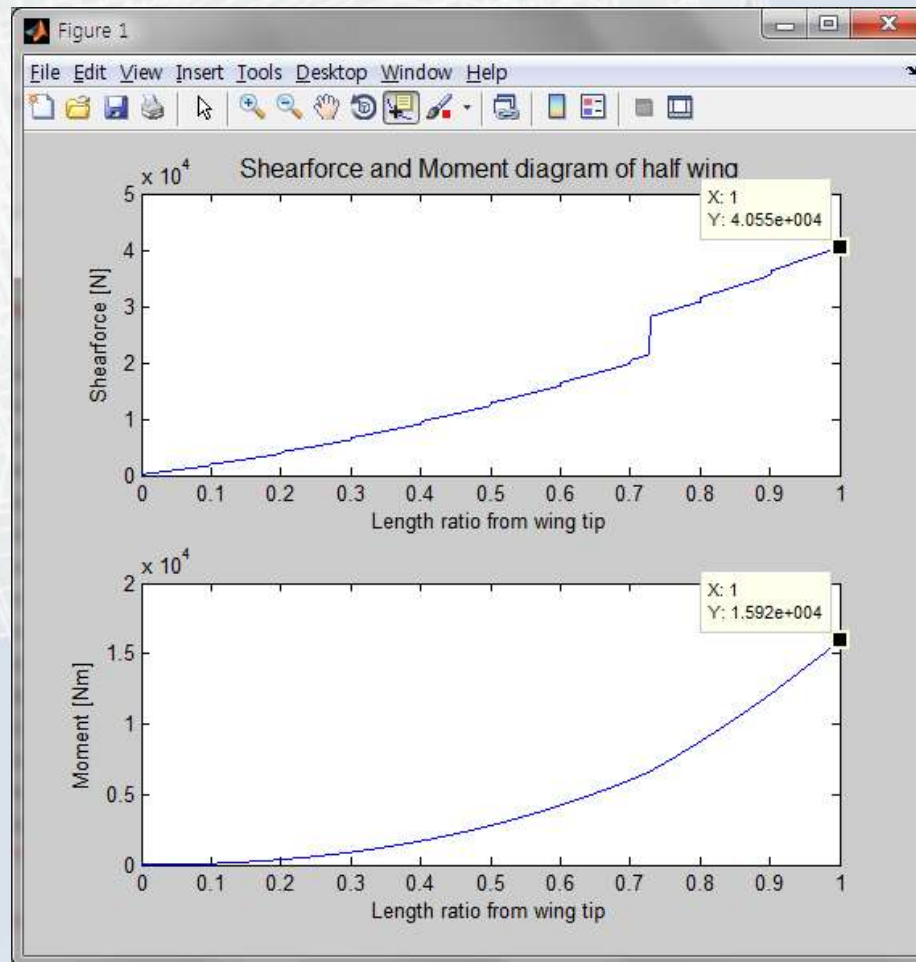
$$V_{fairing} = t_f L_c r_{AR} p_{foil} (1 + \lambda) / 4$$

$$V_{fuel} = A_{foil} L_c r_{AR} - V_{rib} - V_{spar} - V_{fairing}$$

Problem Re-Formulation

STEP 5 / Constraints

- 주익의 길이방향 전단력 및 모멘트에 대한 제한조건



$$\begin{cases} \sigma \leq \sigma_{max} \\ \tau \leq \tau_{max} \end{cases}$$

Problem Re-Formulation

STEP 5

Constraints

$$V_{max} = \sum_{i=0}^{N_r-1} 0.10106 L_{c,i} t_r \rho g + 0.05 \rho g L_c^2 t_s r_{AR} + 0.05053 \rho_{fuel} g L_c^2 r_{AR} \\ + 1.0232 \rho g L_c^2 r_{AR} + g(m_{engine} + m_{prop})$$

$$\text{Where, } \sum_{i=0}^{N_r-1} L_{c,i} = L_c (\lambda N_r + (1 - \lambda)(N_r - 1) / 2)$$

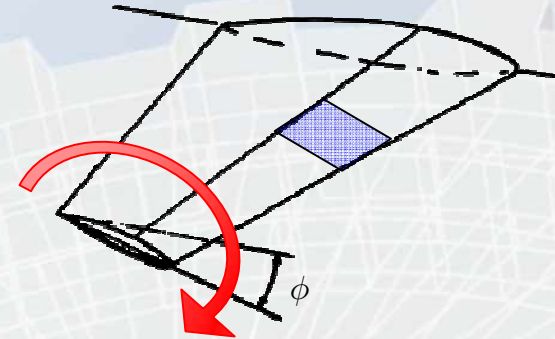
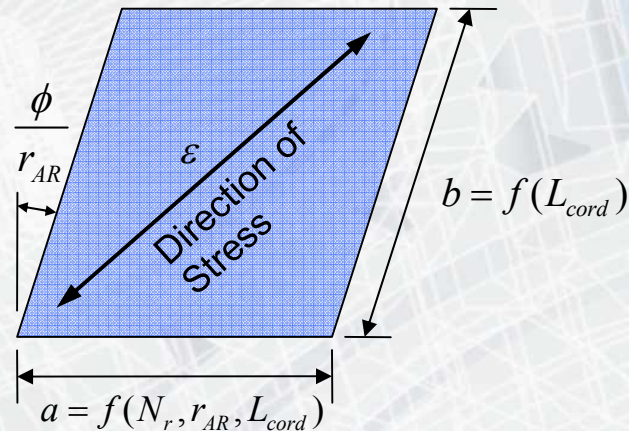
$$M_{max} = 0.10106 t_r \rho L_c g \left[\frac{(N_r - 1)(2N_r - 1)}{6N_r} + \frac{(1 - 2\lambda)(N_r - 1)}{2} + \lambda N_r \right] \\ + 0.025 \rho L_c^2 r_{AR} t_s g + 0.05053 \rho_{fuel} g L_c^2 r_{AR} \left(\frac{\lambda}{2} + \frac{1 - \lambda}{3} \right) \\ + 1.0232 \rho g t_f L_c^2 r_{AR} \left(\frac{\lambda}{2} + \frac{1 - \lambda}{3} \right) + 0.27132 g(m_{engine} + m_{prop})$$

$$\text{g1: } \tau_{max} = \frac{3V}{2A} = \frac{3V_{max}}{2 \times 2 \times 0.1 L_c t_s} \quad \text{g2: } \sigma_{max} = \frac{My}{I} = \frac{M_{max} 0.05 L_c}{2 \times \frac{1}{12} t_s (0.1 L_c)^3}$$

Problem Re-Formulation

STEP 5 / Constraints

- 날개의 비틀림 파손에 대한 제한조건



$$g3: \sigma_{dis} \leq \sigma_{max}$$

Where,
$$\sigma_{dis} = E\epsilon = E\left(\sqrt{1 + \frac{2ab}{a^2 + b^2} \sin \frac{\phi}{r_{AR}}} - 1\right)$$

$$a = \frac{r_{AR} L_c}{2N_r}, b = \frac{1.0335}{3} L_c$$

Problem Re-Formulation

STEP 5

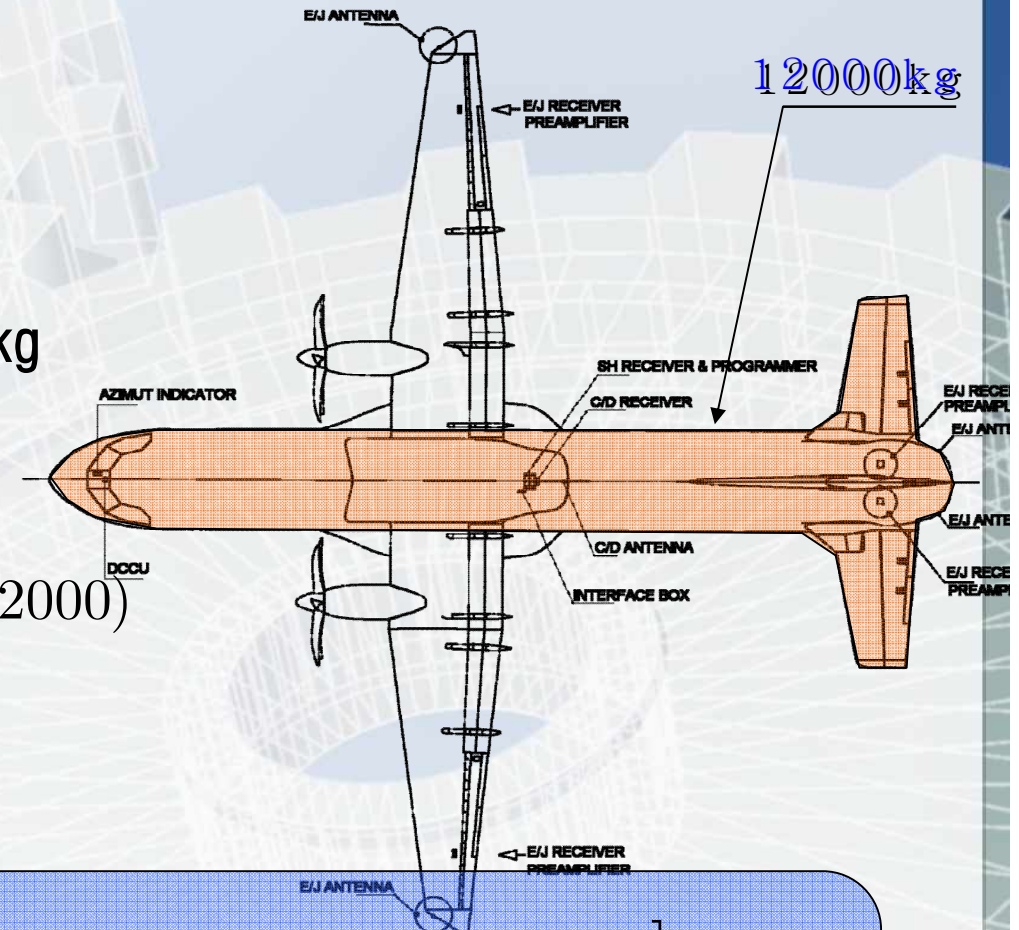
Constraints

- 양력에 대한 제한조건

- 기체 총 중량 : 23000kg

- 동체중량 : 12000kg

- $W_{lift} \geq g(2m_{wing} + 12000)$



g4: $W_{lift} \geq [f(t_s, N_r, r_{AR}, L_c) \times 2 + 12000] \times 9.8$

Problem Re-Formulation

STEP 5 / Constraints

- 항력으로 인한 페어링 변형에 관한 제한조건

– 항력(Total Drag) = 유도항력(Induced Drag) + 압력항력(Pressure Drag)

$$Total\ Drag : \frac{Lift\ Force}{Wing\ Span} \times \sin 5^\circ + Area \times C_D \times \frac{1}{2} \rho v^2$$

출처 : Aircraft Design : A Conceptual Approach (AIAA Education series)

g5:
$$\left[\frac{F_{lift}}{L_c r_{AR}} \sin 5^\circ + 0.10106 r_{AR} L_c \times 0.04 \times \frac{1}{2} \times 0.459 \times 100^2 \right] \leq \sigma$$
$$N_r \times t_f \times L_c$$

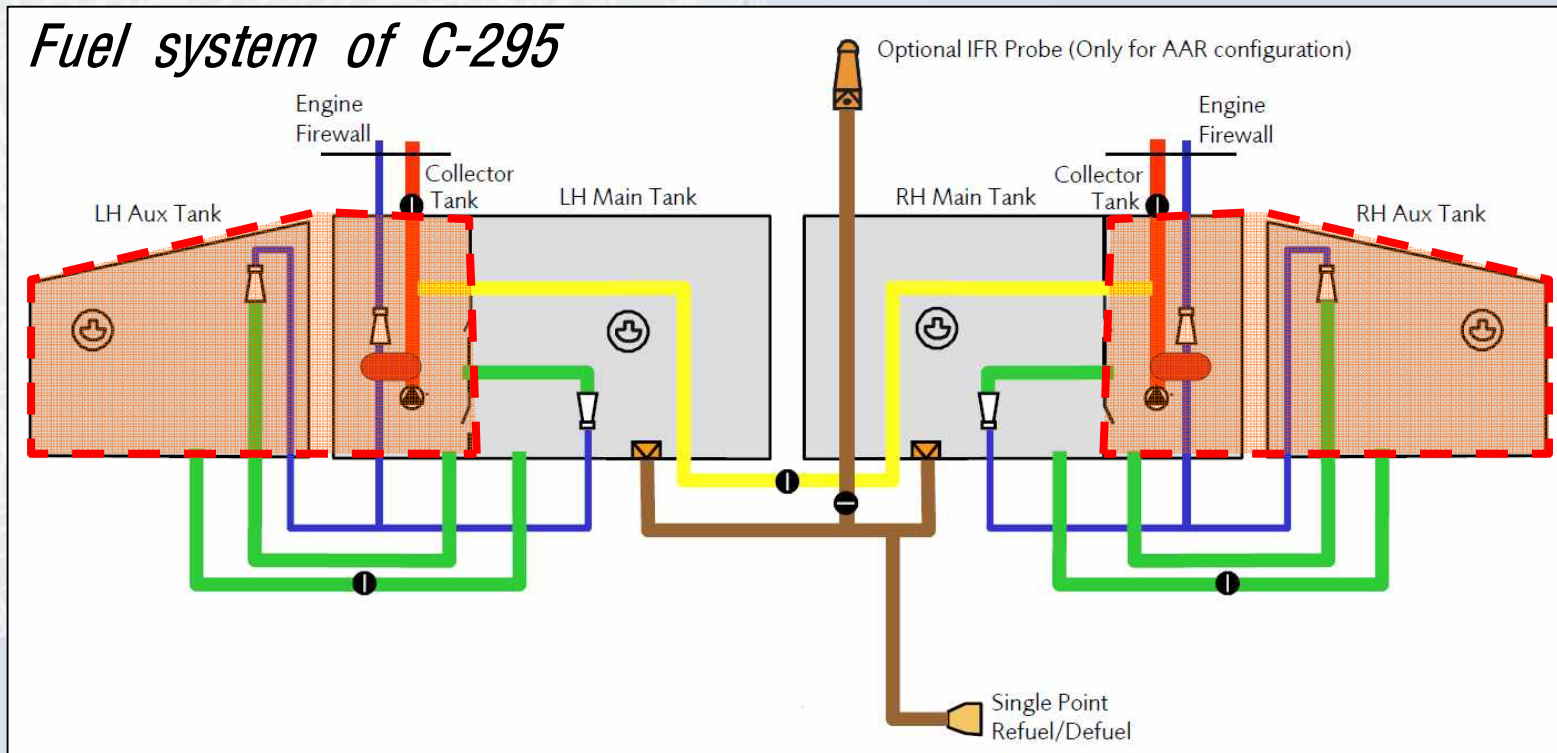
Problem Re-Formulation

STEP 5

Constraints

- 연료탱크 저장용량에 대한 제한조건

Fuel system of C-295



출처 : EADS "C-295 Technical Review"

$$g6: V_{fuel} \geq 5 m^3$$

Analysis and Solution

- MS Excel의 해 찾기 마법사를 이용한 결과

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		t_rib	0.01	m	Aluminium Composition	2024-T3	Unit					
3		λ	0.25		Young's Modulus	7.00E+10	Pa					
4		g	9.8	m/s^2	Shear Modulus	2.75E+10	Pa			1.08E+08		
5		p_alu	2770	kg/m^3	Tensile Strength	2.15E+08	Pa	1.08E+08				
6		p_fl	807.5	kg/m^3	Elongation	13	%			1.08E+08		
7		p_air	0.459	kg/m^3	Yield Strength	1.40E+08	Pa	107500000				
8		m_eng	521.64	kg	Density	2770	kg/m3					
9		m_prop	163.7	kg								
10						Vmax	Mmax	Vrib	Vspar	Vfaring		
11						2095106.846	159588.366	0.003529527	0.91011437	0.956237061		
12		spar(thick)	rib(num)	A.R	Chord	g1 : T_max	g2 : G_max	g3 : G_spar	g4 : G_spar	g5 : fuel	g6 : lift	CostFunc
13		0.05841271	9	8.92209932	3.88045673	107500000	46475062.46	211965791.3	98082596.6	2.5	211398.756	4785.6508
14												
15												
16		$V_{max} = \sum_{i=0}^{N_r-1} 0.1010 L_{cord,i} t_r \rho g + 0.05 \rho g L_{cord}^2 t_r^2 r_{AR} + 0.05053 \rho_{fuel} g$										
17		$M_{max} = 0.1010 \rho g \sum_{i=0}^{N_r-1} L_{cord,i} + 0.05 \rho g L_{cord}^2 r_{AR} t_r^2 + 0.05053 \rho_{fuel} g$										
18		$\text{Where, } V_{rib} = A_r \sum_{i=1}^{N_r} \frac{i}{N_r} t_r A_r (1-\lambda) = t_r A_r (1-(N_r+1)(1-\lambda))$										
19		$V_{spar} = 0.1 \times L_{cord}^2 r_{AR} t_r^2$										
20		$V_{faring} = t_r L_{cord} r_{AR} \rho_{fuel} (1+\lambda) / 4$										
21		$V_{fuel} = A_r L_{cord} r_{AR} - V_{rib} - V_{spar} - V_{faring}$										
22												
23												
24												
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26												
27												
28												
29												
30												
31												
32												
33												
34												
35												

해 찾기 모델 설정

목표 셀(E):

해의 조건: ☐ 최대값(M) ☒ 최소값(N) ☐ 지정값(V):

값을 바꿀 셀(B):

제한 조건(U):

-
-
-
-
-
-

실행(S) 닫기 옵션(O)... 초기화(B) 도움말(H)

추정(G) 추가(A)... 변경(C)... 삭제(D)

$$t_R^* = 0.68413, N_r^* = 9, r_{AR}^* = 8.9221, L_c^* = 3.8805, f^* = 4785.6508$$

Analysis and Solution

- MATLAB의 최적화 Tool을 이용한 결과

The image shows the MATLAB Optimization Tool window. The 'Problem Setup and Results' tab is active. The solver is set to 'fmincon - Constrained nonlinear minimization' with the 'Interior point' algorithm. The objective function is '@fmin' and the start point is '[0.1 12 10 2.5]'. The constraints section shows linear inequalities, linear equalities, and bounds. The 'Options' tab on the right shows stopping criteria: Max iterations (Specify: 5000), Max function evaluations (Specify: 10000), X tolerance (Specify: 1e-10), Function tolerance (Specify: 1e-6), Nonlinear constraint tolerance (Specify: 1e-6), SQP constraint tolerance (Specify: 1e-6), and Unboundedness threshold (Specify: -1e20). The 'Function value check' and 'User-supplied derivatives' sections are also visible.

Problem Setup and Results

Solver: fmincon - Constrained nonlinear minimization
 Algorithm: Interior point
 Problem
 Objective function: @fmin
 Derivatives: Approximated by solver
 Start point: [0.1 12 10 2.5]
 Constraints:
 Linear inequalities: A: b:
 Linear equalities: Aeq: beq:
 Bounds: Lower: 0;0;0;0 Upper:
 Nonlinear constraint function: @fmin_incon
 Derivatives: Approximated by solver
 Run solver and view results
 Start Pause Stop
 Current iteration: 71 Clear Results
 feasible directions, to within the default value of the function tolerance, and constraints were satisfied to within the default value of the constraint tolerance.
 Final point:
 1 2 3 4
 0.068 8.326 8.912 3.881

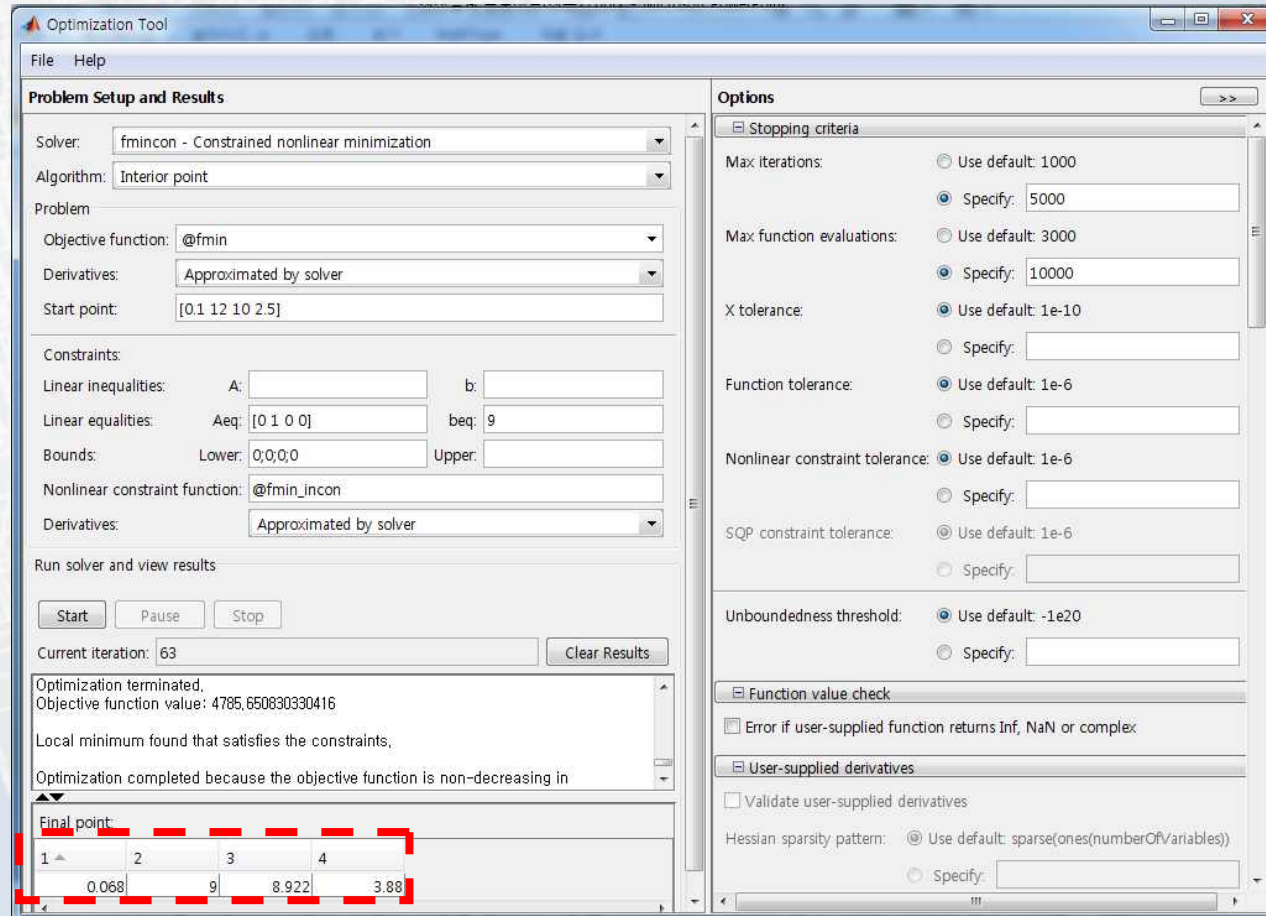
Options

Stopping criteria
 Max iterations: Use default: 1000 Specify: 5000
 Max function evaluations: Use default: 3000 Specify: 10000
 X tolerance: Use default: 1e-10 Specify:
 Function tolerance: Use default: 1e-6 Specify:
 Nonlinear constraint tolerance: Use default: 1e-6 Specify:
 SQP constraint tolerance: Use default: 1e-6 Specify:
 Unboundedness threshold: Use default: -1e20 Specify:
 Function value check
 Error if user-supplied function returns Inf, NaN or complex:
 User-supplied derivatives
 Validate user-supplied derivatives
 Hessian sparsity pattern: Use default: sparse(ones(numberOfVariables)) Specify:

$$t_R^* = 0.68413, N_r^* = 8.3255, r_{AR}^* = 8.9125, L_c^* = 3.8808, f^* = 4776.1602$$

Analysis and Solution

- MATLAB의 최적화 Tool을 이용한 결과

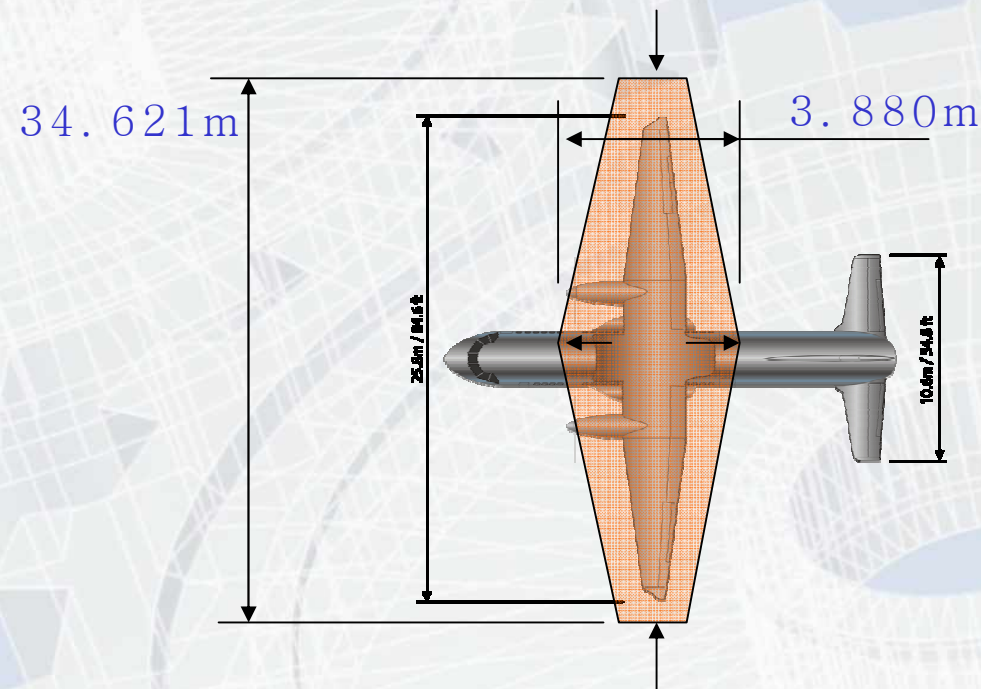


$$t_R^* = 0.68413, N_r^* = 9, r_{AR}^* = 8.9221, L_c^* = 3.8804, f^* = 4785.6508$$

Discussion

- 최적해를 구한 결과

- Aspect ratio는 감소하고 Chord길이는 증가함
- 전체적으로 주익의 크기는 증가하였지만 본래 설계된 기체의 주익보다 경량화된 주익을 얻을 수 있었음



$M_{\text{before}} = 11000\text{kg}$

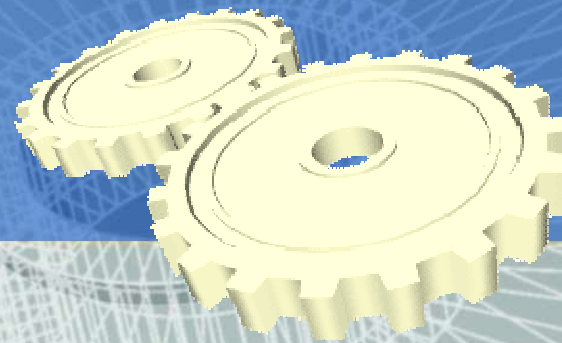


$W_{\text{after}} = 9571\text{kg}$

Discussion

- Feasible Region 확인 불가
 - 설계변수가 4개임에 따라 3차원 공간에 나타낼 수 없음
- MATLAB Optimization Toolbox 사용 제한
 - 설계변수는 실수와 정수의 혼합된 형태
 - 제약조건은 모두 비선형 형태로 존재
 - ▶ 비선형 제약조건에 의해 `fmincon`을 이용하였을 경우 직접적으로 정수조건 부여 불가
 - ▶ 정수형 설계변수를 직접 변화시키면서 해의 수렴 양상 관찰
- 해석모델 보완 필요
 - Aeroelastic(공탄성)에 의한 제한조건 도입 필요
 - 실 기체 설계에 사용되는 요소 도입 및 반영
 - 실제 Rib 형상 및 구조, Control Surface 등

Thank you!
Q & A



Thank you!

