

# Numerical Methods for Engineers

*With Software and Programming Applications* • Fourth Edition

# Parachutist Problem (1)

- Mathematical model

$$F = ma \rightarrow a = \frac{F}{m} \rightarrow \frac{dv}{dt} = \frac{mg - cv}{m} \rightarrow \frac{dv}{dt} = g - \frac{c}{m}v$$

$\frac{dv}{dt} + \frac{c}{m}v = g$  : differential equation

$$v_h = c_1 e^{\lambda t} \rightarrow \lambda = -\frac{c}{m}$$

$$v = c_1 e^{\left(-\frac{c}{m}\right)t} + c_2 \rightarrow \frac{c}{m}c_2 = g \rightarrow c_2 = \frac{gm}{c}$$

$$v = c_1 e^{\left(-\frac{c}{m}\right)t} + \frac{gm}{c} \leftarrow (t=0, v=0) \Rightarrow c_1 = -\frac{gm}{c}$$

$$v = \frac{gm}{c} \left[ 1 - e^{\left(-\frac{c}{m}\right)t} \right] : \text{analytical (or exact) solution}$$



```
>>syms v g c m  
>>dsolve('Dv=g-c/m*v', 'v(0)=0')
```

# Parachutist Problem (2)

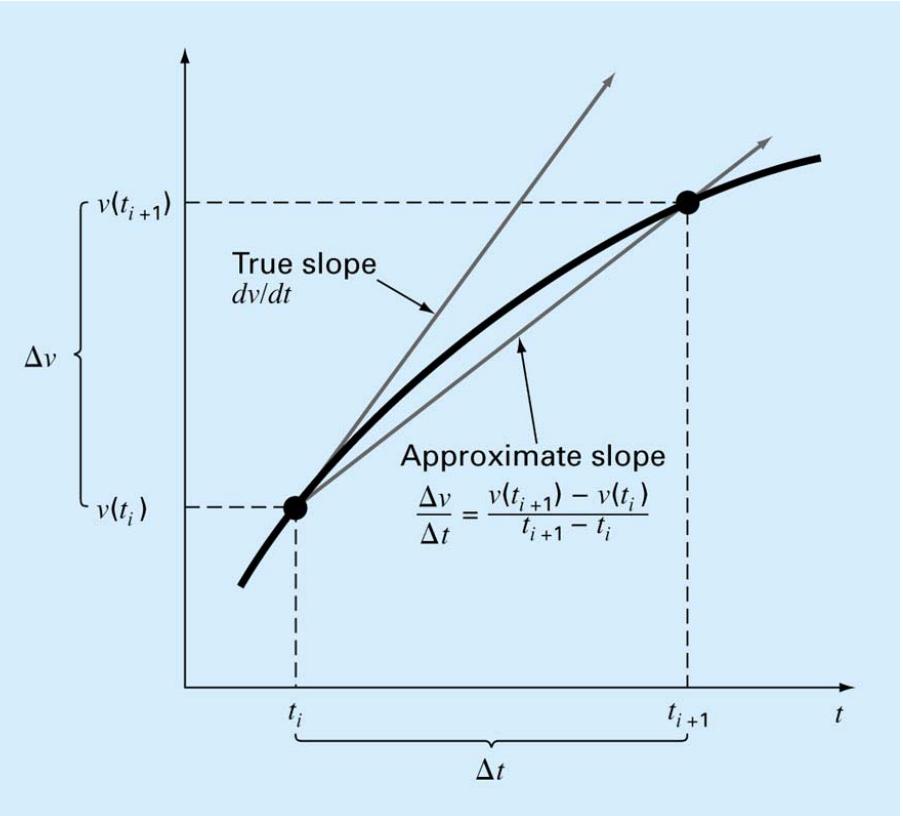
- Numerical methods

$$\frac{dv}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} \cong \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$

$$\frac{dv}{dt} = g - \frac{c}{m} v \rightarrow \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{c}{m} v(t_i)$$

$$v(t_{i+1}) = \underbrace{v(t_i)}_{\text{old}} + \underbrace{\left[ g - \frac{c}{m} v(t_i) \right]}_{\text{slope}} \underbrace{(t_{i+1} - t_i)}_{\text{step size}}$$

: Euler's method



# Packages and Programming

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- Software users
  - Those who take what they are given
  - Power user: write Excel VBA, MATLAB M-files
- Computer programs
  - A set of instructions that direct the computer to perform a certain task
- Programming topics
  - Simple information representation: variables, type declaration
  - Advanced information representation: data structure, array
  - Mathematical formulas: assignment, intrinsic functions
  - Input/output
  - Logical representation: sequence, selection, repetition
  - Modular programming: functions, subroutines

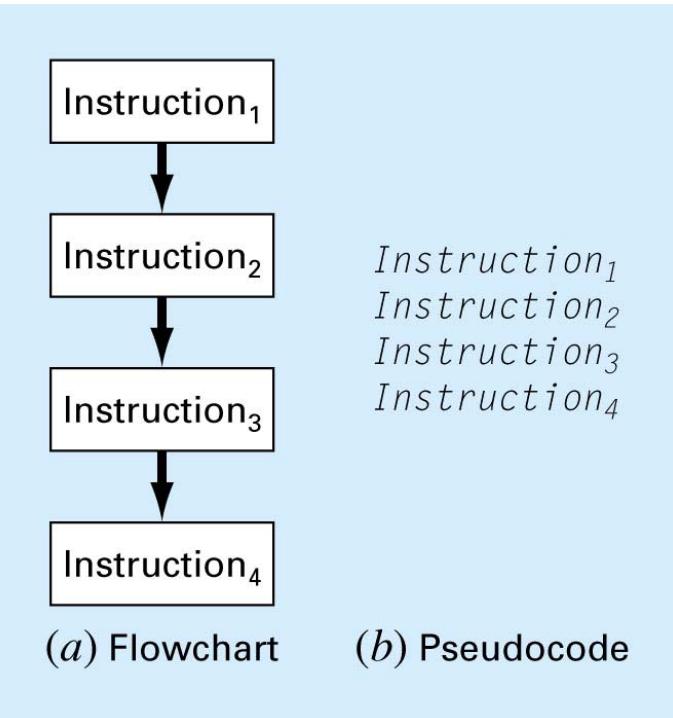
# Structured Programming

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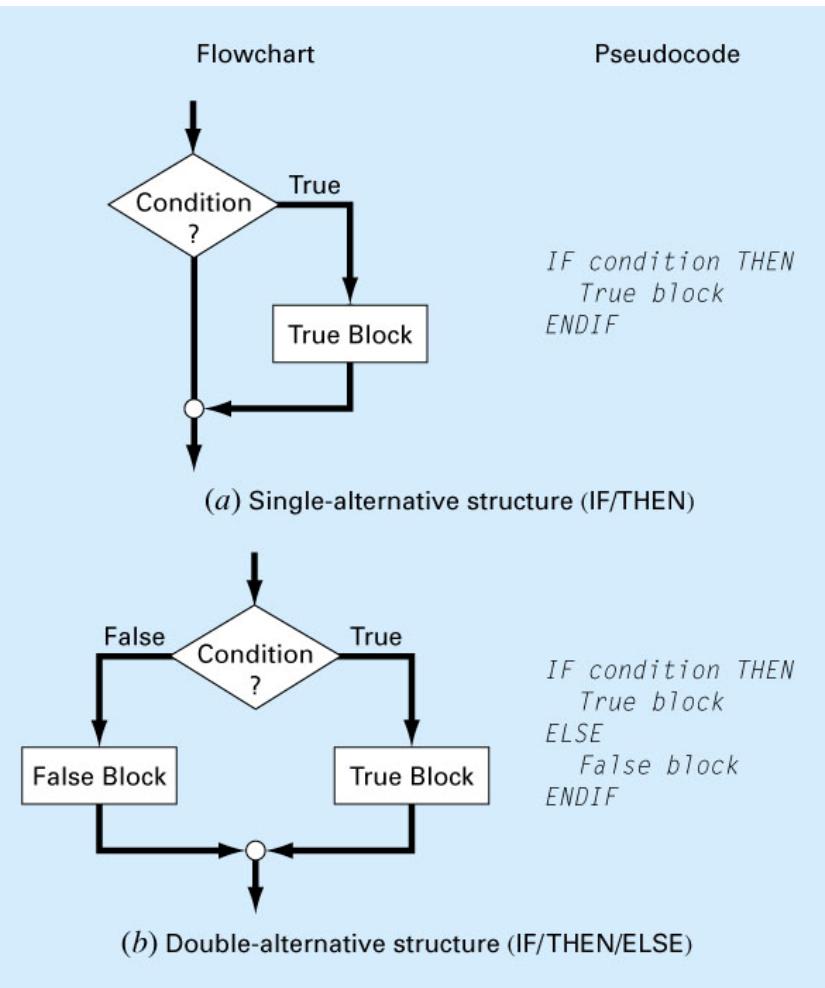
- Benefits to writing organized, well-structured code
  - Shorter time to develop, test, and update
- Three fundamental control structures
  - Sequence, selection and repetition
- Flowchart
  - Visual or graphical representation of an algorithm
- Pseudocode
  - Code-like statements in place of the graphical symbols of the flowchart
  - Bridges the gap between flowcharts and computer code

# Logical Representation (1)

- Sequence

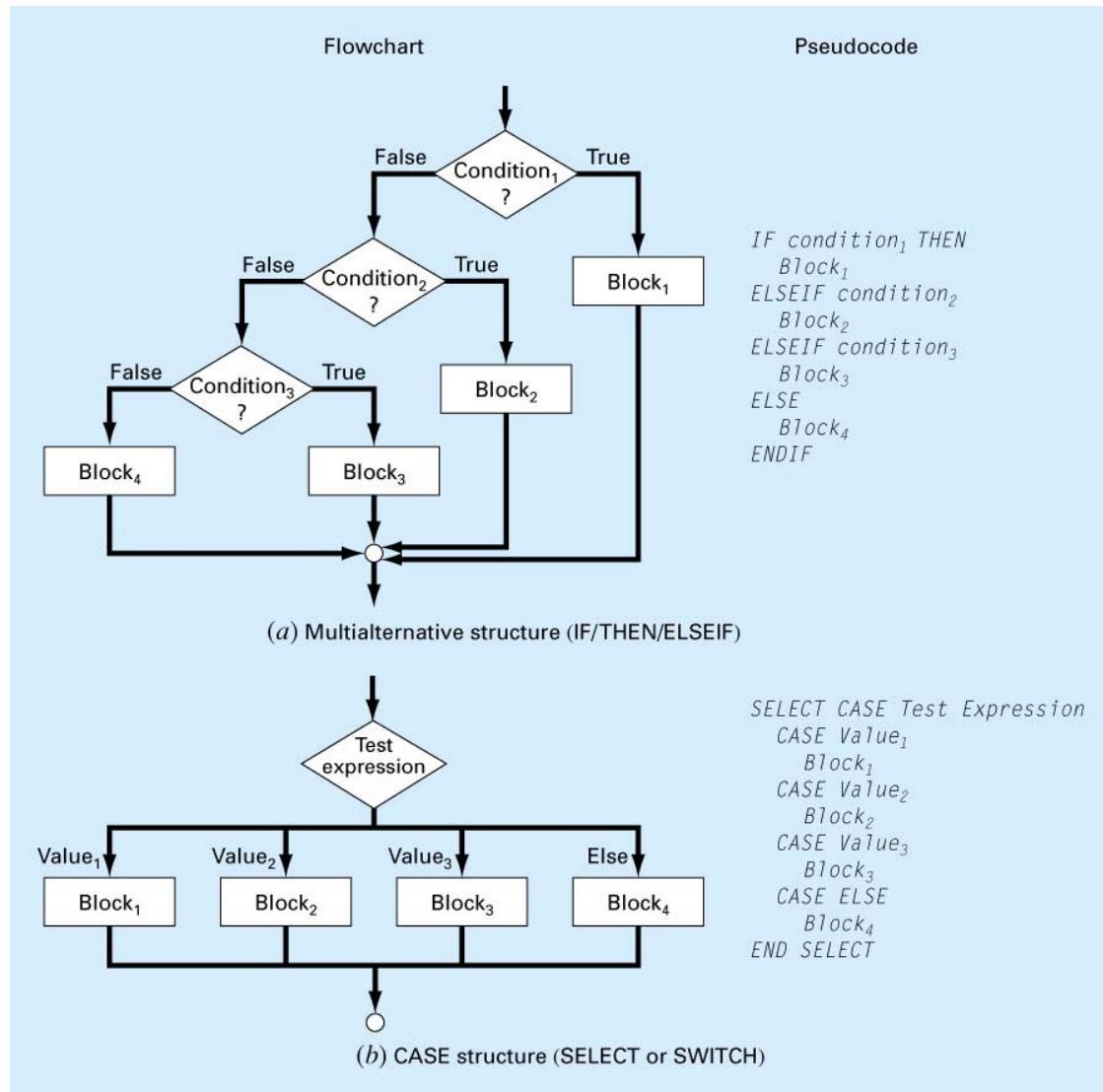


- Selection (1)



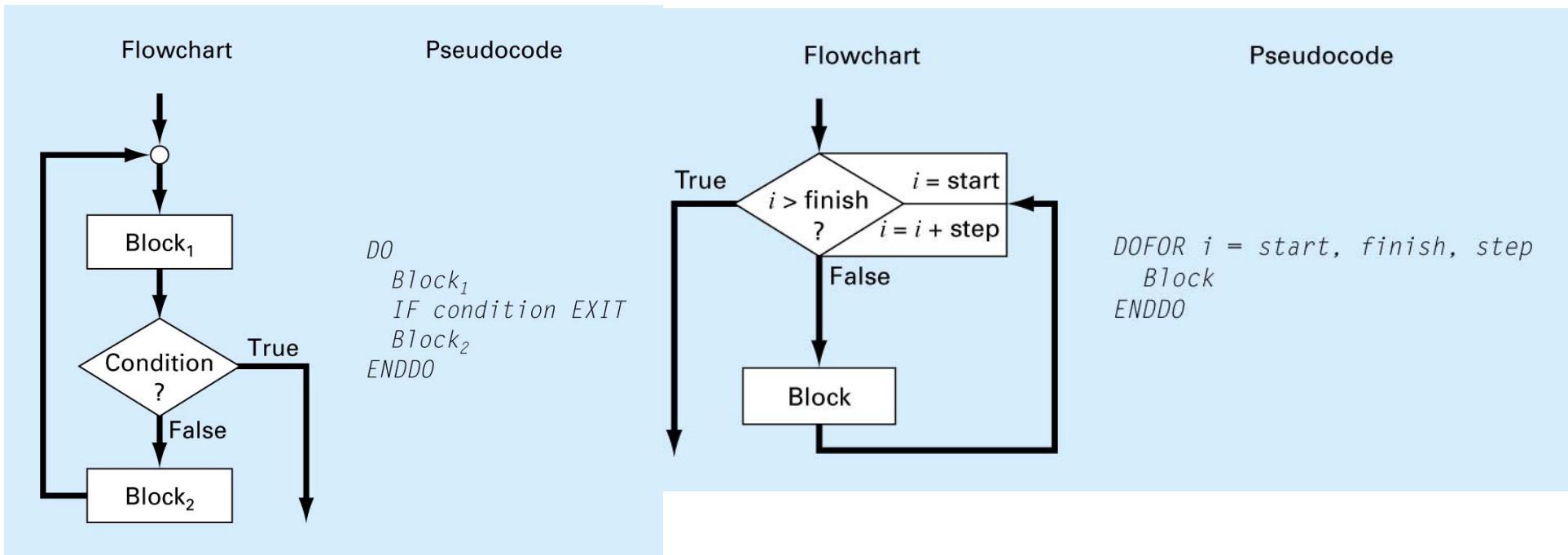
# Logical Representation (2)

- Selection (2)



# Logical Representation (3)

- Repetition



# Modular Programming

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- Divide into small subprograms, or modules
  - Independent and self-contained as possible
  - Designed to perform a specific, well-defined function
  - Short and highly focused
- High-level languages such as Fortran 90 or C
  - Functions: single return
  - Subroutines: several returns
- Software packages like Excel and MATLAB
  - Subprograms

# Control Structures (1)

| Pseudocode   | Excel VBA   | MATLAB   |
|--|---|--|
| <u>IF/THEN:</u><br><b>IF</b> condition <b>THEN</b><br><i>True block</i><br><b>ENDIF</b>  | If b<>0 then<br>r1 = -c / b<br>End If   | if b ~= 0<br>r1 = -c / b;<br>end   |
| <u>IF/THEN/ELSE:</u><br><b>IF</b> condition <b>THEN</b><br><i>True block</i><br><b>ELSE</b><br><i>False block</i><br><b>ENDIF</b>  | If a < 0 Then<br>b = Sqr (Abs (a))<br>Else<br>b = Sqr (a)<br>End If                                   | if a < 0<br>b = sqrt (abs(a));<br>else<br>b = sqrt (a);<br>end                             |
| <u>IF/THEN/ELSEIF:</u><br><b>IF</b> condition1 <b>THEN</b><br><i>Block1</i><br><b>ELSEIF</b> condition2<br><i>Block2</i><br><b>ELSE</b><br><i>Block3</i><br><b>ENDIF</b> | If class = 1 Then<br>x = x + 8<br>Else If class < 1 Then<br>x = x - 8<br>Else<br>x = x - 64<br>End If | if case == 1<br>x = x + 8;<br>elseif class < 1<br>x = x - 8;<br>else<br>x = x - 64;<br>end |

# Control Structures (2)

| Pseudocode   | Excel VBA  | MATLAB  |
|--|--|---|
| <u>CASE:</u><br>SELECT CASE <i>test expression</i><br>CASE <i>value1</i><br><i>block1</i><br>CASE <i>value2</i><br><i>block2</i><br>CASE ELSE<br><i>block3</i><br>END SELECT | Select Case <i>a + b</i><br>Case <i>Is &lt; -50</i><br><i>x = -5</i><br>Case <i>Is &lt; 0</i><br><i>x = -5 - (a + b) / 10</i><br>Case Else<br><i>x = 5</i><br>End Select | switch <i>a + b</i><br>case 1<br><i>x = -5;</i><br>case 2<br><i>x= -5 - (a + b) / 10</i><br>otherwise<br><i>x = 5;</i><br>end |
| <u>DOEXIT:</u><br>DO<br><i>block1</i><br>IF <i>condition</i> EXIT<br><i>block2</i><br>ENDIF  | Do<br><i>i = i + 1</i><br>IF <i>i &gt;= 10</i> Then Exit Do<br><i>j = i*x</i><br>Loop  | while (1)<br><i>i = i + 1;</i><br>if <i>i &gt;= 10</i> , break, end<br><i>j = i*x</i><br>end                                  |
| <u>COUNT-CONTROLLED LOOP:</u><br>DOFOR <i>i = start, finish, step</i><br><i>Block</i><br>ENDDO   | For <i>i = 1 To 10 Step 2</i><br><i>x = x + i</i><br>Next <i>i</i>   | for <i>i = 1:10:2</i><br><i>x = x + i</i><br>end  |

# Parachutist Problem: Excel VBA



```
g = 9.8
INPUT cd, m
INPUT ti, vi, tf, dt
t = ti
v = vi
n = (tf - ti) / dt
DOFOR i = 1 TO n
    dvdt = g - (cd / m) * v
    v = v + dvdt * dt
    t = t + dt
ENDDO
DISPLAY v
```

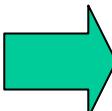
```
h = dt
DO
    IF t + dt > tf THEN
        h = tf - t
    ENDIF
    dvdt = g - (cd / m) * v
    v = v + dvdt * h
    t = t + h
    IF t ≥ tf EXIT
ENDDO
DISPLAY v
```

```
FUNCTION Euler(dt, ti, tf, yi)
t = ti
y = yi
h = dt
DO
    IF t + dt > tf THEN
        h = tf - t
    ENDIF
    dydt = dy(t, y)
    y = y + dydt * h
    t = t + h
    IF t ≥ tf EXIT
ENDDO
Euler = y
END
```

# Parachutist Problem: MATLAB

```
g=9.8;
m=input('mass (kg):');
cd=12.5;
ti=0;
tf=2;
vi=0;
dt=0.1;
t = ti;
v = vi;
h = dt;
while (1)
    if t + dt > tf
        h = tf - t;
    end
    dvdt = g - (cd / m) * v;
    v = v + dvdt * h;
    t = t + h;
    if t >= tf, break, end
end
disp('velocity (m/s):')
disp(v)
```

```
>> file_name
mass (kg): 100
velocity (m/s):
17.4381
```



```
function euler = f(dt,ti,tf,yi,m,cd)
t = ti;
y = yi;
h = dt;
while (1)
    if t + dt > tf
        h = tf - t;
    end
    dydt = dy(t, y, m, cd);
    y = y + dydt * h;
    t = t + h;
    if t >= tf, break, end
end
euler = y;
```

```
function dy = f(t, v, m, cd)
g = 9.8;
dy = g - (cd / m) * v;
```

```
>> m=68.1;
>> cd=12.5;
>> ti=0;
>> tf=2.;
>> vi =0;
>> vt=0.1;
>> euler (dt,ti,tf,vi,m,cd)
```

# MATLAB

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- Data Analysis
  - 다양한 연산기능을 이용하여 각종 데이터 해석이 가능
- Visualization and Image Processing
  - 강력한 가시화 기능을 활용한 직감적인 해석결과 파악
- Programming & Algorithm Development
  - 구조화된 고급 언어로 알고리즘 개발을 지원
- Application Deployment
  - 작업 효율화를 지원하는 독립 실행 애플리케이션 배포
- Modeling & Simulation
  - 시뮬레이션 및 모델링을 위한 다양한 함수
- Test & Measurement
  - 계측기기로부터 데이터 수집 및 임의 신호 생성

# Summary

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| Item                            | Excel  | MATLAB   |
|---------------------------------|--|--|
| Root location                   | Goal Seek<br>Solve   | <code>fzero(f,x0,options)</code><br><code>Roots(a)</code>  |
| Linear Algebraic Equations      | <code>minverse</code><br><code>mmult</code>  |  |
| Curve Fitting                   | <code>Forecast</code> , <code>growth</code> ,<br><code>intercept</code> , <code>linest</code> ,<br><code>logest</code> , <code>slope</code> , <code>trend</code> | <code>Polyfit</code> , <code>interp1</code> ,<br><code>interp2</code> , <code>spline</code> , <code>fft</code> |
| Integration and Differentiation |  | <code>quad</code><br><code>trapz</code><br><code>diff</code>   |
| Ordinary Differential Equations | <code>macro</code>   | <code>ode23</code> , <code>ode45</code><br><code>ode15s</code> , <code>ode23s</code><br><code>eig</code>       |
| Partial Differential Equations  | <code>cell ↔ rectangular grids</code>  |  |

# Root Location / Polynomial Manipulation

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- Excel
  - Goal Seek, Solver
- MATLAB

| Function | Description                               |
|----------|---|
| fzero    | Root of single function                   |
| roots    | Find polynomial roots                     |
| poly     | Construct polynomial with specified roots |
| polyval  | Evaluate polynomial                       |
| polyvalm | Evaluate polynomial with matrix argument  |
| residue  | Partial-fraction expansion                |
| polyder  | Differentiate polynomial                  |
| conv     | Multiply polynomials                      |
| deconv   | Divide polynomials                        |

# Linear Algebra Equations (1)

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- Excel
  - Functions: minverse, mmult, mdeterm, transpose
- MATLAB
  - Matrix analysis

| Function | Description                                    |
|----------|--|
| cond     | Matrix condition number                        |
| norm     | Matrix or vector norm                          |
| rcond    | LINPACK reciprocal condition number            |
| rank     | Number of linearly independent rows or columns |
| det      | Determinant                                    |
| trace    | Sum of diagonal elements                       |
| null     | Null space                                     |
| orth     | Orthogonalization                              |
| rref     | Reduced row echelon form                       |

# Linear Algebra Equations (2)

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## – Linear Equations

| Function | Description                                       |
|----------|---|
| \ and /  | Linear equation solution; >>help slash            |
| chol     | Cholesky factorization                            |
| lu       | Factors from Gauss elimination                    |
| inv      | Matrix inverse                                    |
| qr       | Orthogonal-triangular decomposition               |
| qrdelete | Delete a column from the QR factorization         |
| qrinsert | Insert a column from the QR factorization         |
| nnls     | Nonnegative least squares                         |
| pinv     | Pseudoinverse                                     |
| lscov    | Least squares in the presence of known covariance |

# Curve Fitting

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- Excel

| Function  | Description   |
|-----------|---|
| FORECAST  | Returns a value along a linear trend                |
| GROWTH    | Returns values along an exponential trend           |
| INTERCEPT | Returns the intercept of the linear regression line |
| LINEST    | Returns the parameters of a linear trend            |
| LOGEST    | Returns the parameters of an exponential trend      |
| SLOPE     | Returns the slope of the linear regression line     |
| TREND     | Returns values along a linear trend                 |

- MATLAB

| Function | Description                          |
|----------|--------------------------------------|
| polyfit  | Fit polynomial to data               |
| interp1  | 1-D interpolation (1-D table lookup) |
| interp2  | 2-D interpolation (2-D table lookup) |
| spline   | Cubic spline data interpolation      |
| fft      | Discrete Fourier transform           |