Simulink 기초
2^{nd} order system

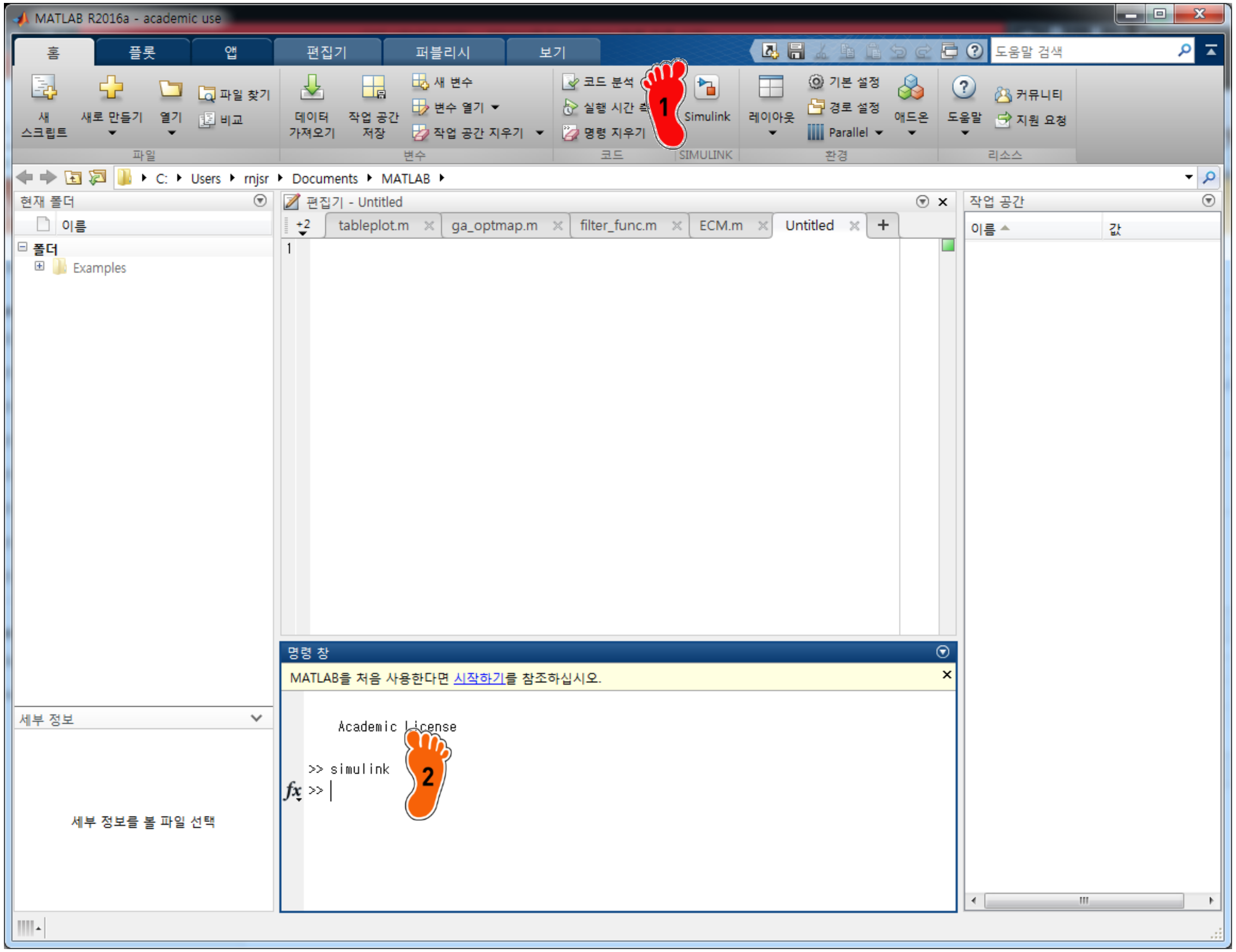
Computational Design Laboratory
Department of Automotive Engineering
Hanyang University, Seoul, Korea

CONTENTS

OVERVIEW

- **Simulation and Model-Based Design**
  - **Block diagram environment for multi-domain simulation and Model-Based Design**
  - **Integrated with MATLAB, enabling to incorporate MATLAB algorithms into models and export simulation results to MATLAB for further analysis**
  - **Video : Simulink Overview**

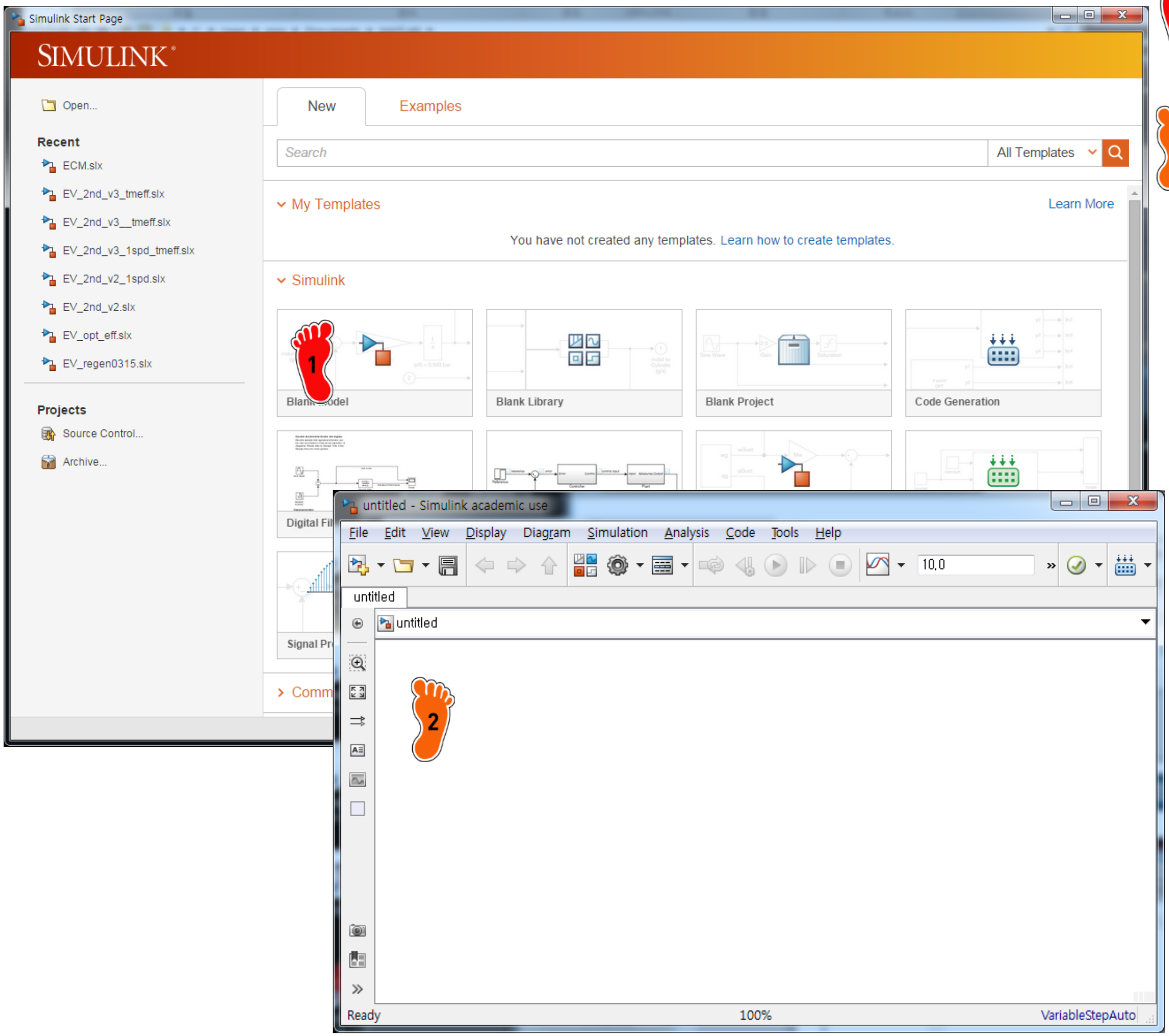
SIMULINK ENVIRONMENT



1 Simulink Icon 클릭 또는

2 명령 창에 'simulink' 입력

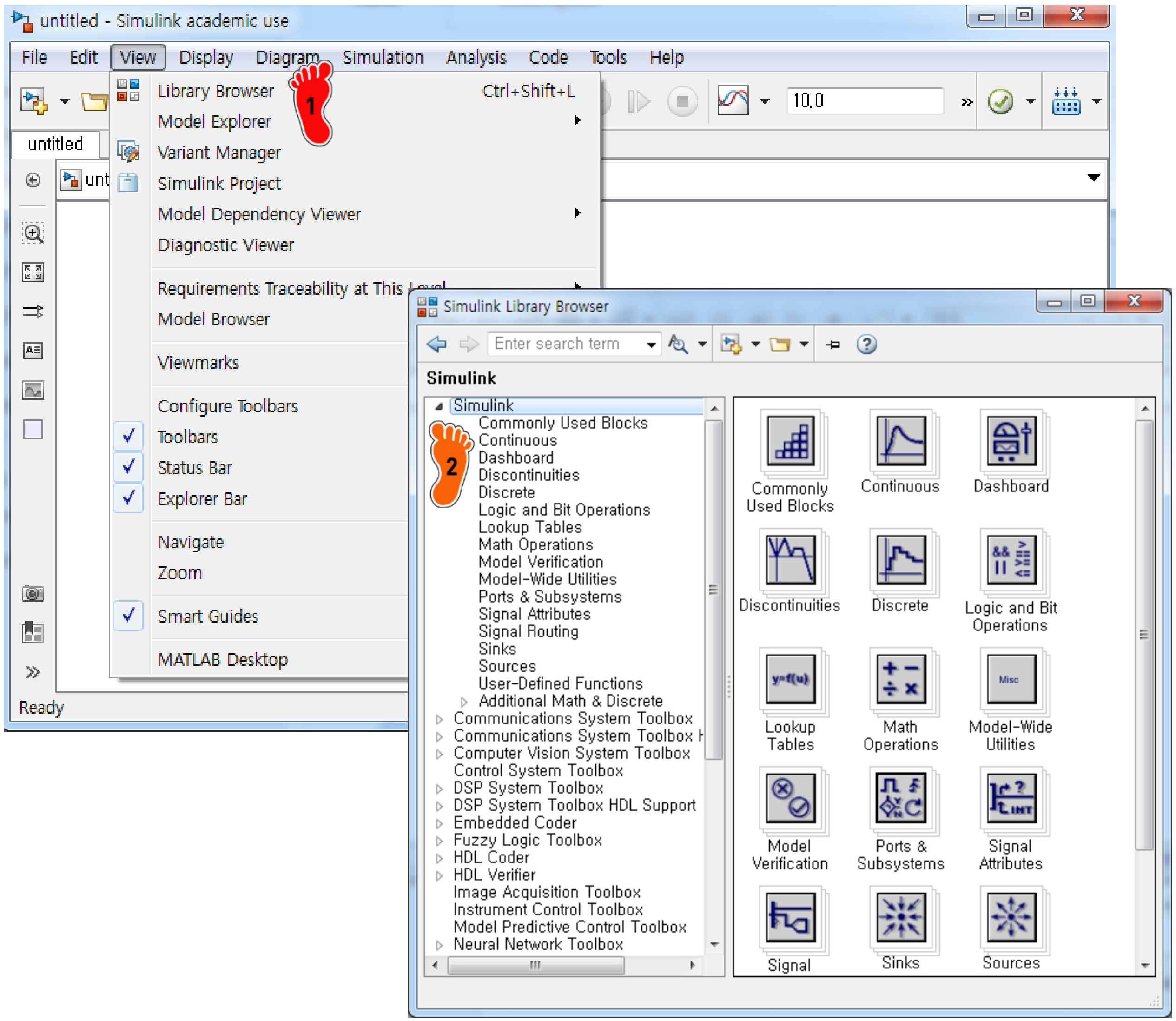
SIMULINK ENVIRONMENT



1 Blank Model 클릭

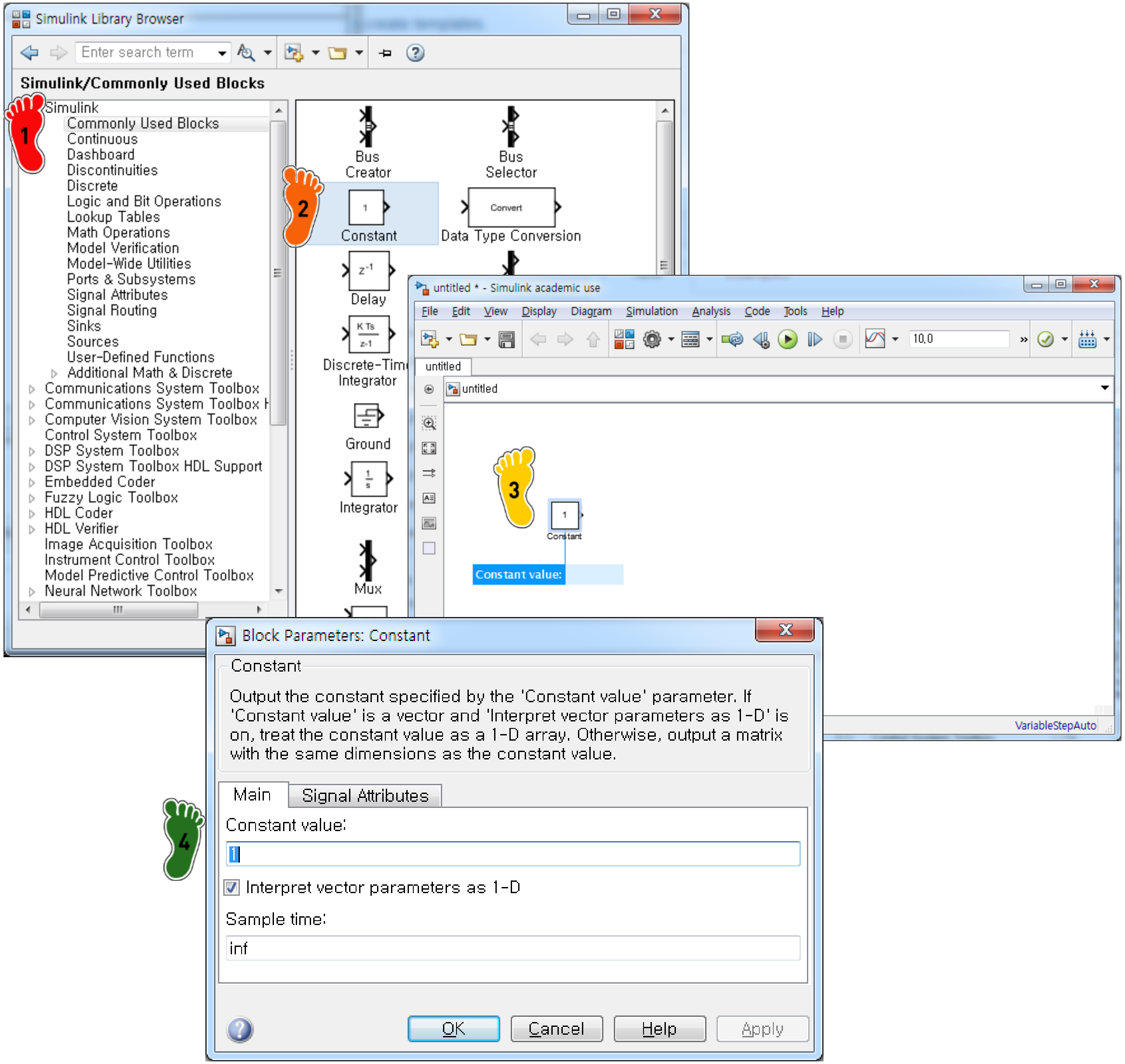
2 모델링 화면 생성

SIMULINK ENVIRONMENT



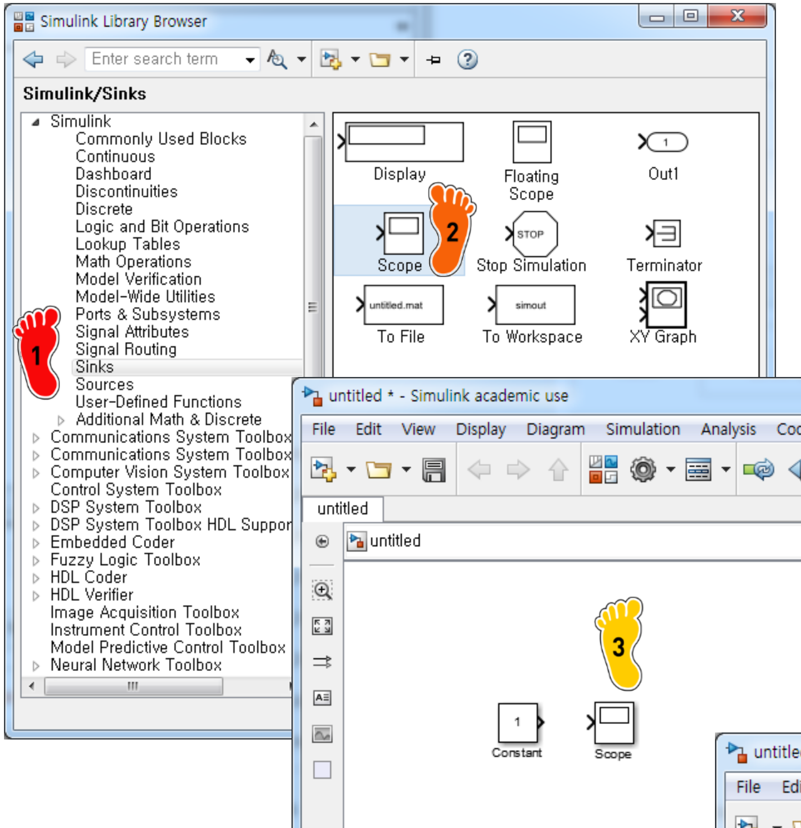
1. View → Library Browser 클릭
2. 원하는 모듈 선택

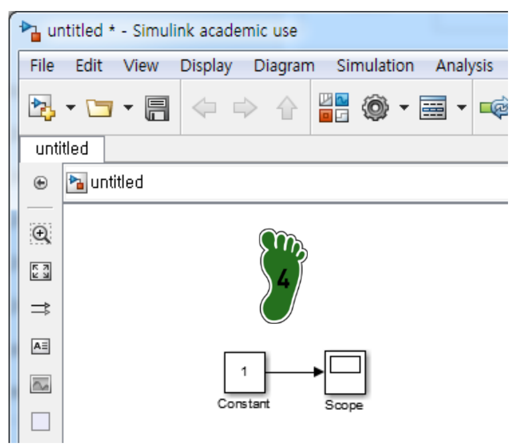
SIMULINK ENVIRONMENT



1. 'Commonly Used Blocks' 클릭
2. 'Constant' block 클릭
3. 모델링 화면으로 Drag&Drop
4. 'Constant' block 더블클릭 Parameter 창 생성 원하는 값 입력

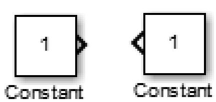
SIMULINK ENVIRONMENT





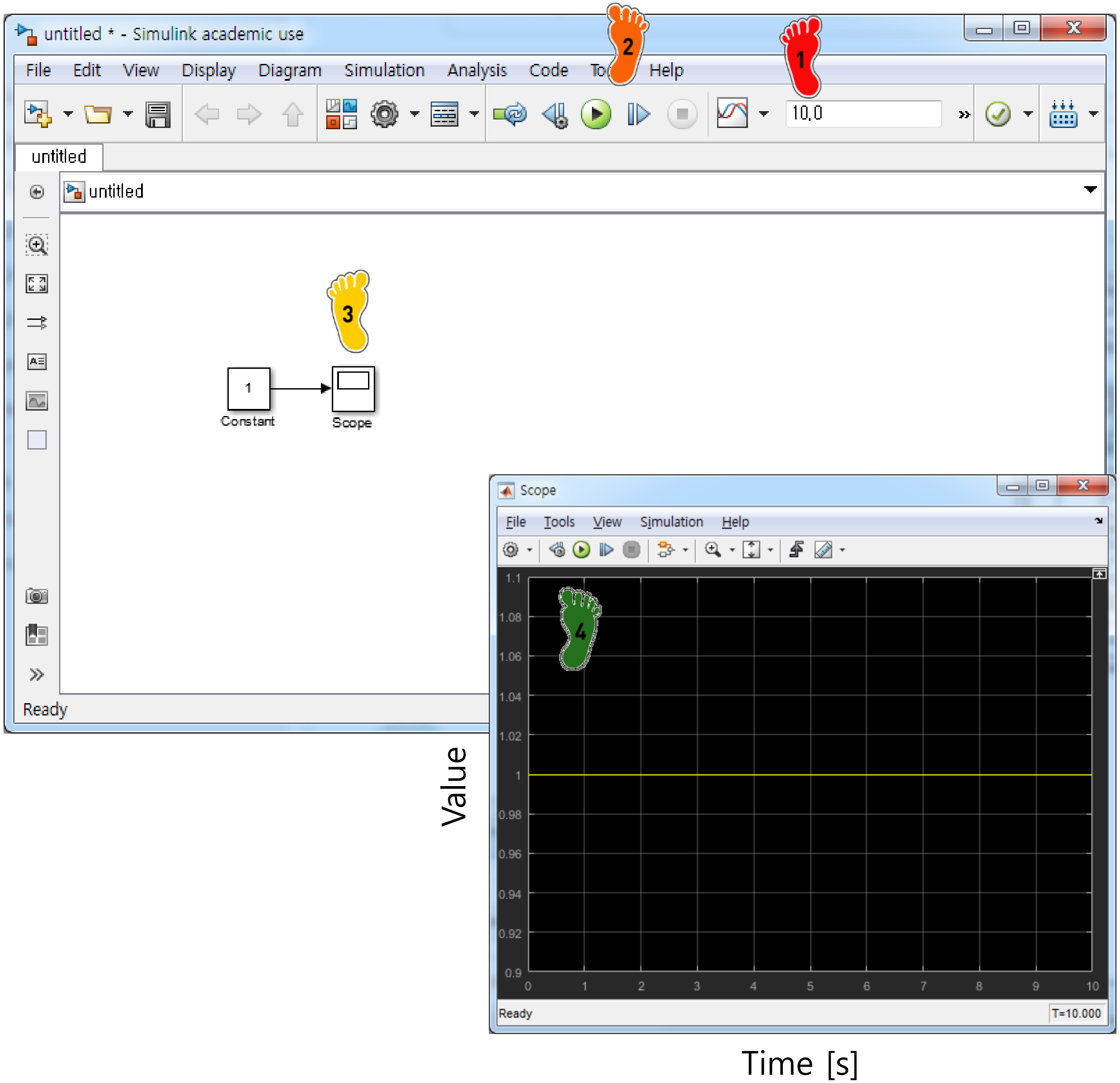
1. 'Sinks' 클릭
2. 'Scope' block 클릭
3. 모델링 화면으로 Drag&Drop
4. 'Constant' block 의 오른쪽 화살표를 누른 상태에서 'Scope' block 왼쪽 화살표에 Drag&Drop

Ctrl+I : block flip



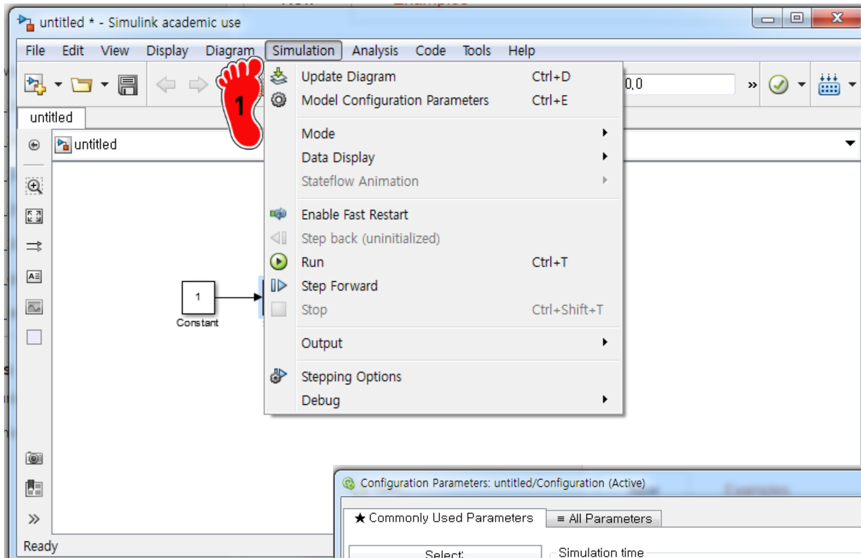
Ctrl+R : block rotate (CCW)

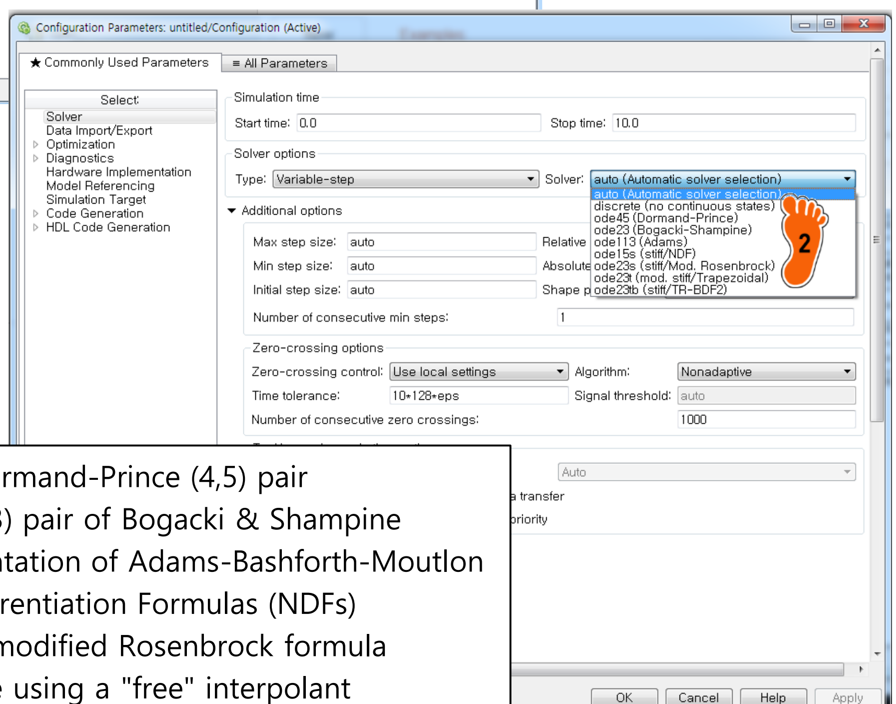
SIMULINK ENVIRONMENT



1. Simulation Time [s] 설정
2. ‘Run’ 클릭
3. ‘Scope’ block 더블클릭
4. Scope window에서 결과 확인

SIMULINK ENVIRONMENT

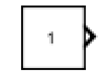
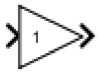
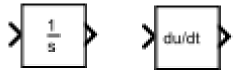
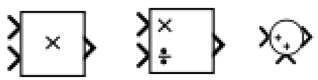
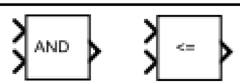
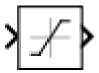
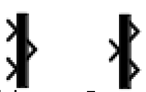
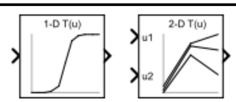




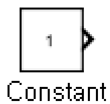
1. Simulation-Model Configuration Parameters 클릭
2. Solver Type 및 다양한 옵션 설정 가능

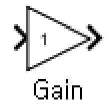
ode45 : Runge-Kutta, Dormand-Prince (4,5) pair
ode23 : Runge-Kutta (2,3) pair of Bogacki & Shampine
ode113 : PECE Implementation of Adams-Bashforth-Moutlon
ode15s : Numerical Differentiation Formulas (NDFs)
ode23s : Second-order, modified Rosenbrock formula
ode23t : Trapezoidal rule using a "free" interpolant
ode23tb : implicit Runge-Kutta formula with two stages

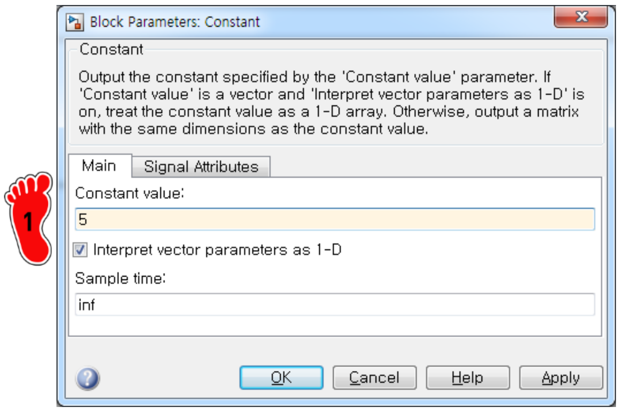
COMMONLY USED BLOCKS

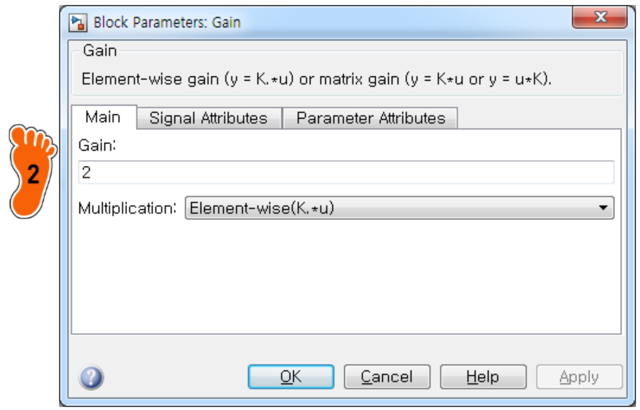
ICON	Description
 Constant	상수 입력
 Gain	y(output)=a*u(input), a : gain
 Integrator, Derivative	적분기 : $y = \int u\,dt$, 미분기 : $y = \frac{du}{dt}$
 Product, Divide, Sum	사칙연산자 (곱하기, 나누기, 더하기, 빼기)
 Logical Operator, Relational Operator	비교문 (and, or, less than, more than, equal)
 Saturation	입력신호 범위 제한
 Mux, Demux	단일 → 벡터 합성, 벡터 → 단일 분해
 1-D Lookup Table, 2-D Lookup Table	입력에 대한 출력을 사전에 정의한 table(1-D or 2-D)값에 의해 정의
 Fcn	입력에 대한 출력 신호를 사용자가 지정한 함수에 의해 정의 $y = f(\mathbf{x})$

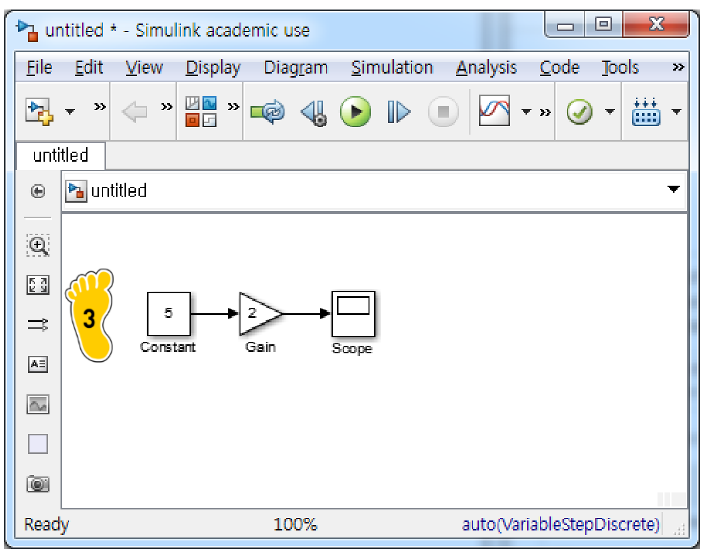
COMMONLY USED BLOCKS



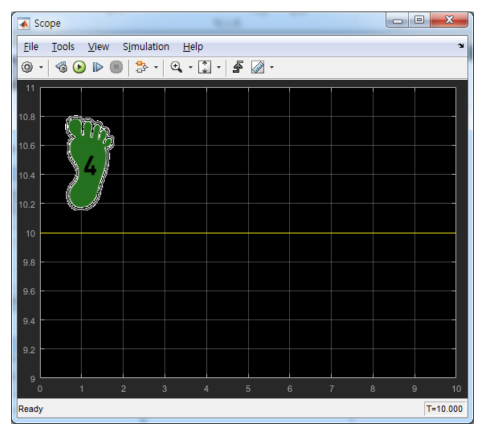








$5 \times 2 = 10$



1 Constant value '5' 입력

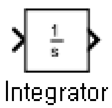
2 Gain '2' 입력

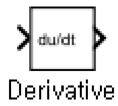
3 Block Diagram 구성

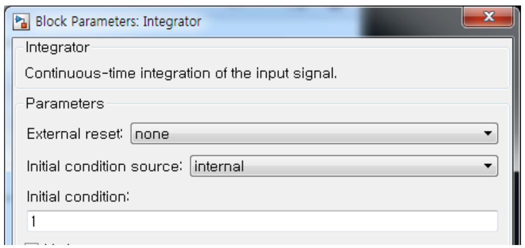
4 Scope window에서 결과 확인

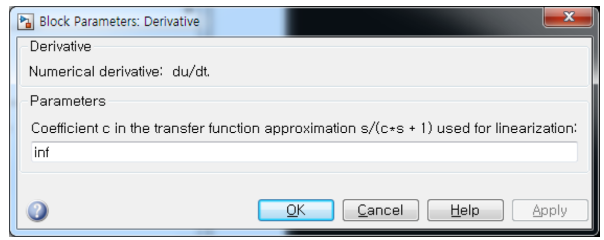
COMMONLY USED BLOCKS

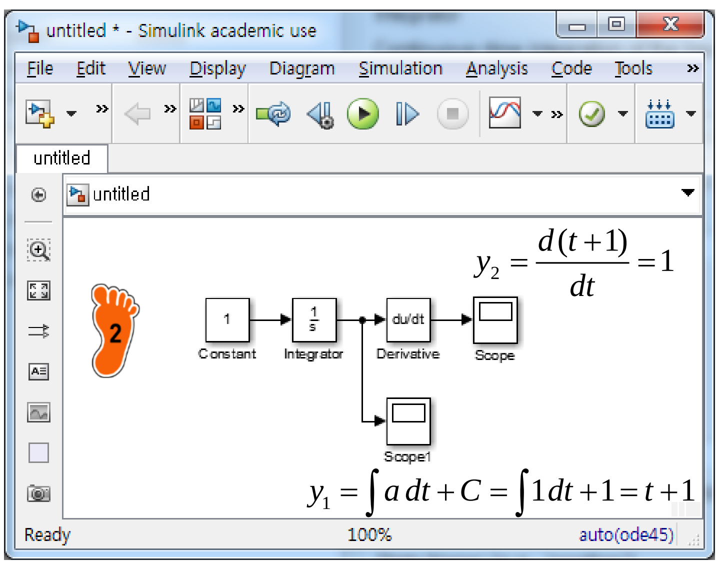
1. Initial Condition '1' 입력
2. Block Diagram 구성
3. Scope window에서 결과 확인





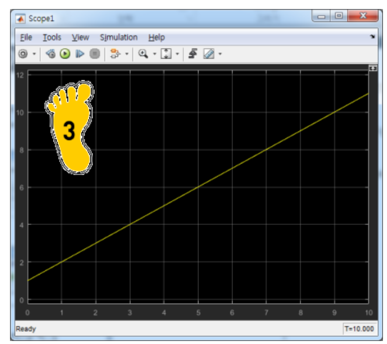




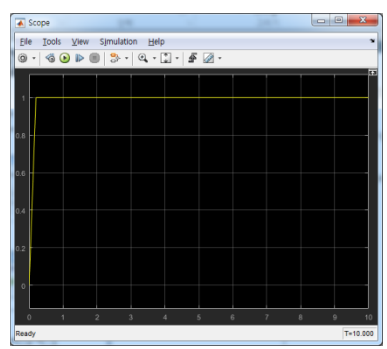


$$y_2 = \frac{d(t+1)}{dt} = 1$$

$$y_1 = \int a\,dt + C = \int 1 dt + 1 = t + 1$$

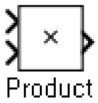


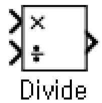
y_1

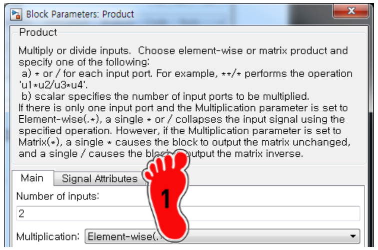


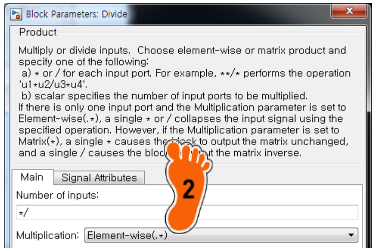
y_2

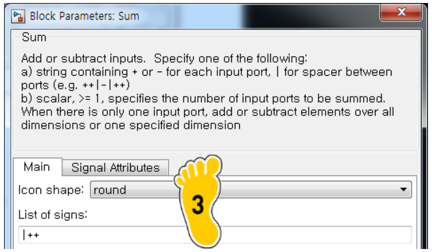
COMMONLY USED BLOCKS







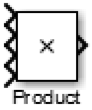


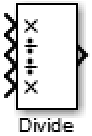


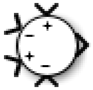
Ex1) Input Port 4

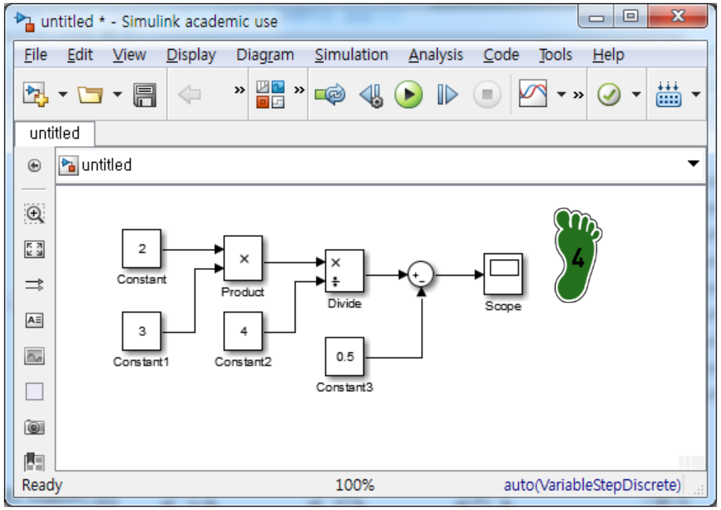
Ex2) Input 연산자 '*//*'

Ex3) Input 연산자 '+-+-'

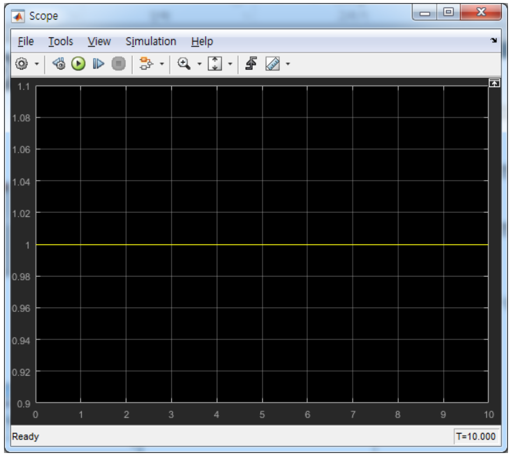






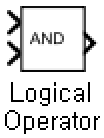


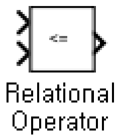
$$y = \{(2\times3)\div4\} - 0.5 = 1$$

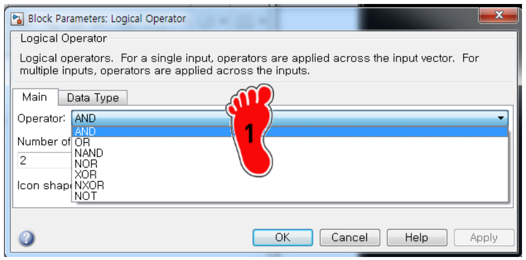


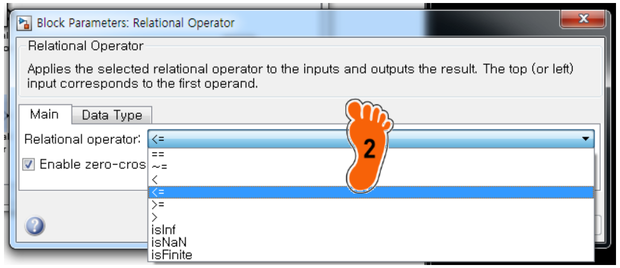
1. Input Port 숫자 (입력 변수의 개수) 입력
2. 각 입력변수에 대한 */입력
3. 각 입력변수에 대한 +-입력
4. Block Diagram 구성 및 결과 확인

COMMONLY USED BLOCKS

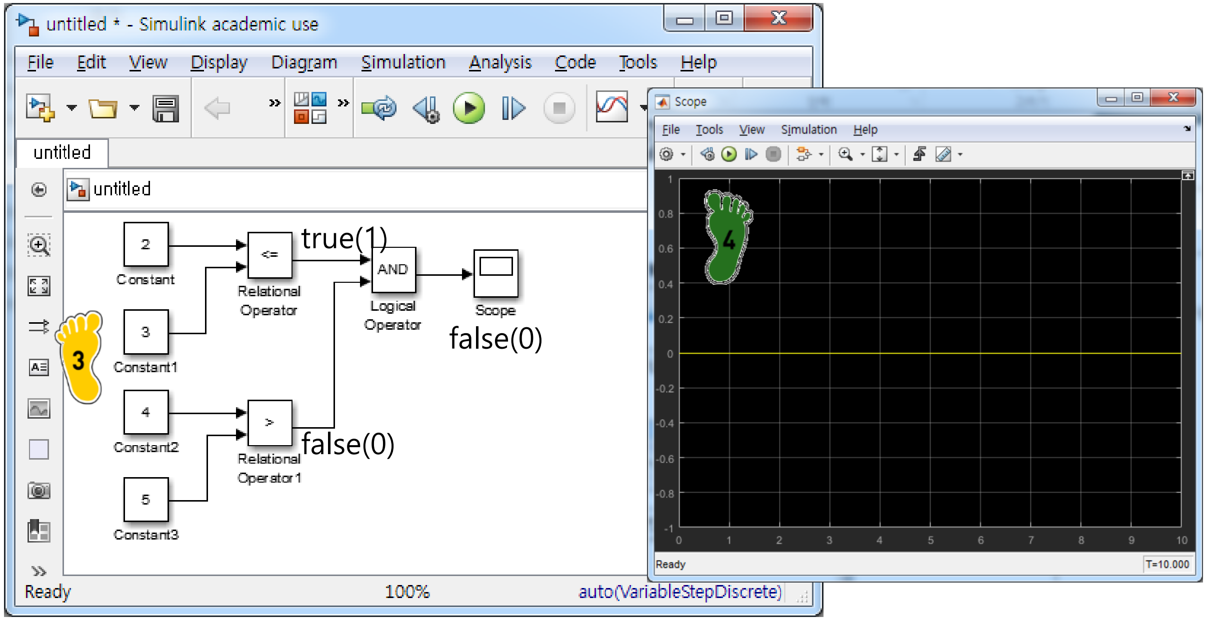








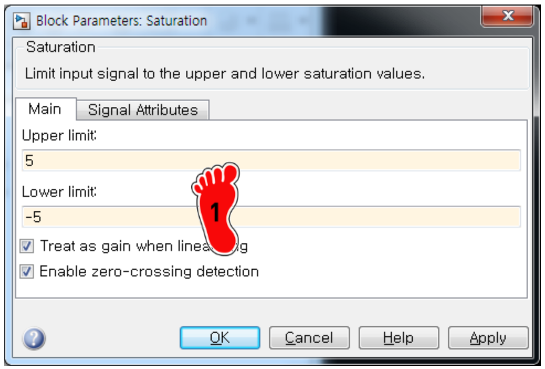
출력값 (true인 경우 : 1, false인 경우 : 0)

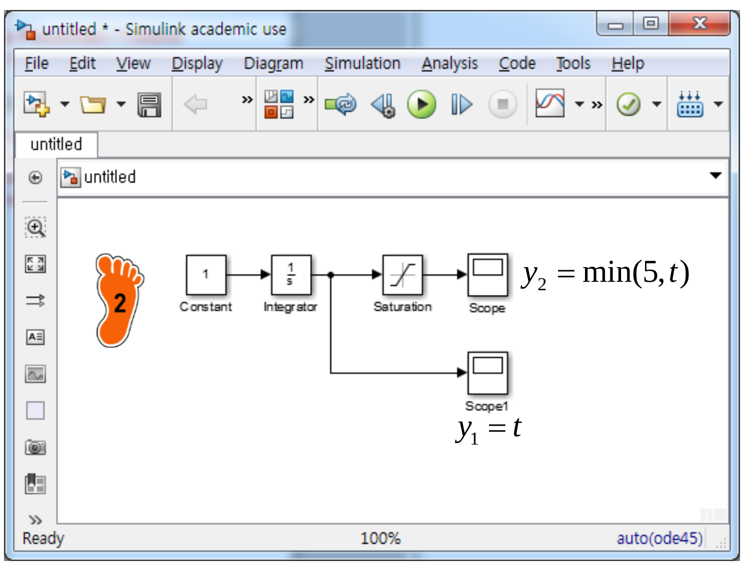


1. 로직(and, or, not….)과 입력 변수 개수 입력
2. 로직(>=, ==, ~=….) 입력
3. Block Diagram 구성
4. 결과 확인

COMMONLY USED BLOCKS

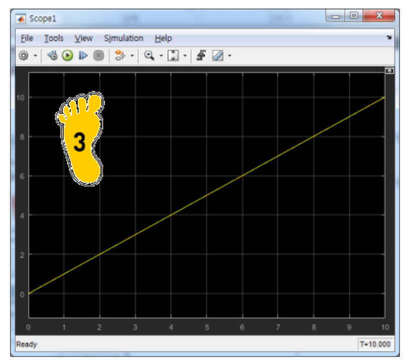
1. Upper/lower limit 입력
2. Block Diagram 구성
3. 결과 확인



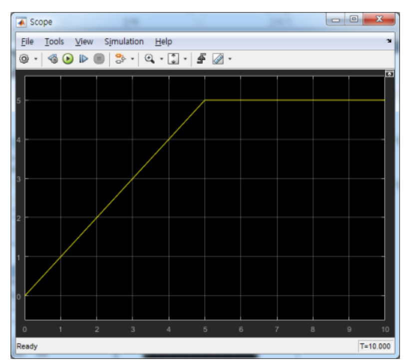


$y_2 = \min(5, t)$

$y_1 = t$



y_1



y_2

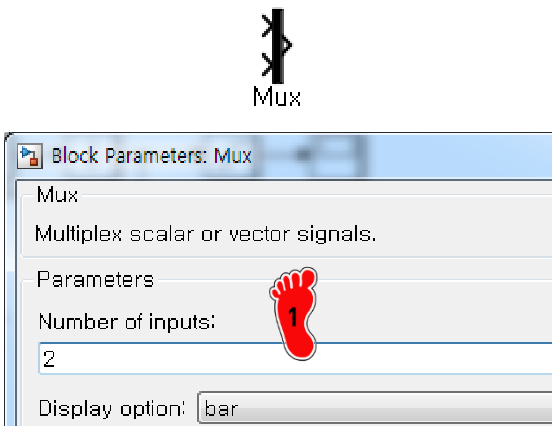
COMMONLY USED BLOCKS

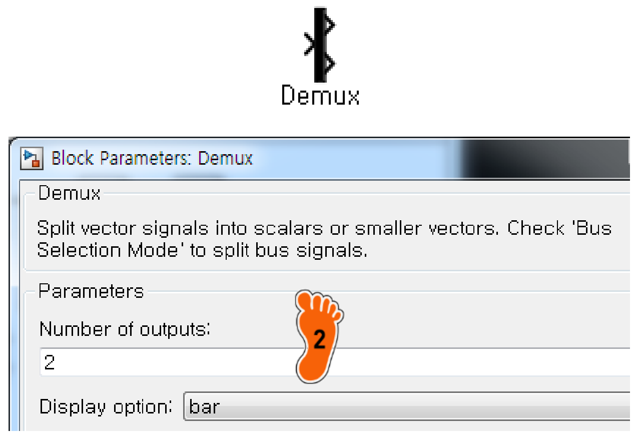
1 입력 신호 수 입력

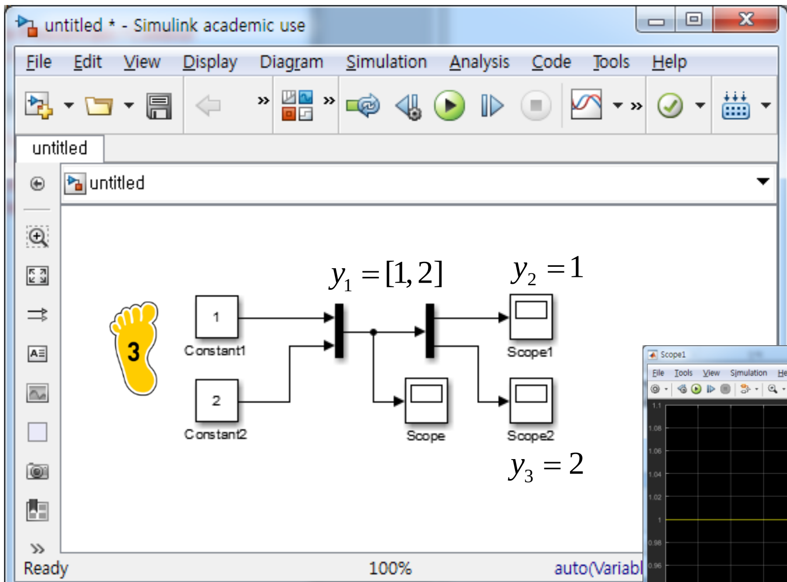
2 출력 신호 수 입력

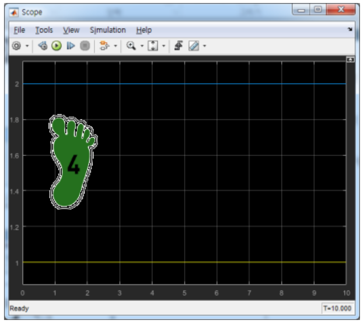
3 Block Diagram 구성

4 결과 확인

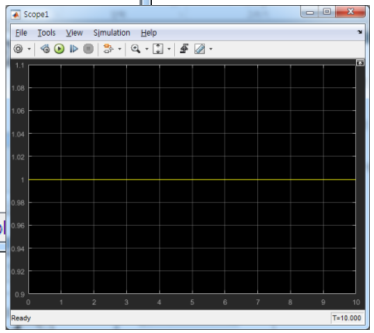




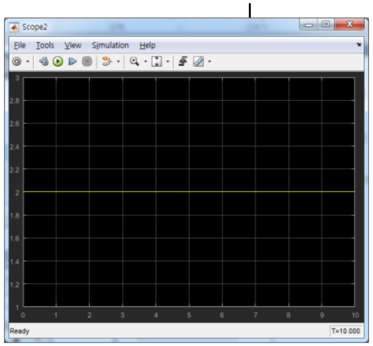




y_1

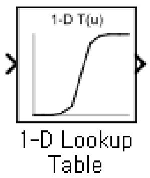


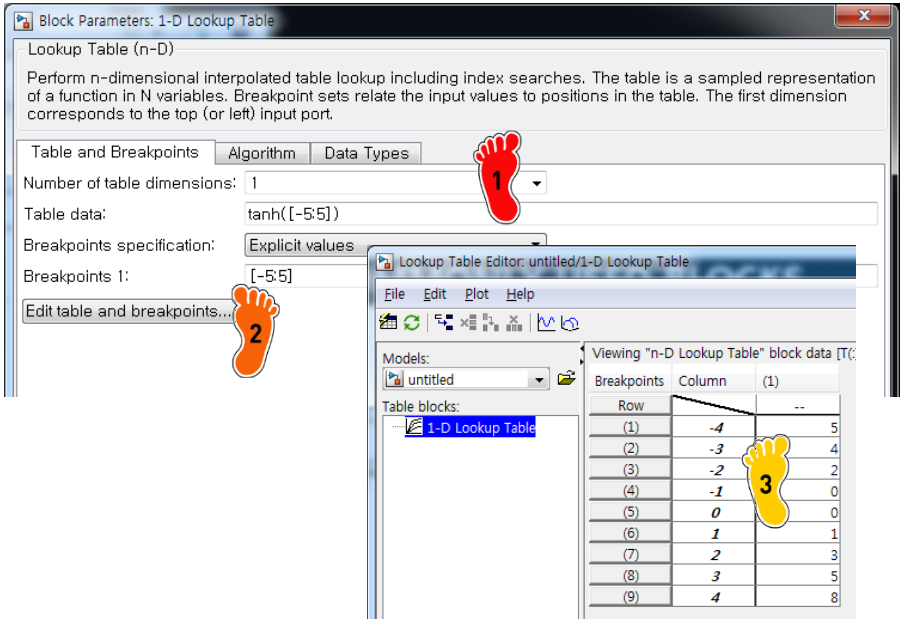
y_2



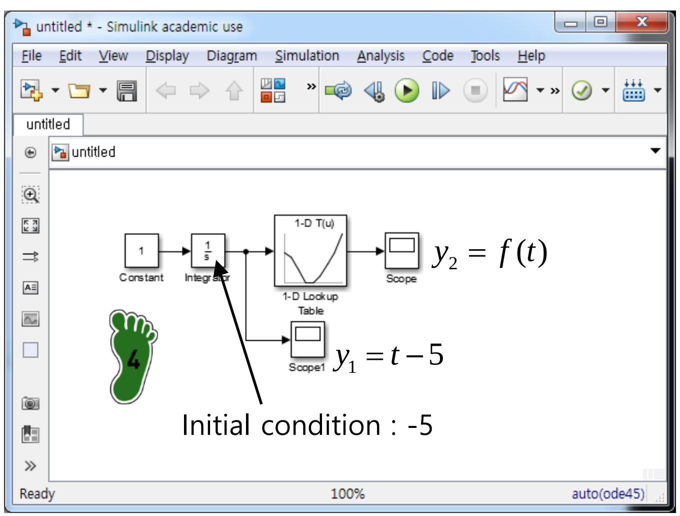
y_3

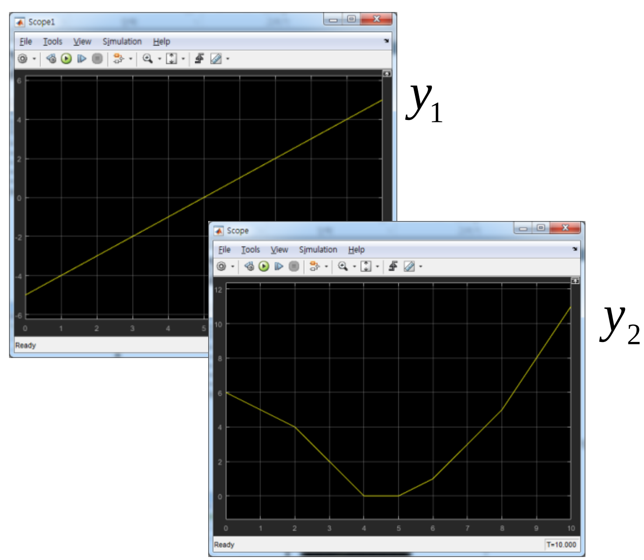
COMMONLY USED BLOCKS





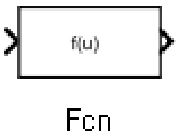
- 1 Table 차원 입력
- 2 'edit table' 클릭
- 3 Table 값 입력
- 4 Block diagram 구성 및 결과 확인

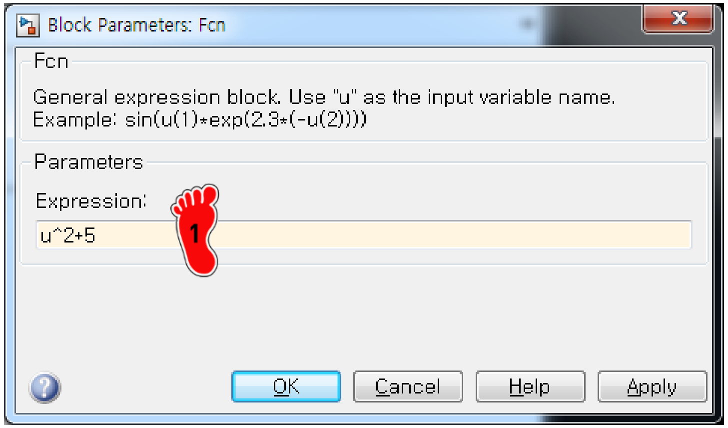


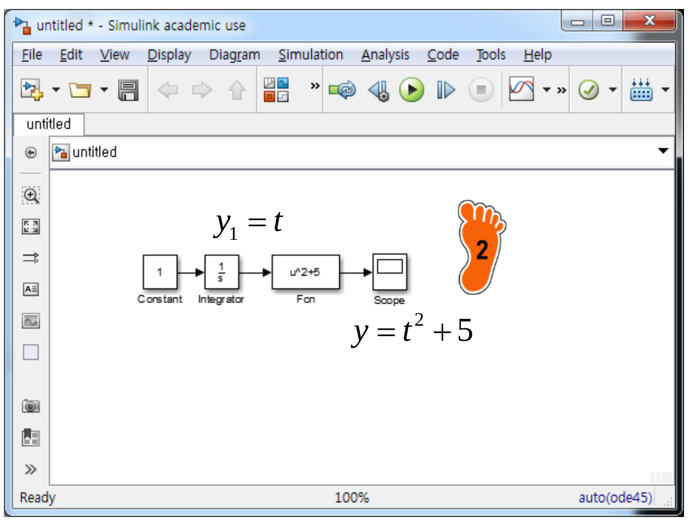


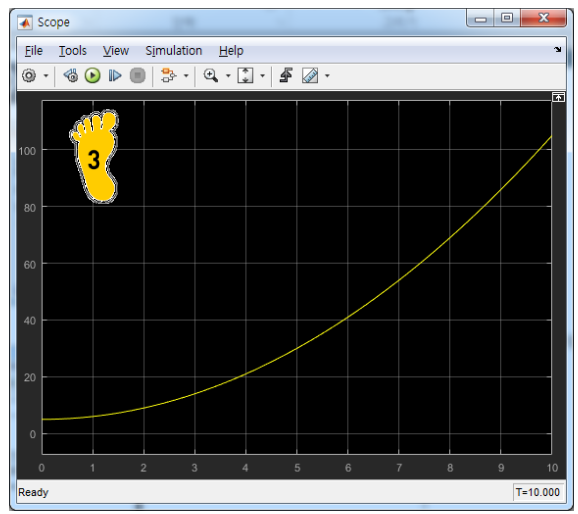
COMMONLY USED BLOCKS

1. 출력 함수 입력
2. Block diagram 구성
3. 결과 확인





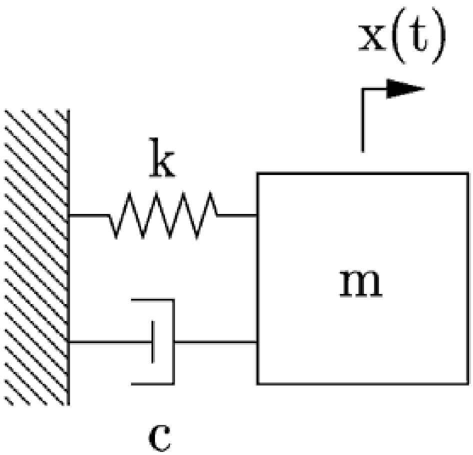




y_2

2ND ORDER SYSTEM

Mass-Spring-Damper로 구성된 2차시스템 (1DOF)

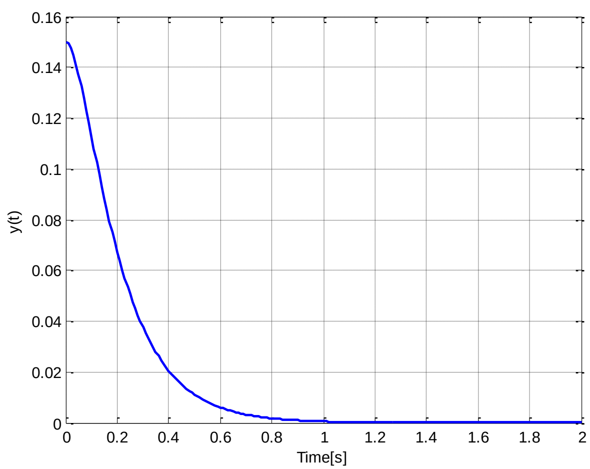


$$m\frac{d^2x}{dt}+c\frac{dx}{dt}+kx=F(t)$$

m=9.072 kg , c=200 kg/s, k=889.96 N/m, x(0)=0.15 m

※ Analytic Sol.

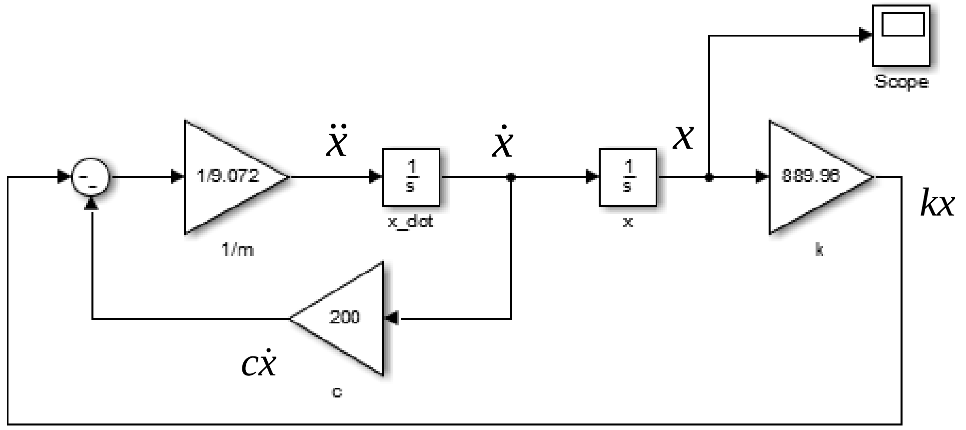
$$x(t)=0.2463e^{-6.190t}-0.0963e^{-15.83t}$$

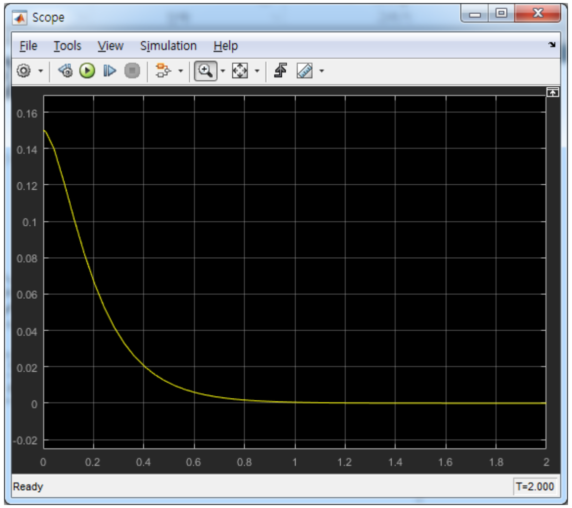


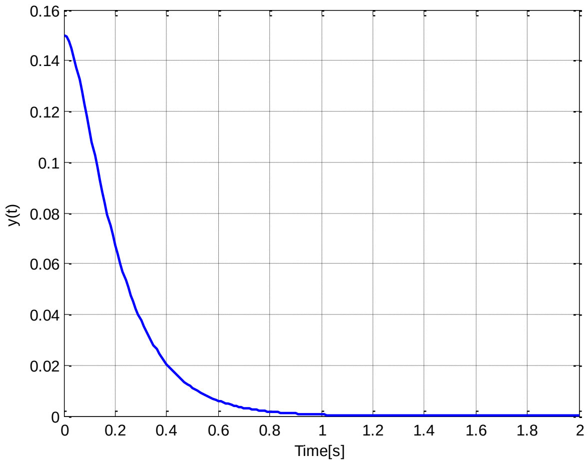
2ND ORDER SYSTEM

$$m\frac{d^2x}{dt}+c\frac{dx}{dt}+kx=0 \quad \Rightarrow \quad \ddot{x}=-\frac{c}{m}\dot{x}-\frac{k}{m}x=\frac{1}{m}(-c\dot{x}-kx)$$

$\ddot{x}$ 에 대하여 정리 후 모델링한 방법

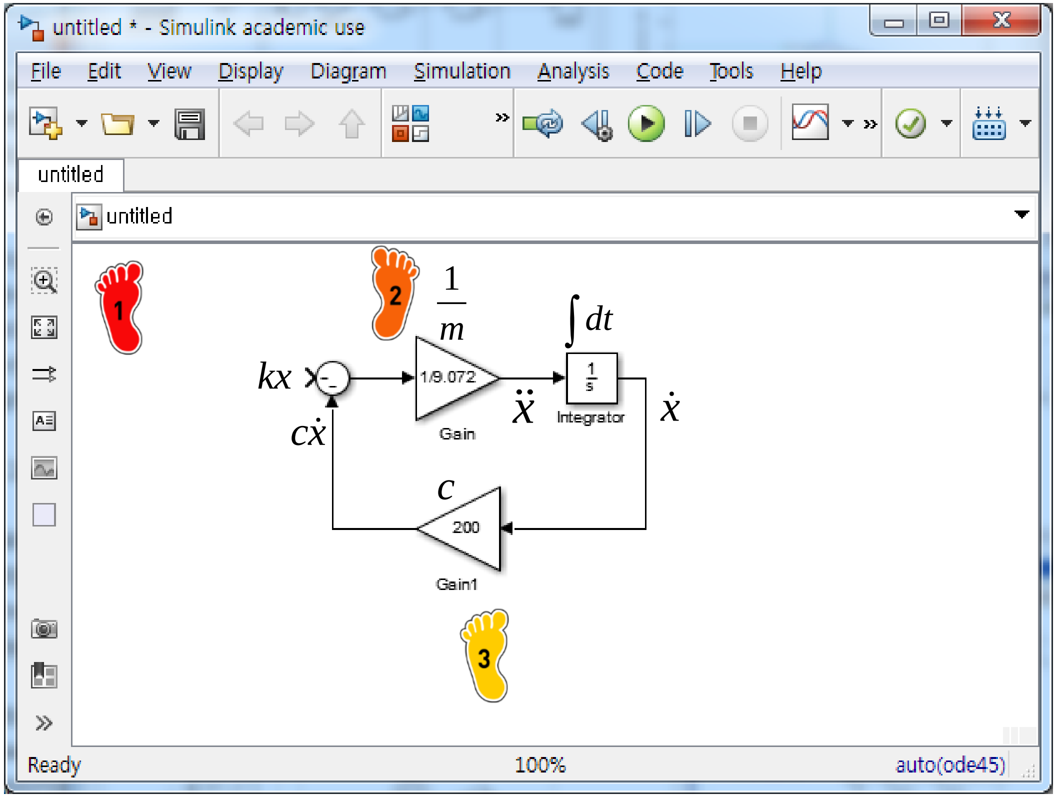






2ND ORDER SYSTEM

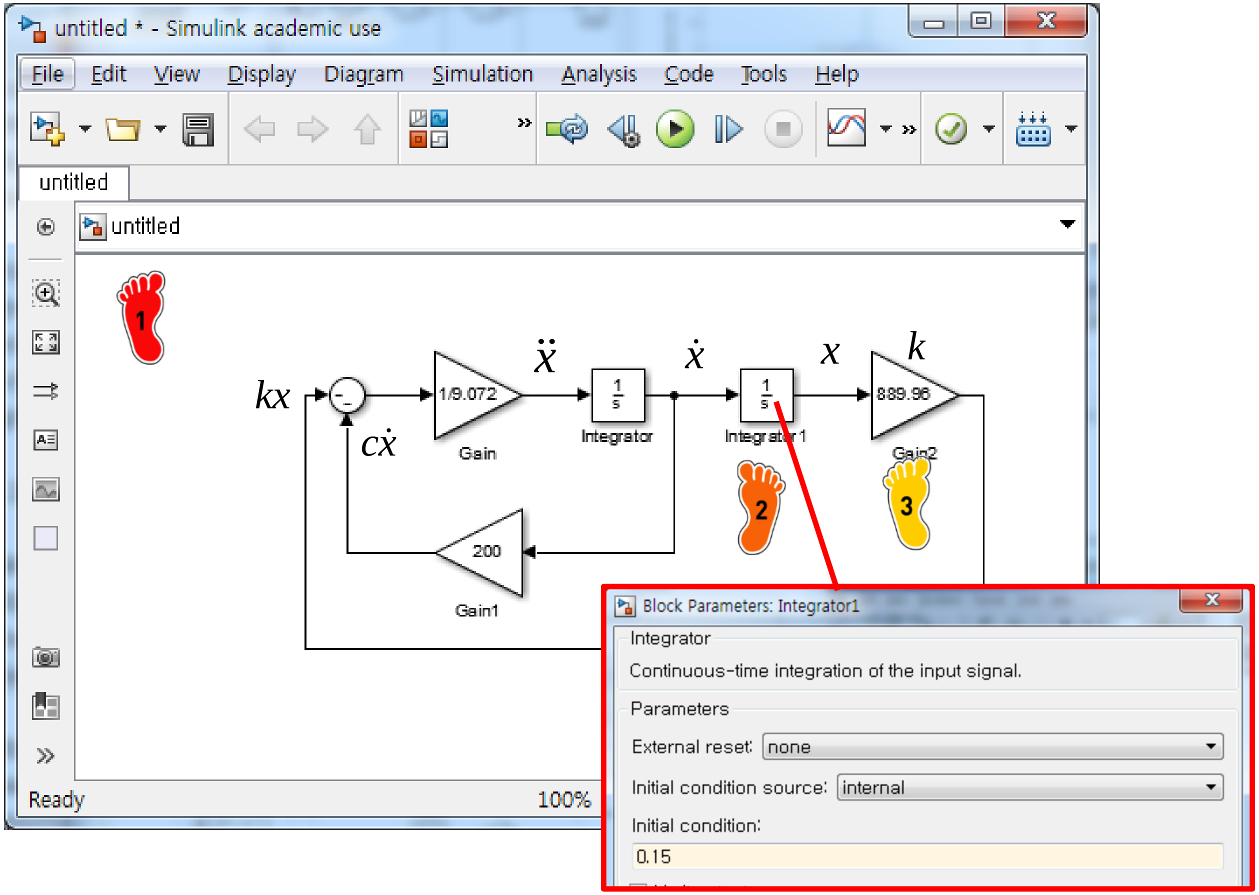
$$\ddot{x} = \boxed{\frac{1}{m}\left(-c\dot{x}\right.}\left.-kx\right)$$



1. Block diagram 구성
2. Mass값 입력(1/m)
3. Damper값 입력

2ND ORDER SYSTEM

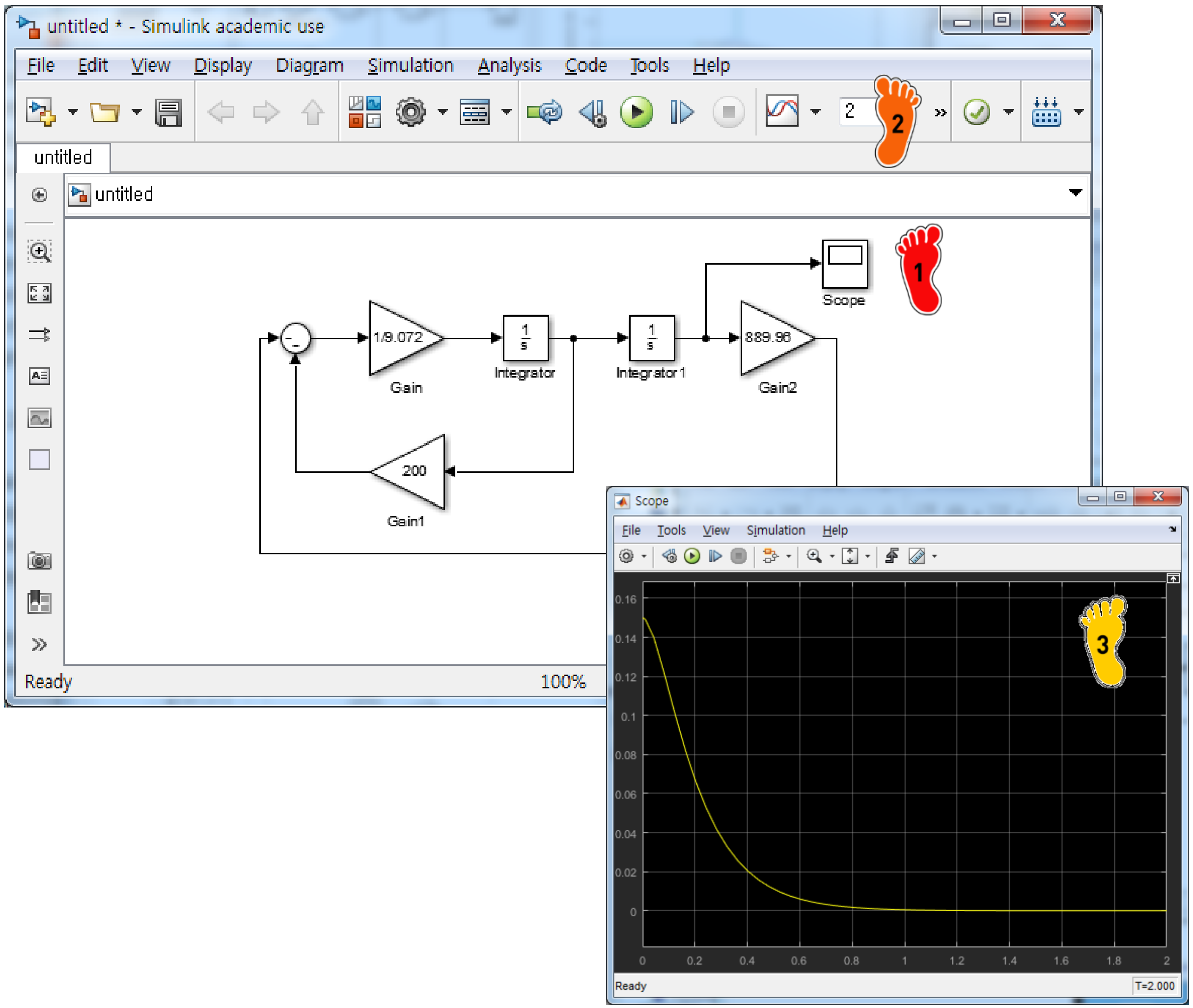
$$\ddot{x} = \frac{1}{m}\left(-c\dot{x} - kx\right)$$



1. Block diagram 구성
2. 초기 변위 입력
3. spring값 입력

2ND ORDER SYSTEM

$$\ddot{x} = -\frac{c}{m}\dot{x} - \frac{k}{m}x$$



1 Scope 구성 (x)

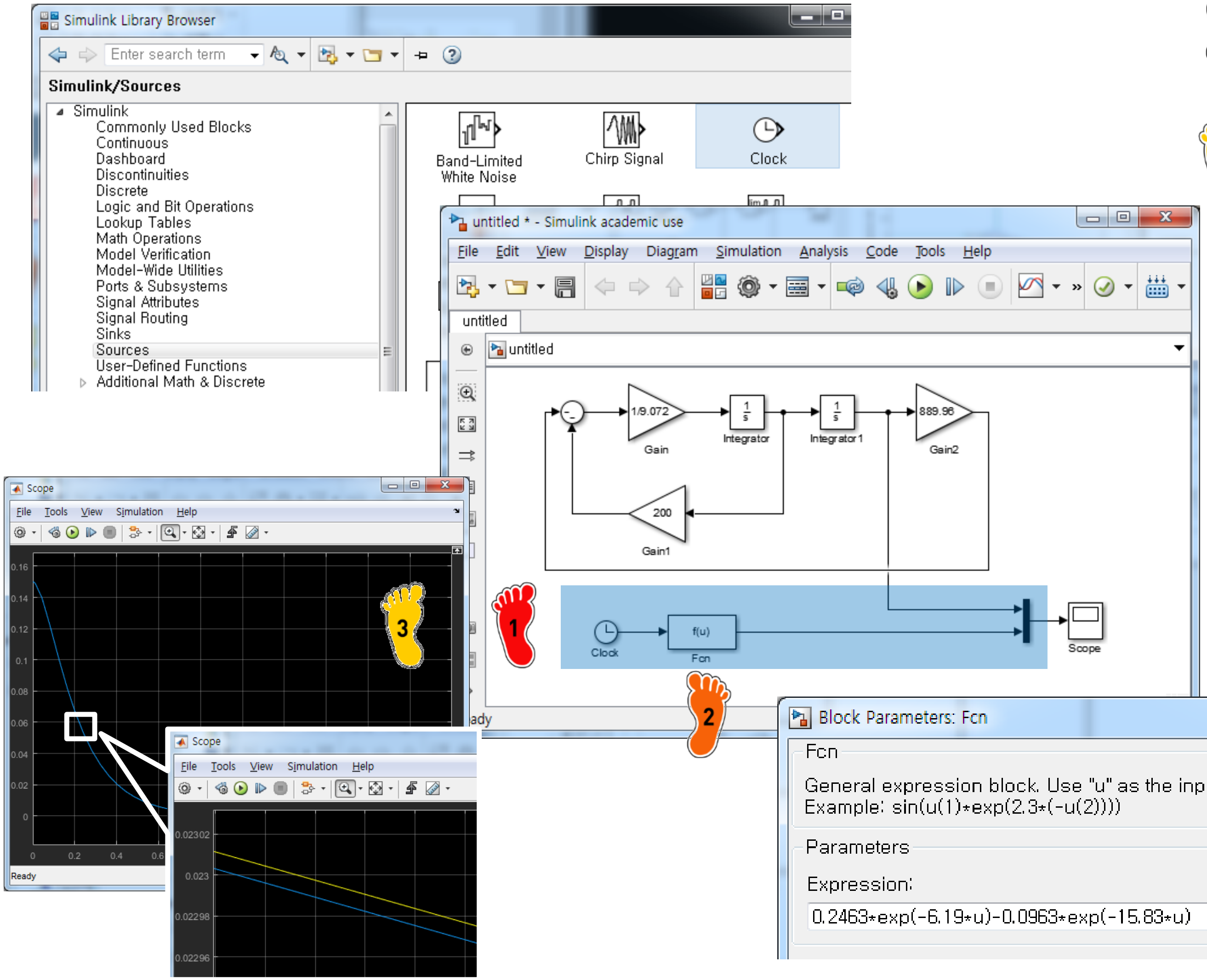
2 시뮬레이션 시간 조정 (2 s)

3 Run & 결과 확인

2^{ND} ORDER SYSTEM

※ Analytic Sol.

$$x(t) = 0.2463e^{-6.190t} - 0.0963e^{-15.83t}$$

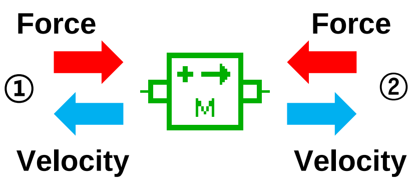


1. Block diagram 구성
2. Fcn block내 analytic sol. 입력
3. Run & 결과 확인

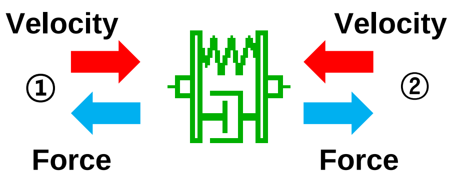
MODEL BASED DESIGN

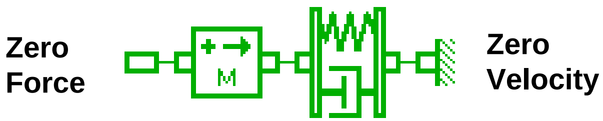
Model Based Design 개념

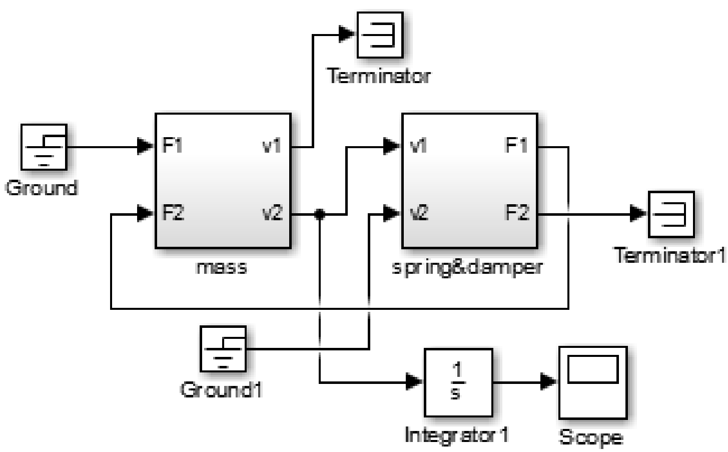
Mass Component

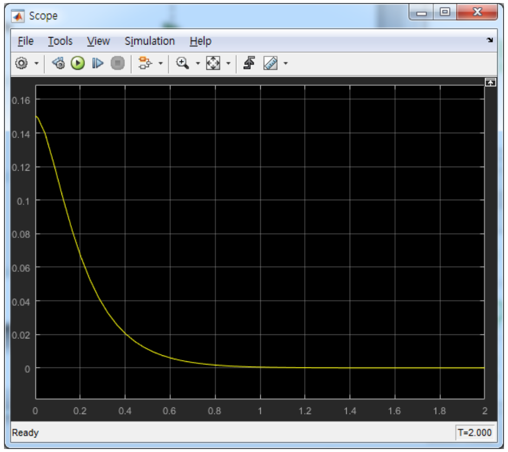


Spring&Damper Component



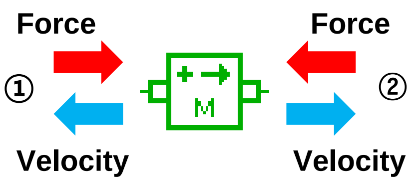






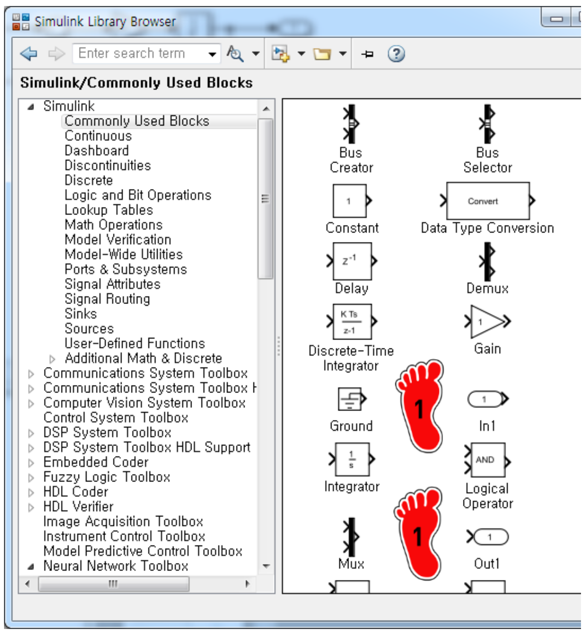
MODEL BASED DESIGN

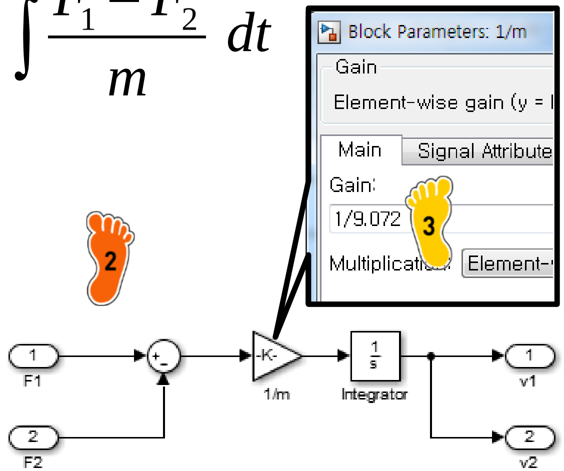
Mass Component

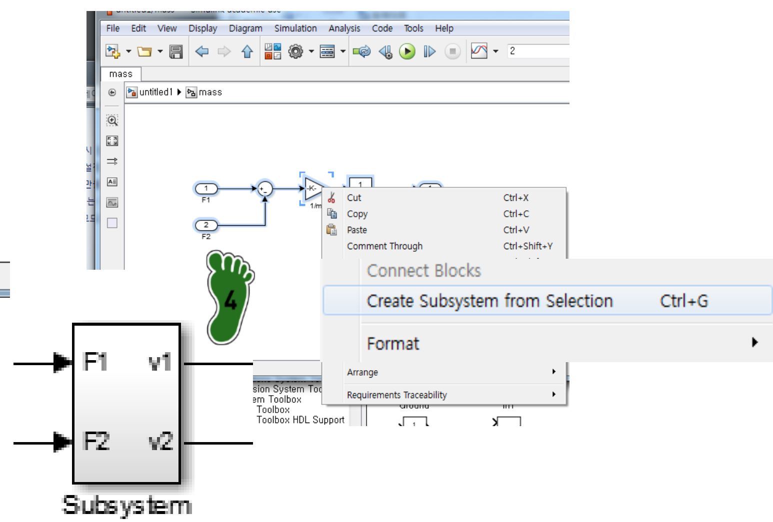


$$F_1 - F_2 = m\ddot{x} \quad \ddot{x} = \frac{(F_1 - F_2)}{m}$$

$$\dot{x} = \int \frac{F_1 - F_2}{m}\, dt$$



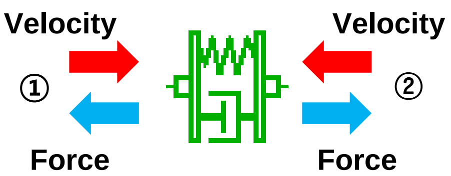




1. Commonly Used Blocks에서 In1, Out1 block 이용
2. Block Diagram 구성
3. Parameter 값 입력
4. 모델 전체 선택 후, 우클릭 Create Subsystem 클릭

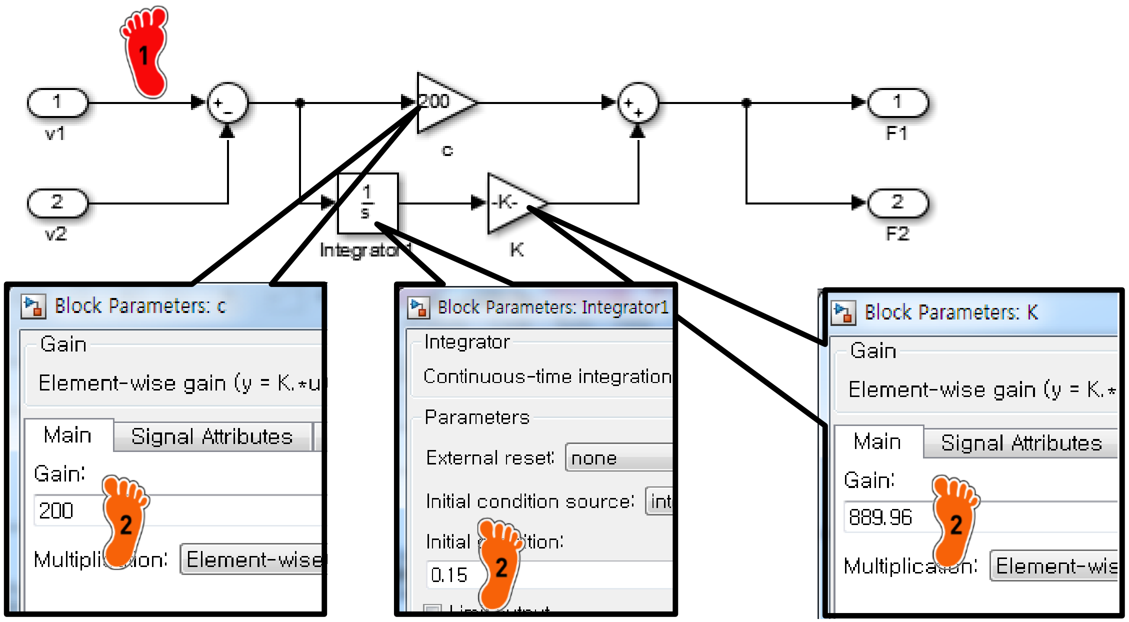
MODEL BASED DESIGN

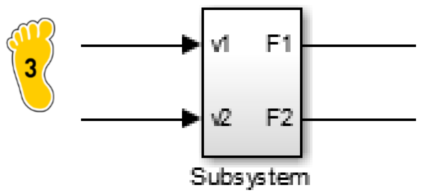
Spring&Damper Component



$$F = c(\dot{x}_1 - \dot{x}_2) + k(x_1 - x_2)$$

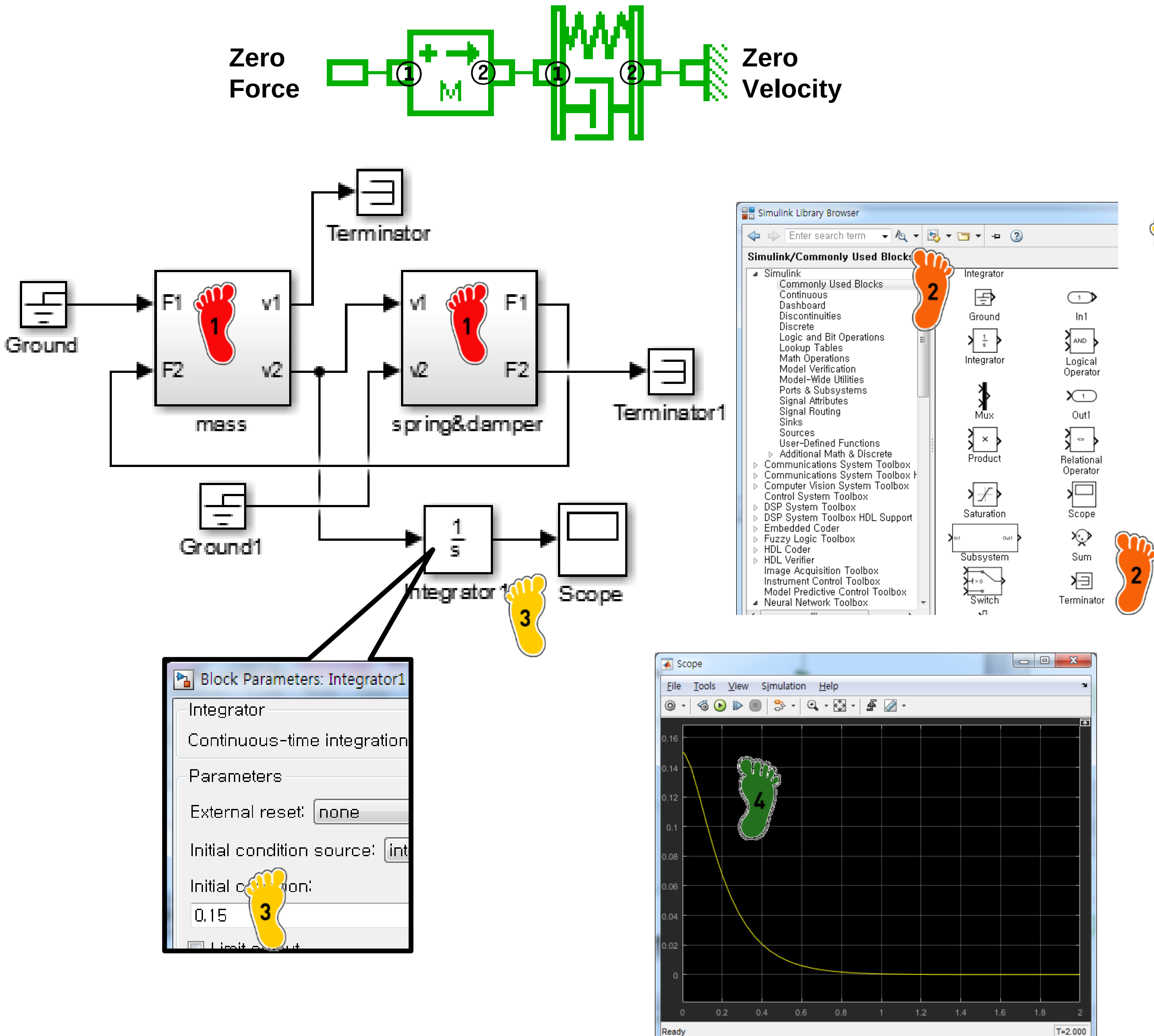
$$F = c(\dot{x}_1 - \dot{x}_2) + k\int(\dot{x}_1 - \dot{x}_2)\,dt$$





1. Block Diagram 구성
2. Parameter 값 입력
3. 모델 전체 선택 후, 우클릭 Create Subsystem 클릭

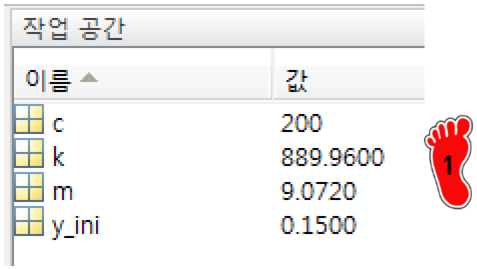
MODEL BASED DESIGN

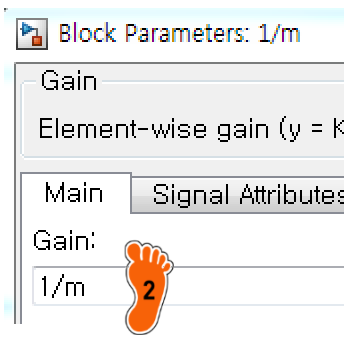


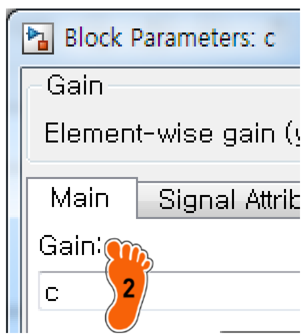
1. 구성한 Mass, Spr.&Damp. system을 이용, In, Out에 맞게 포트 연결
2. Mass 1번, Spr.&Damp. 2번에 Ground, Terminal 연결
3. Mass의 v1 or v2를 이용해서 변위 출력
4. Run 및 결과 확인

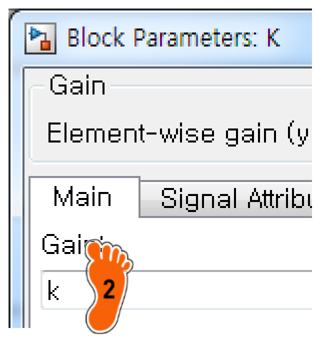
MODEL BASED DESIGN

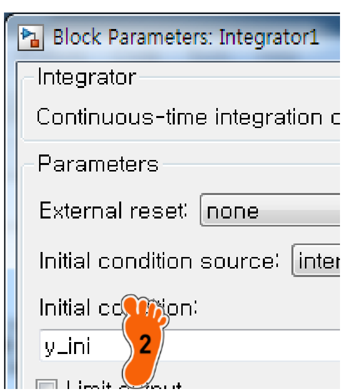
MATLAB WorkSpace에 Parameter 설정

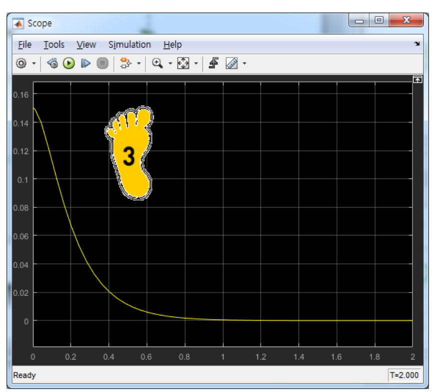












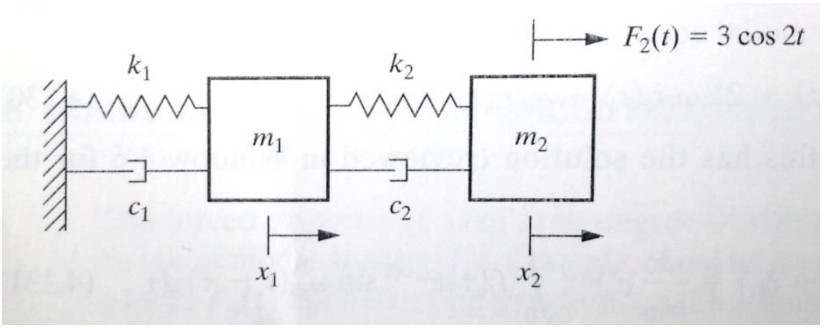
1 Command Window에 Parameter값 입력

2 모델 Parameter값에 설정한 변수명 입력

3 Run 및 결과 확인

CASE STUDY

2-DOF system



$$m_1 = 9\,\text{kg}, m_2 = 1\,\text{kg}$$
$$k_1 = 24\,\text{N/m}, k_2 = 3\,\text{N/m}$$
$$c_1 = 2.4\,\text{Ns/m}, c_2 = 0.3\,\text{Ns/m}$$
$$F_2(t) = 3\cos 2t$$

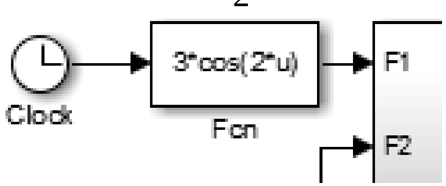
※ Ref. : Daniel J. Inman, "Engineering Vibration",
Prentice Hall International, Inc., pp 296-298, 2001

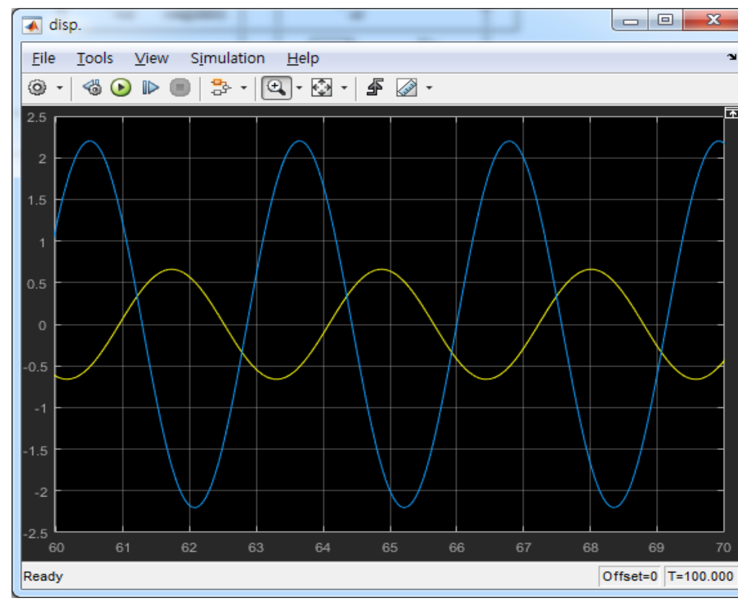
$$m_1\ddot{x}_1 = -k_2(x_1 - x_2) - c_2(\dot{x}_1 - \dot{x}_2) - k_1 x_1 - c_1\dot{x}_1$$
$$m_2\ddot{x}_2 = k_2(x_1 - x_2) + c_2(\dot{x}_1 - \dot{x}_2) + F(t)$$

※ Analytic Solution (Steady State)

$$x_1(t) = 0.2451\cos(2t - 0.1974) - 0.6249\sin 2t$$
$$x_2(t) = 0.7354\cos(2t - 0.1974) + 1.8749\sin 2t$$

F_2





ASSIGNMENT(3RD WEEK: ODE)

Background. Electric circuits where the current is time-variable rather than constant are common. A transient current is established in the right-hand loop of the circuit shown in Fig. 28.11 when the switch is suddenly closed.

Equations that describe the transient behavior of the circuit in Fig. 28.11 are based on Kirchhoff's law, which states that the algebraic sum of the voltage drops around a closed loop is zero (recall Sec. 8.3). Thus,

$$L\frac{di}{dt} + Ri + \frac{q}{C} - E(t) = 0 \tag{28.9}$$

where $L(di/dt)$ = voltage drop across the inductor, L = inductance (H), R = resistance (Ω), q = charge on the capacitor (C), C = capacitance (F), $E(t)$ = time-variable voltage source (V), and

$$i = \frac{dq}{dt} \tag{28.10}$$

$$E = E_0 \sin(\omega t)$$

$$L = 2 \text{ H}$$

$$E_0 = 5 \text{ V}$$

$$C = 0.25 \text{ F}$$

$$\omega = 5 \text{ rad/s}$$

$$R = 3\text{Ohm}$$