Solid Mechanics (beam)

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CONTENTS

- **Beam model: 2D**
  - ✓ **Textbook of “Solid Mechanics”**
- **Beam model: 3D**
- **Assignment**

STRUCTURAL MECHANICS MODULE

PHYSICS	DEFAULT NAME	DEPENDENT VARIABLES	GEOMETRY LEVEL POINTS	EDGES	BOUNDARIES	DOMAINS
STRUCTURAL MECHANICS						
Solid Mechanics	solid	$\mathbf{u}$, (p)	√	√	√	√
Shell	shell	$\mathbf{u}$, a_x, a_y, a_z	√	√	√	
Plate	plate	$\mathbf{u}$, a_x, a_y	√		√	√
Beam	beam	$\mathbf{u}$, θ	√	√		
Truss	truss	$\mathbf{u}$	√	√		
Membrane	mem	$\mathbf{u}$	√	√	√	
Thermal Stress	ts	$\mathbf{u}$, (p), T	√	√	√	√
Joule Heating and Thermal Expansion	tem	$\mathbf{u}$, (p), T, V	√	√	√	√
Piezoelectric Devices	pzd	$\mathbf{u}$,V	√	√	√	√
Fluid Flow						
Fluid-Structure Interaction	fsi	$\mathbf{u}_{\text{solid}}$, $\mathbf{u}_{\text{fluid}}$, p	√	√	√	√

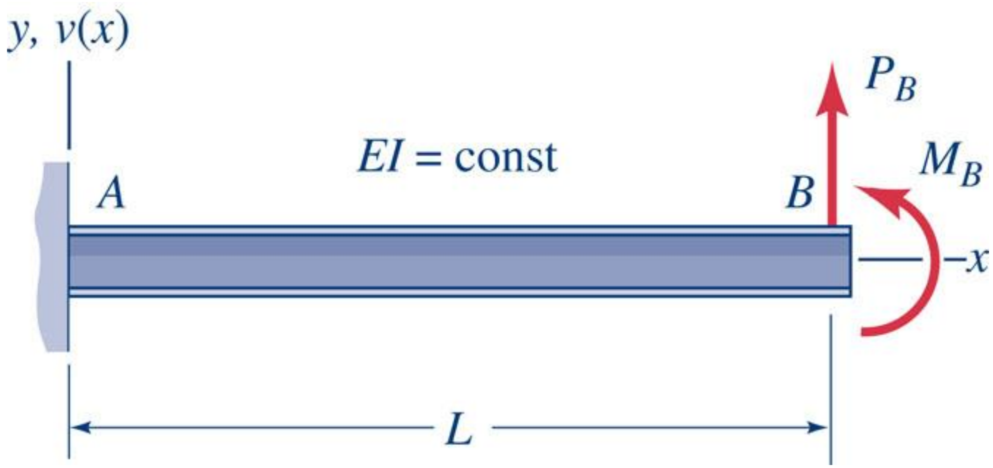
- Beam model: 2D
  - ✓ Textbook of “Solid Mechanics”
- Beam model: 3D
- Assignment

BEAMS

BEAMS

The Beam interface ( ) is intended for the modeling of slender structures (beams) that can be fully described by cross-section properties, such as area and moments of inertia. The Beam interface defines stresses and strains using Hermitian elements and Euler-Bernoulli theory. Beam elements are used to model frame structures, both planar and three-dimensional. It is also suitable for modeling reinforcements of solid and shell structures. The Beam interface includes a library for rectangular, box, circular, pipe, H-profile, U-profile, and T-profile beam sections. Additional features include damping, thermal expansion, and initial stresses and strains. The preset studies for this physics interface are almost the same as for the Solid Mechanics interface, with two exceptions—it does not include the Linear Buckling or Prestressed study types.

EXAMPLE 7.1 (PP.471)



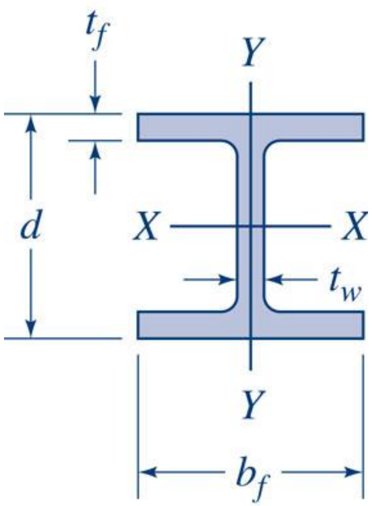
- Geometry
  - $L = 10$ ft
  - $P_B = 4.5$ kips
  - $M_B = 50$ kips-in
- Material Properties
  - A-36 steel
  - $E = 29 \times 10^3$ ksi
- Element Properties
  - Wide-flange beam
  - W10x30
  - $I = 170$ in^4

kips: kilo pounds
ksi: kilo pounds per square inch

Analytical solution

$$\begin{cases} \delta_B = \dfrac{1}{EI}\left[M_B\left(\dfrac{L^2}{2}\right) + P_B\left(\dfrac{L^3}{3}\right)\right] = 0.0730 + 0.5258 = 0.5988 \text{ in} \\ \theta_B = \dfrac{1}{EI}\left[M_B\left(L\right) + P_B\left(\dfrac{L^2}{2}\right)\right] = 0.00122 + 0.00657 = 0.00779 \text{ rad} \end{cases}$$

WIDE-FLANGE BEAM

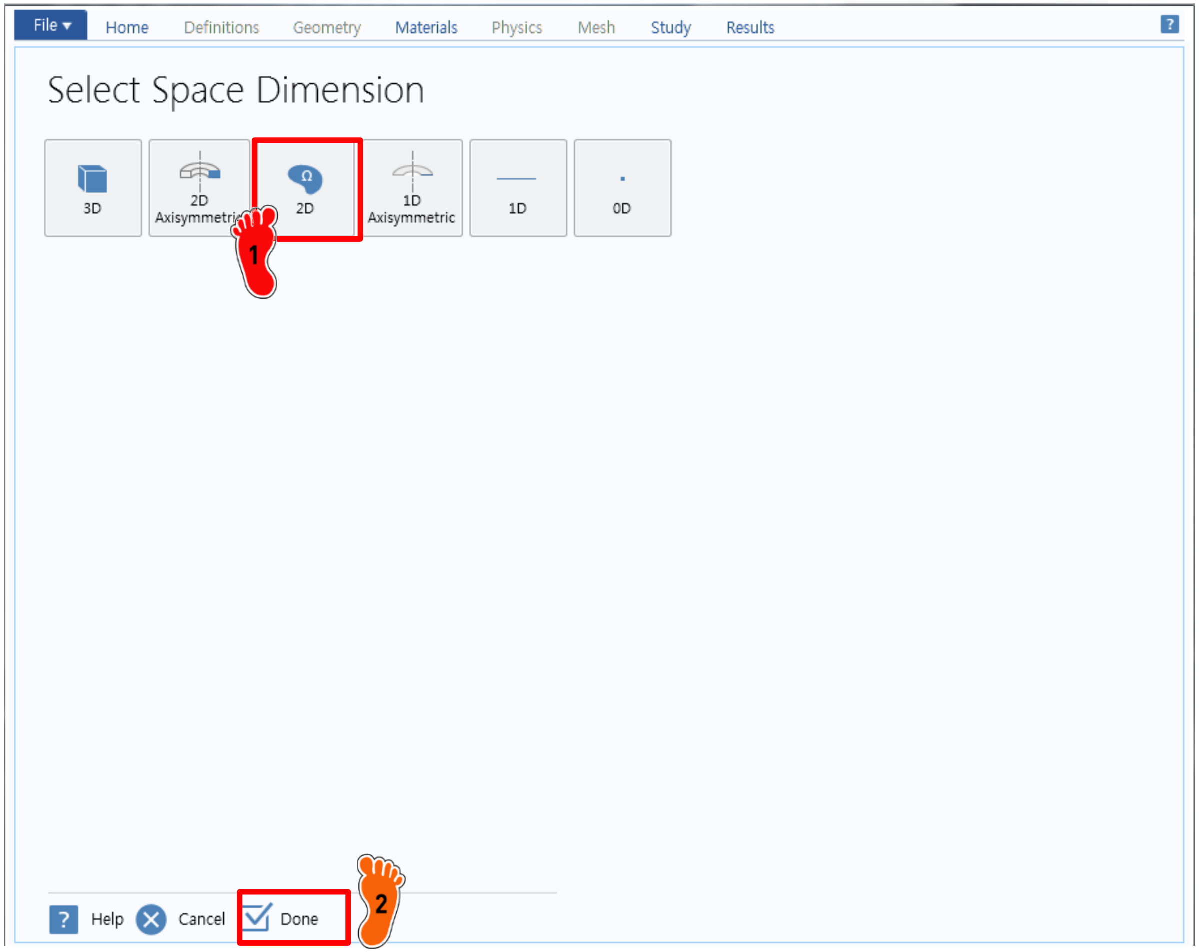


D.1. Properties of Steel Wide-Flange (W) Shapes (U.S. Customary Units)

Designation*	Area A in^2	Depth d in.	Flange Width b_f in.	Flange Thickness t_f in.	Web Thickness t_w in.	Elastic Properties, Axis $X-X$: I_x in^4	Axis $X-X$: S_x in^3	Axis $X-X$: r_x in.	Axis $Y-Y$: I_y in^4	Axis $Y-Y$: S_y in^3	Axis $Y-Y$: r_y in.	Plastic Modulus Z_x in^3
W36×230	67.6	35.90	16.470	1.260	0.760	15000	837	14.9	940	114	3.73	943
×150	44.2	35.85	11.975	0.940	0.625	9040	504	14.3	270	45.1	2.47	581
W33×201	59.1	33.68	15.745	1.150	0.715	11500	684	14.0	749	95.2	3.56	772
×130	38.3	33.09	11.510	0.855	0.580	6710	406	13.2	218	37.9	2.39	467
W30×173	50.8	30.44	14.985	1.065	0.655	8200	539	12.7	598	79.8	3.43	605
× 90	26.4	29.53	10.400	0.610	0.470	3620	245	11.7	115	22.1	2.09	283
W27×146	42.9	27.38	13.965	0.975	0.605	5630	411	11.4	443	63.5	3.21	461
× 84	24.8	26.71	9.960	0.640	0.460	2850	213	10.7	106	21.2	2.07	244
W24× 94	27.7	24.31	9.065	0.875	0.515	2700	222	9.87	109	24.0	1.98	254
× 62	18.2	23.74	7.040	0.590	0.430	1550	131	9.23	34.5	9.80	1.38	153
W21×101	29.8	21.36	12.290	0.800	0.500	2420	227	9.02	248	40.3	2.89	253
× 73	21.5	21.24	8.295	0.740	0.455	1600	151	8.64	70.6	17.0	1.81	172
× 50	14.7	20.83	6.530	0.535	0.380	984	94.5	8.18	24.9	7.64	1.30	110
W18×130	38.2	19.25	11.160	1.200	0.670	2460	256	8.03	278	49.9	2.70	291
× 76	22.3	18.21	11.035	0.680	0.425	1330	146	7.73	152	27.6	2.61	163
W18× 60	17.6	18.24	7.555	0.695	0.415	984	108	7.47	50.1	13.3	1.69	123
× 50	14.7	17.99	7.495	0.570	0.355	800	88.9	7.38	40.1	10.7	1.65	101
W16×100	29.4	16.97	10.425	0.985	0.585	1490	175	7.10	186	35.7	2.51	198
× 67	19.7	16.33	10.235	0.665	0.395	954	117	6.96	119	23.2	2.46	130
× 50	14.7	16.26	7.070	0.630	0.380	659	81.0	6.68	37.2	10.5	1.59	92.0
× 40	11.8	16.01	6.995	0.505	0.305	518	64.7	6.63	28.9	8.25	1.57	72.9
W14×176	51.8	15.22	15.650	1.310	0.830	2140	281	6.43	838	107	4.02	320
×120	35.3	14.48	14.670	0.940	0.590	1380	190	6.24	495	65.7	3.74	212
× 82	24.1	14.31	10.130	0.855	0.510	882	123	6.05	148	29.3	2.48	139
× 53	15.6	13.92	8.060	0.660	0.370	541	77.8	5.89	57.7	14.3	1.92	87.1
× 26	7.69	13.91	5.025	0.420	0.255	245	35.3	5.65	8.91	3.54	1.08	40.2
W12×152	44.7	13.71	12.480	1.400	0.870	1430	209	5.66	454	72.8	3.19	243
× 96	28.2	12.71	12.160	0.900	0.550	833	131	5.44	270	44.4	3.09	147
× 65	19.1	12.12	12.000	0.605	0.390	533	87.9	5.28	174	29.1	3.02	96.8
× 50	14.7	12.19	8.080	0.640	0.370	394	64.7	5.18	56.3	13.9	1.96	72.4
× 35	10.3	12.50	6.560	0.520	0.300	285	45.6	5.25	24.5	7.47	1.54	51.2
× 22	6.48	12.31	4.030	0.425	0.260	156	25.4	4.91	4.66	2.31	0.847	29.3
W10× 60	17.6	10.22	10.080	0.680	0.420	341	66.7	4.39	116	23.0	2.57	74.6
× 45	13.3	10.10	8.020	0.620	0.350	248	49.1	4.32	53.4	13.3	2.01	54.9
× 30	8.84	10.47	5.810	0.510	0.300	170	32.4	4.38	16.7	5.75	1.37	36.6
× 12	3.54	9.87	3.960	0.210	0.190	53.8	10.9	3.90	2.18	1.10	0.785	12.6
W 8× 48	14.1	8.50	8.110	0.685	0.400	184	43.3	3.61	60.9	15.0	2.08	49.0
× 40	11.7	8.25	8.070	0.560	0.360	146	35.5	3.53	49.1	12.2	2.04	39.8
× 35	10.3	8.12	8.020	0.495	0.310	127	31.2	3.51	42.6	10.6	2.03	34.7
× 21	6.16	8.28	5.270	0.400	0.250	75.3	18.2	3.49	9.77	3.71	1.26	20.4
× 15	4.44	8.11	4.015	0.315	0.245	48.0	11.8	3.29	3.41	1.70	0.876	13.6
W 6× 25	7.34	6.38	6.080	0.455	0.320	53.4	16.7	2.70	17.1	5.61	1.52	18.9
× 20	5.87	6.20	6.020	0.365	0.260	41.1	13.4	2.66	13.3	4.41	1.50	14.9
× 16	4.74	6.28	4.030	0.405	0.260	32.1	10.2	2.60	4.43	2.20	0.966	11.7
× 15	4.43	5.99	5.990	0.260	0.230	29.1	9.72	2.56	9.32	3.11	1.46	10.8

*W(nominal depth in inches) × (weight in pounds per foot)

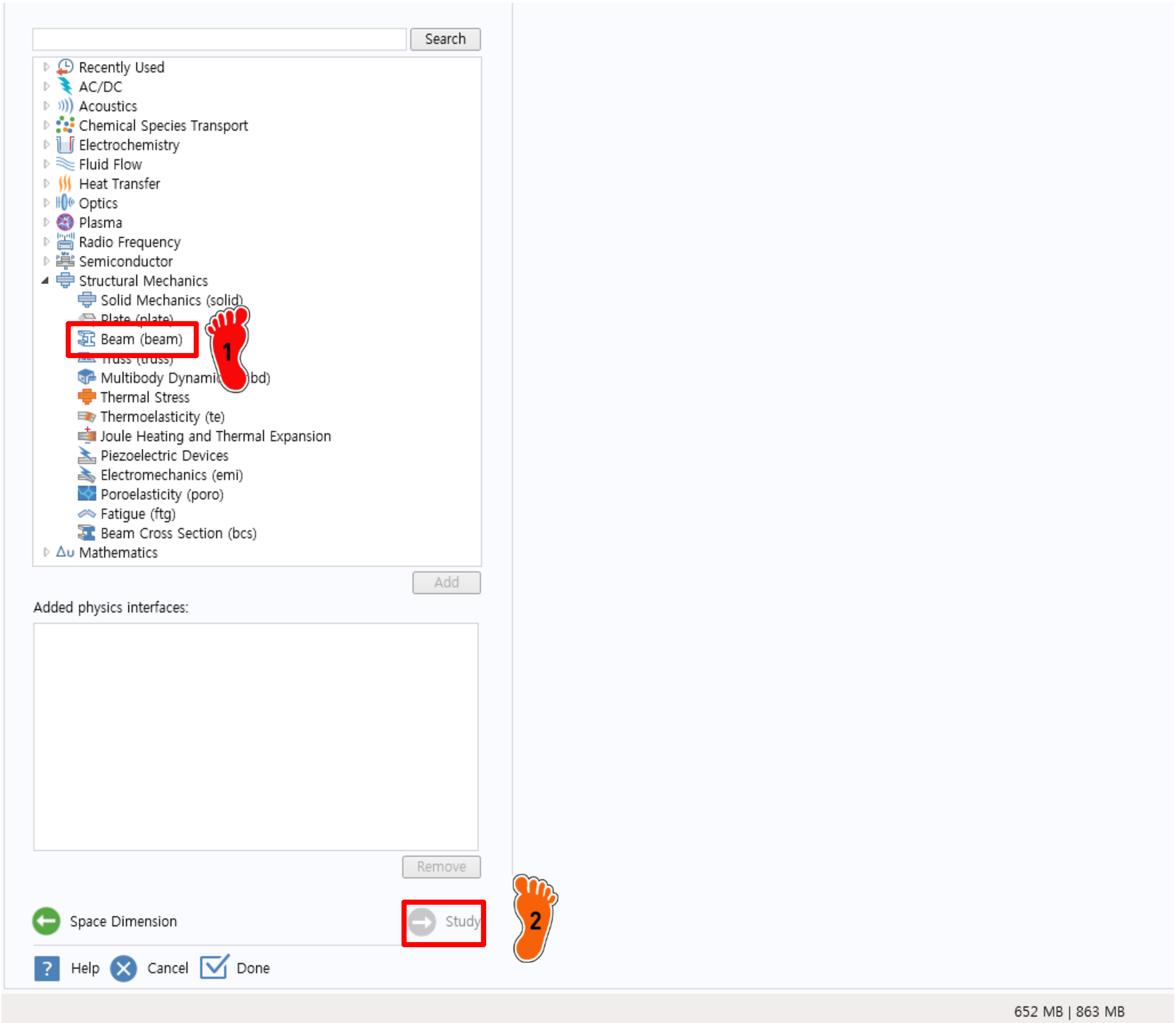
DIMENSION SELECTION



1 2D 선택

2 Done 클릭

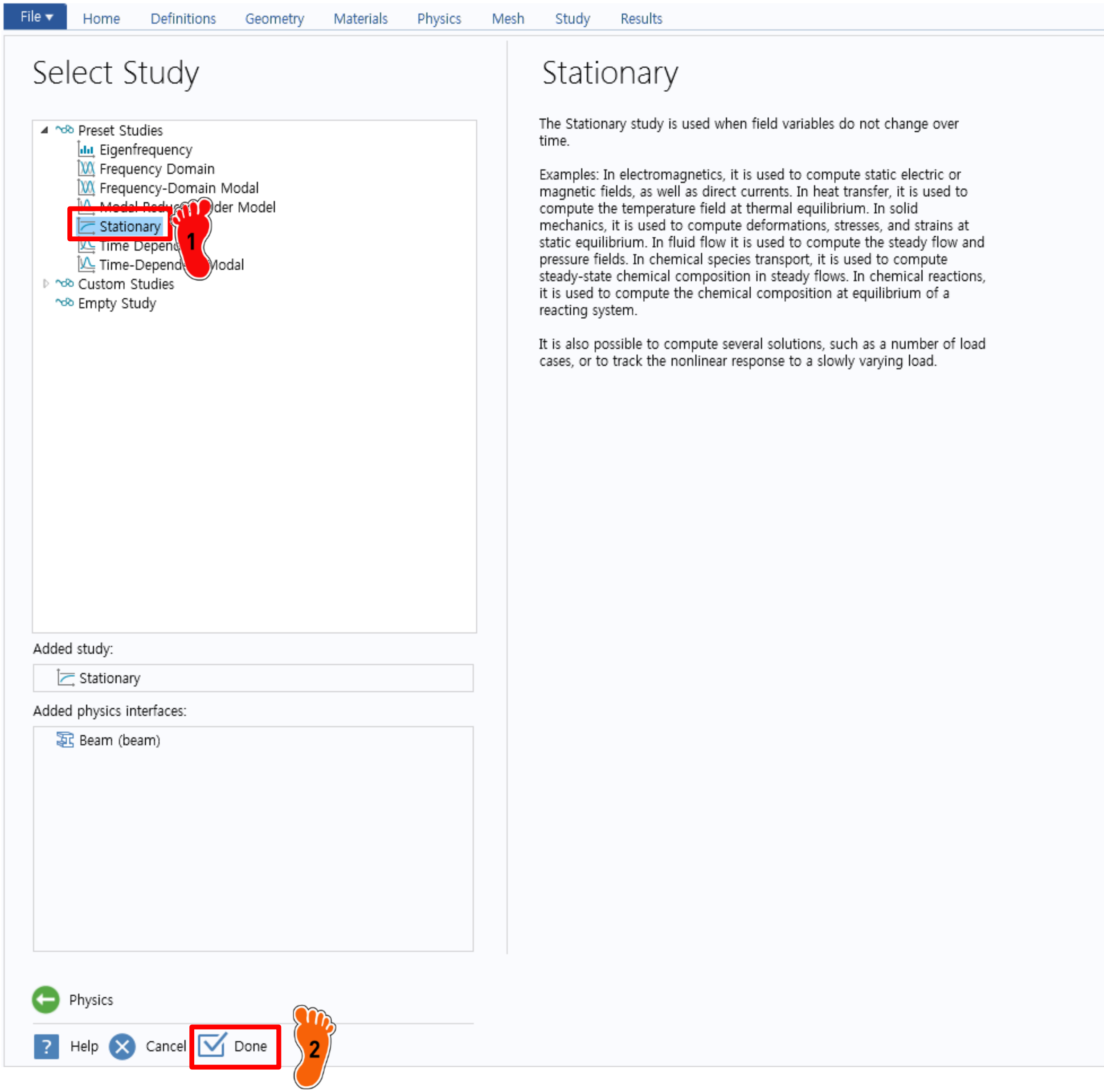
PHYSICS SELECTION



1 Structural Mechanics 의 Beam 추가

2 Study 클릭

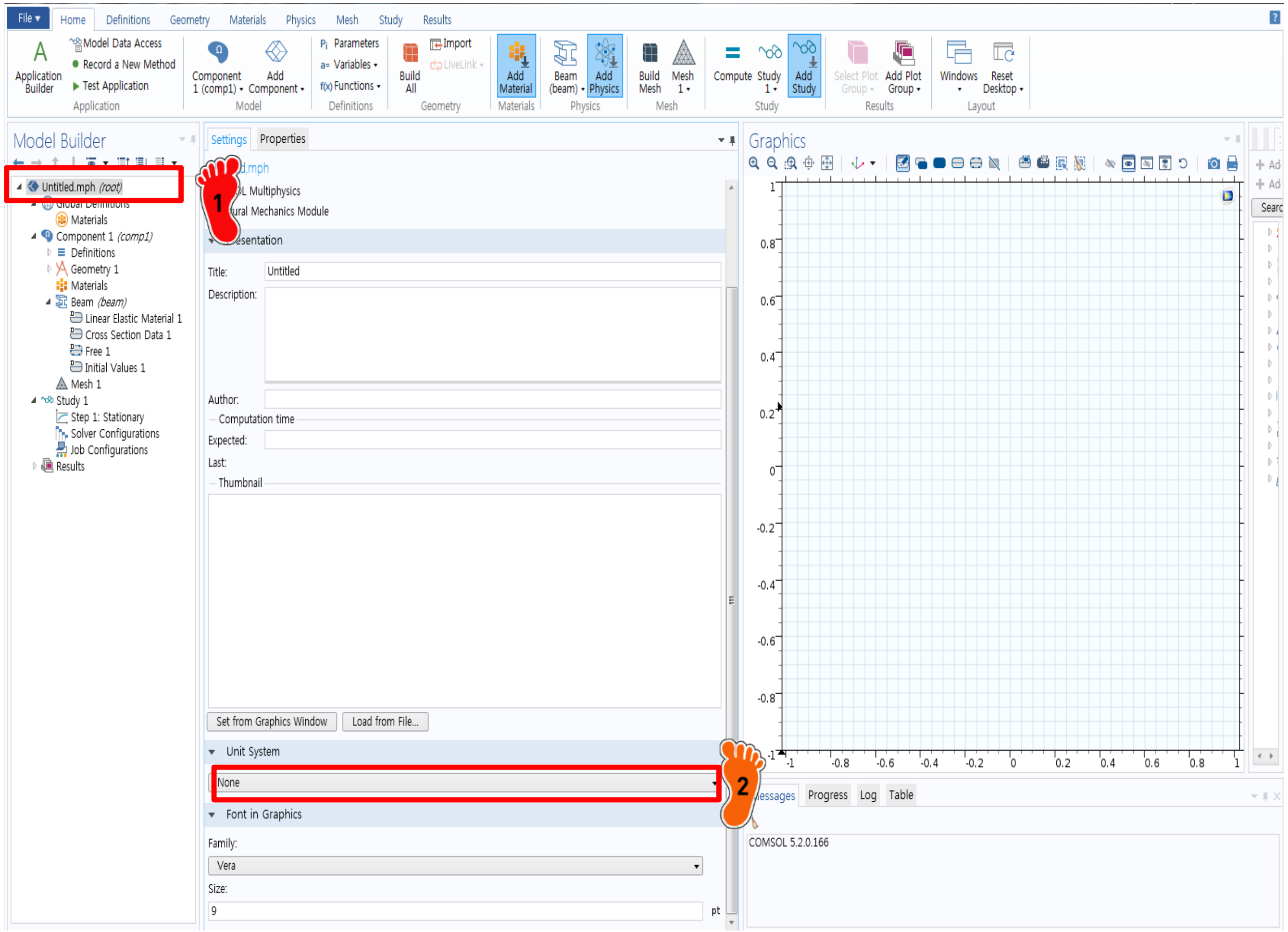
STUDY TYPE SELECTION



1 Stationary 선택

2 Done 클릭

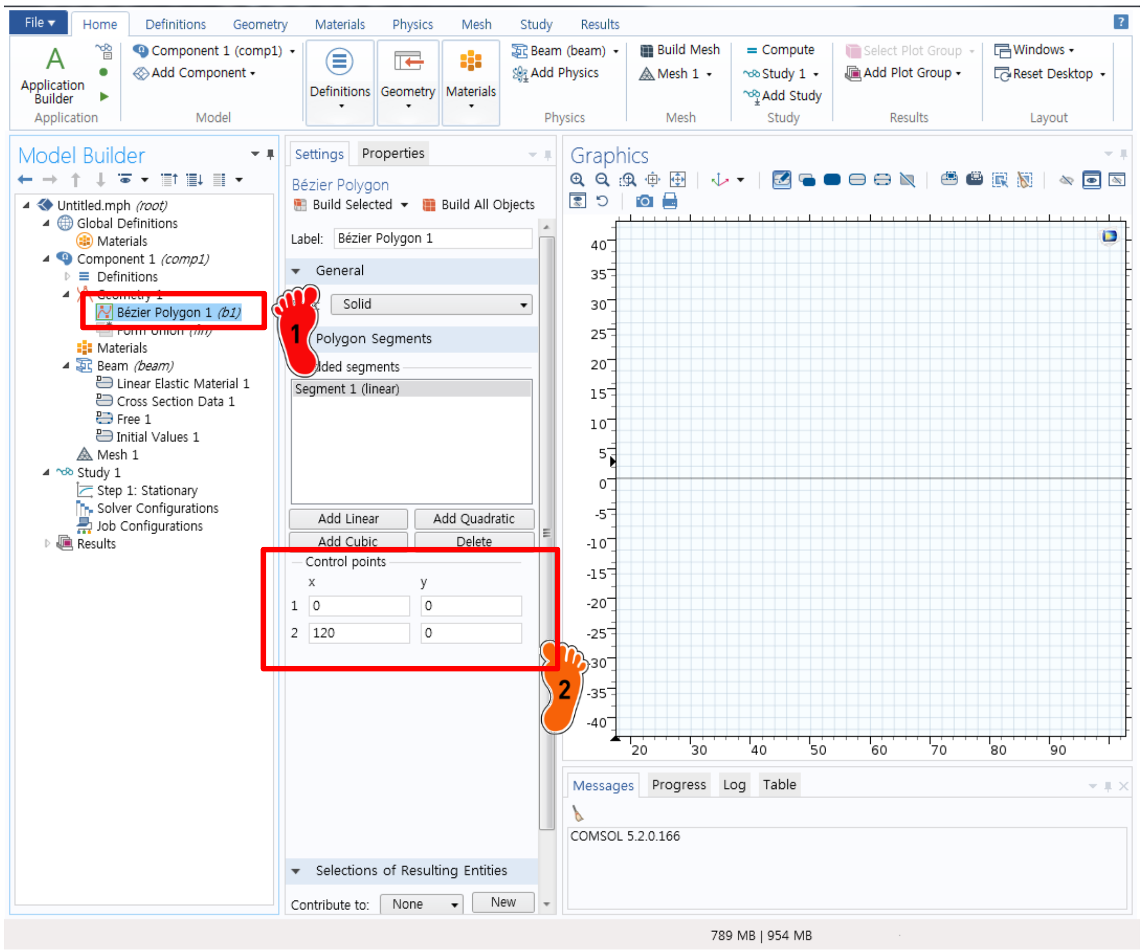
DIMENSIONLESS UNIT



1 Untitled.mph 클릭

2 Unit System 을 None 으로 변경

GEOMETRY CREATION

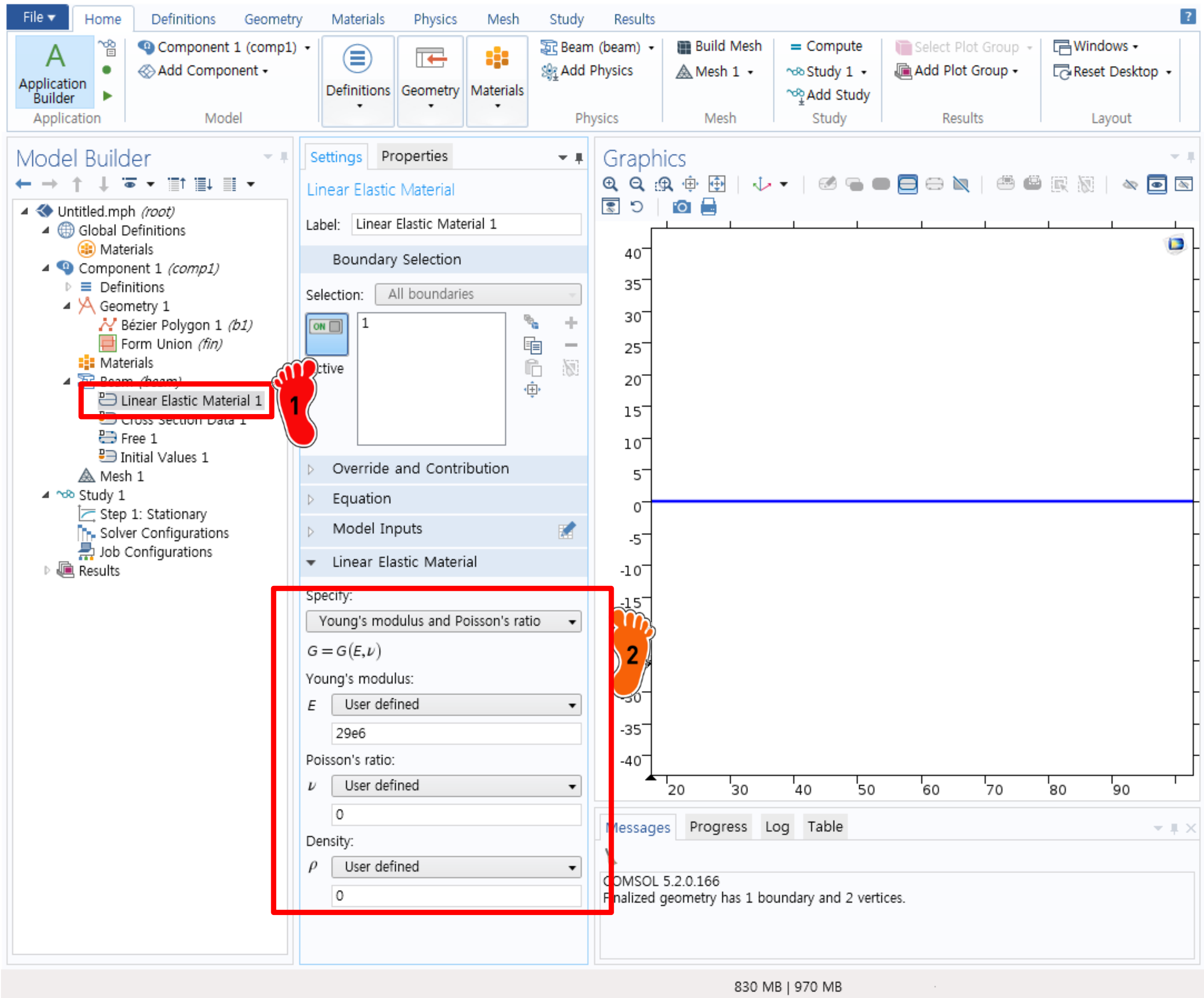


1 Bezier Polygon 생성

2 120 길이의 선분 생성

lb-inch 단위로 계산할 예정
따라서 10 ft = 120 in.

MATERIAL PROPERTY



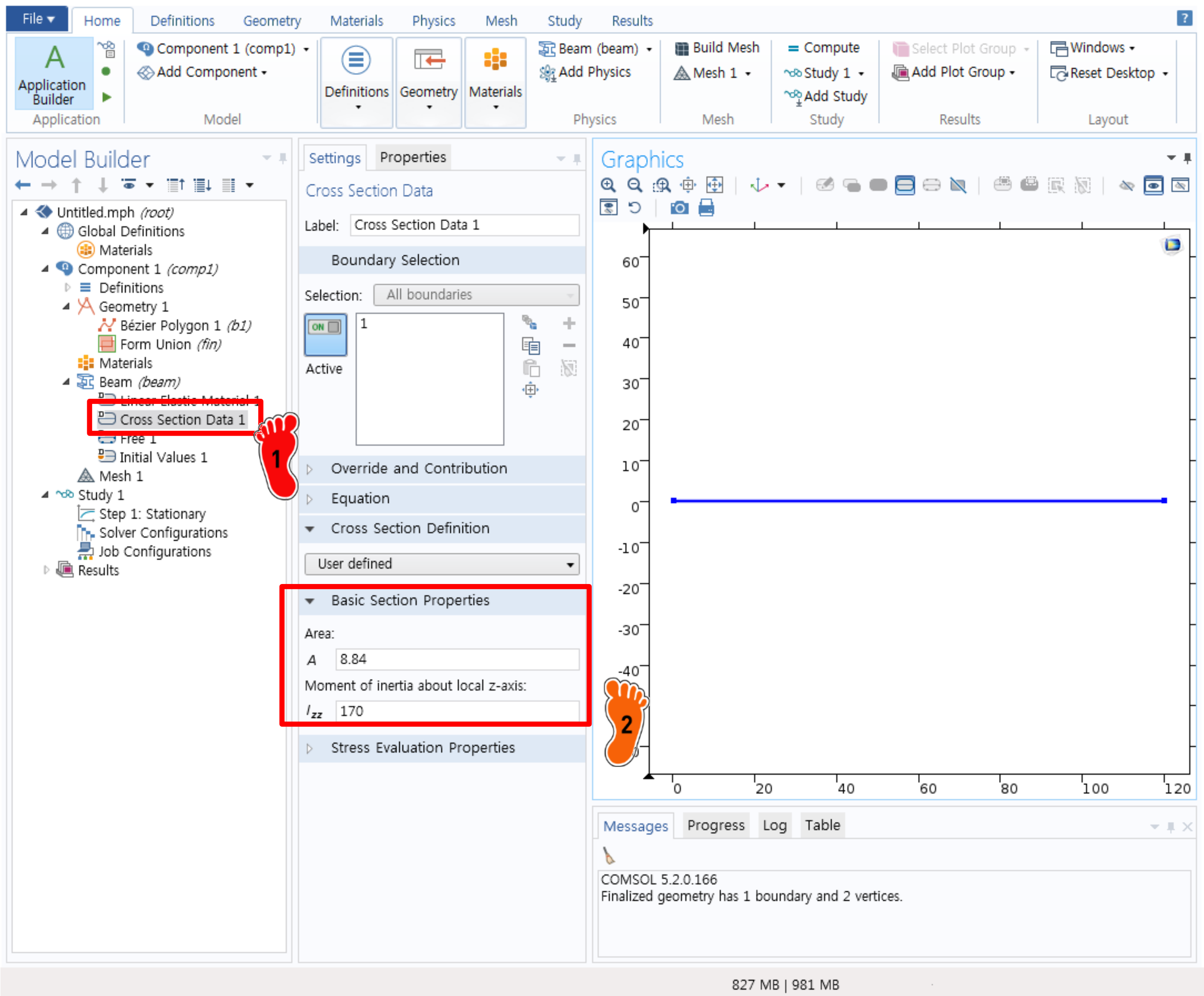
1 Linear Elastic Material 클릭

2 E: 29e6
mu: 0
rho : 0

입력

lb-inch 단위
E = 29e3 ksi = 29e6 psi
= 29e6 lb/in.^{2}

CROSS SECTION

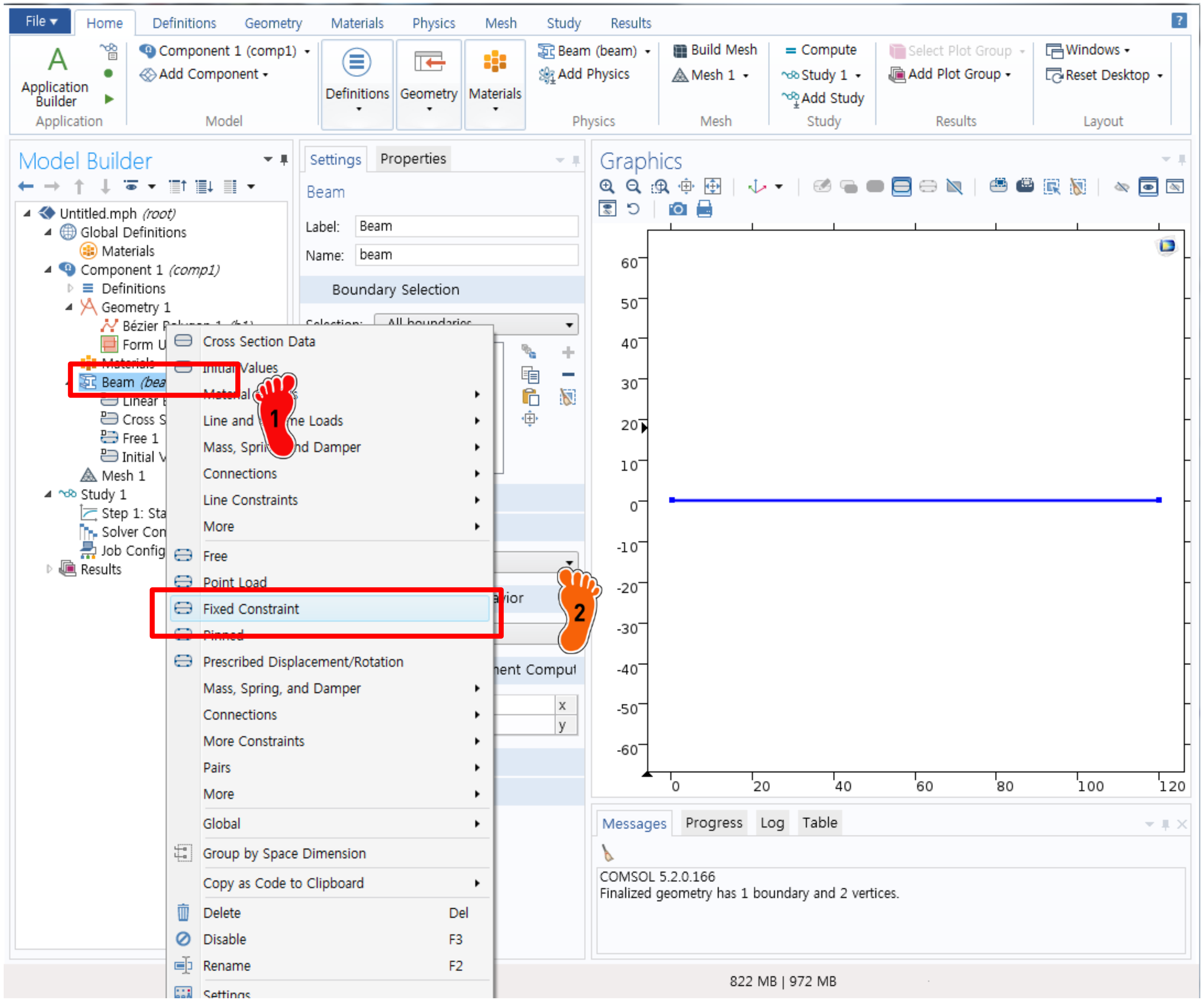


1. Cross Section Data 클릭

2. A: 8.84
I: 170

입력

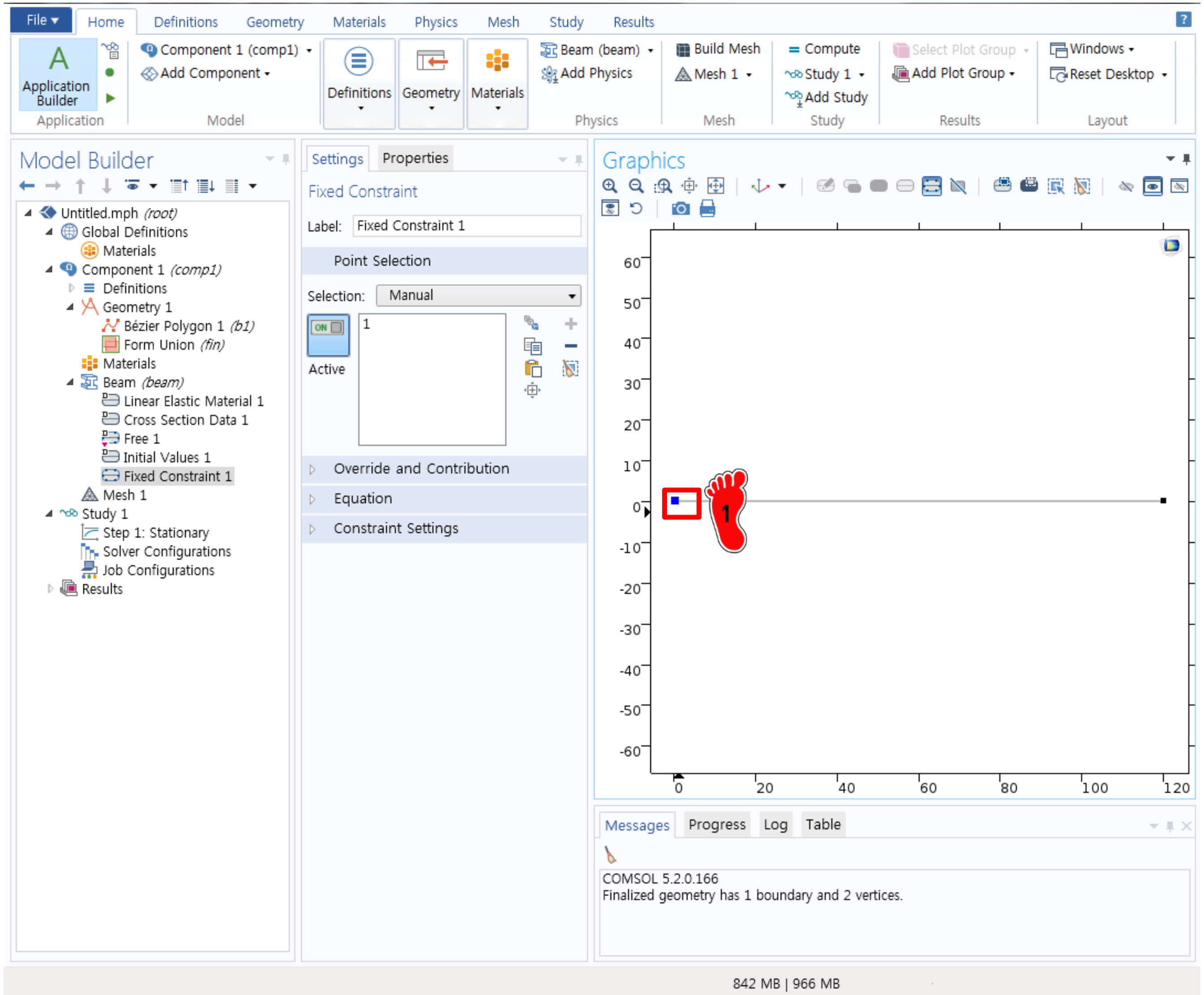
BOUNDARY CONDITION



1 Beam 마우스 우클릭

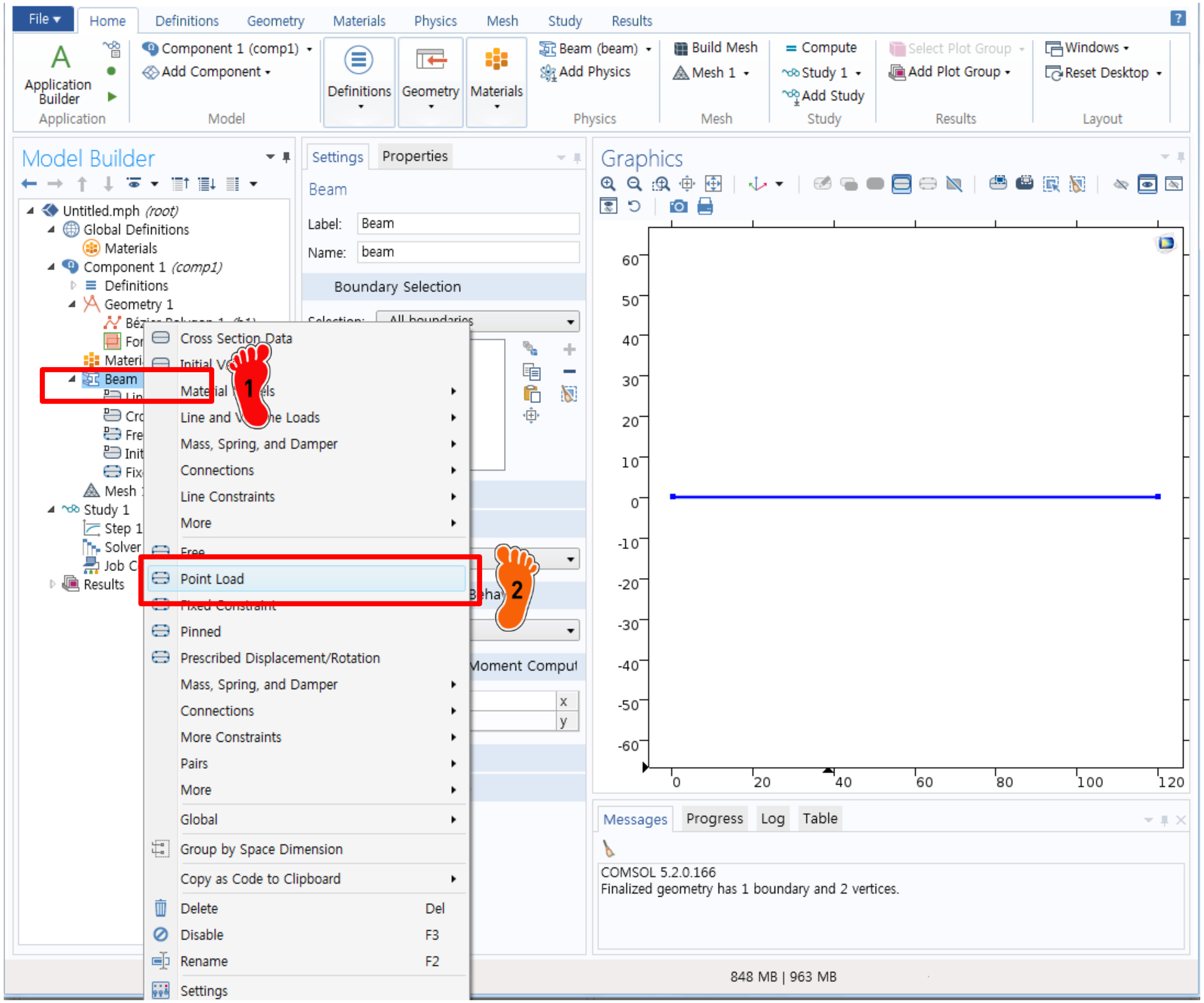
2 Fixed Constraint 클릭

BOUNDARY CONDITION



1 왼쪽 절점 선택

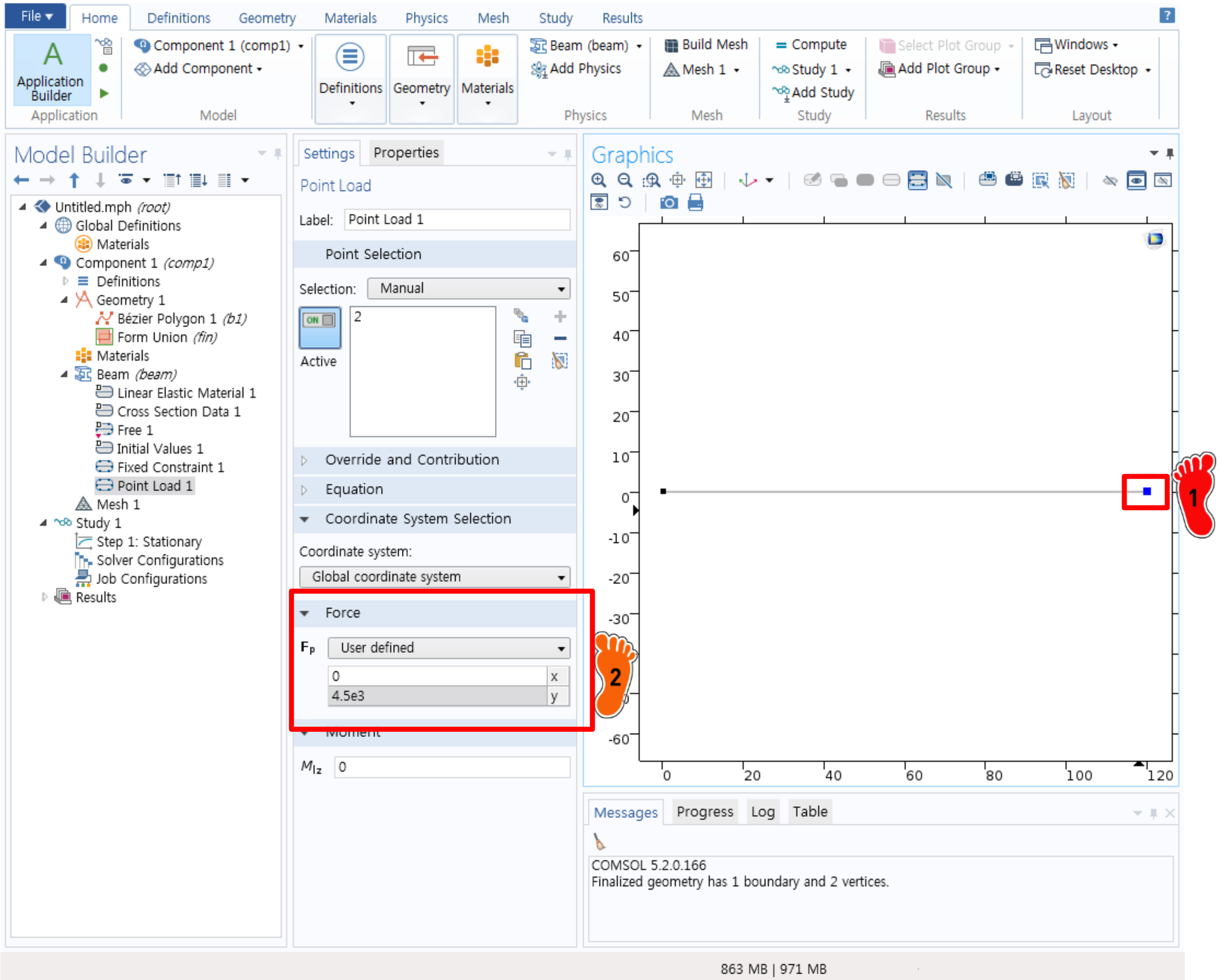
LOADING CONDITION



1 Beam 마우스 우클릭

2 Point Load 클릭

LOADING CONDITION



1 오른쪽 절점 선택

2 y 방향에 4.5e3 입력

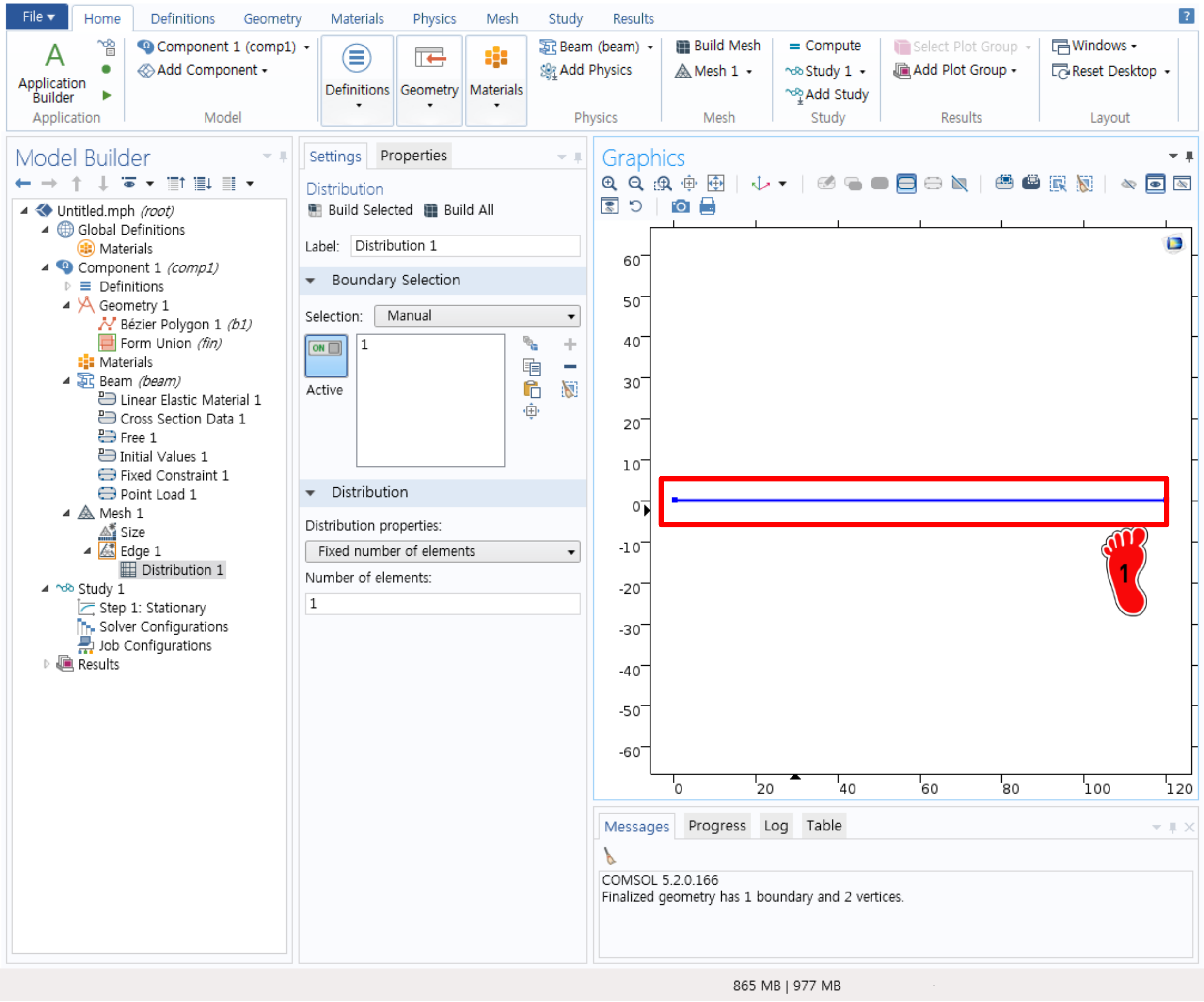
lb-inch 단위

P_B = 4.5 kips = 4.5e3 lb

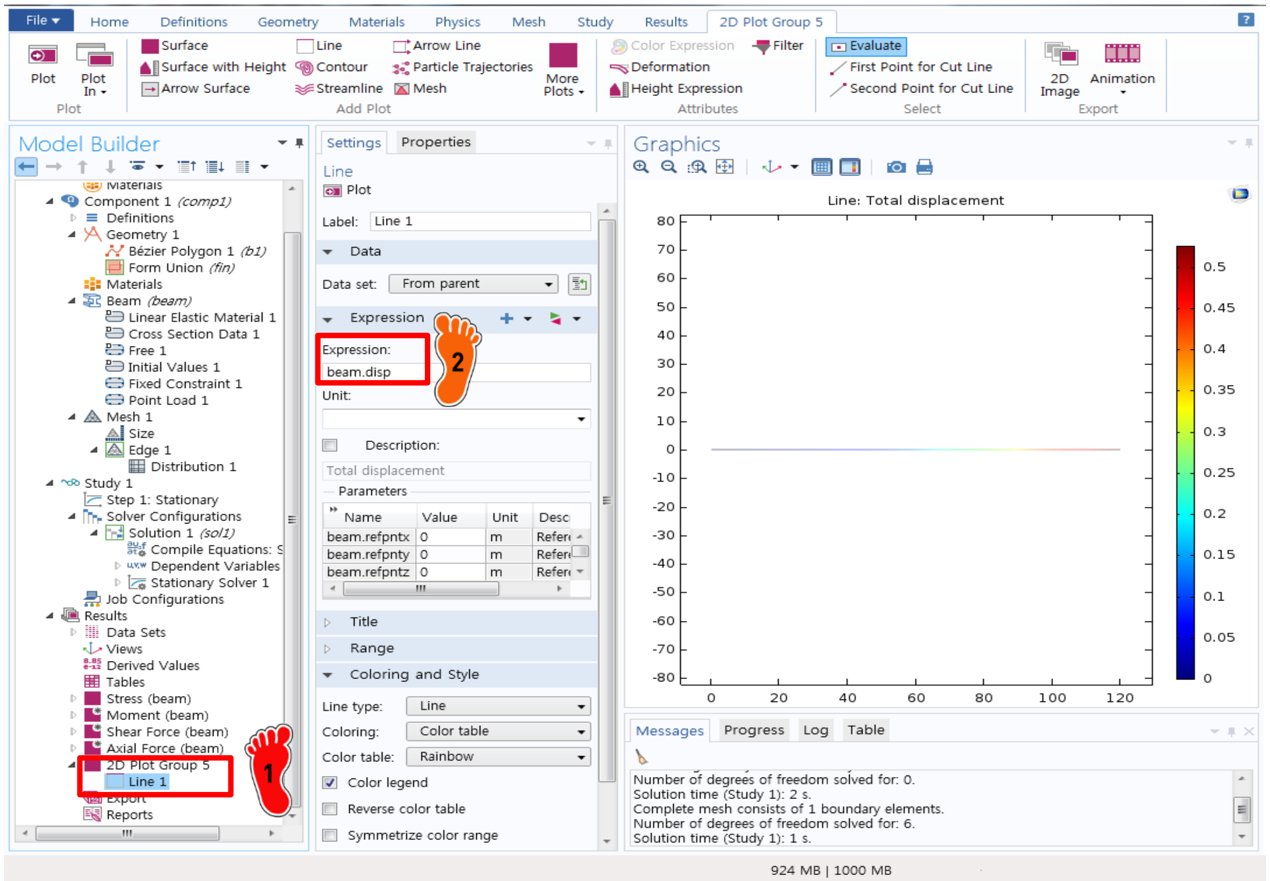
MESH

한 개의 요소가 되도록 메시 생성

그 후 compute 로 유한요소 해석 수행



POST-PROCESSING



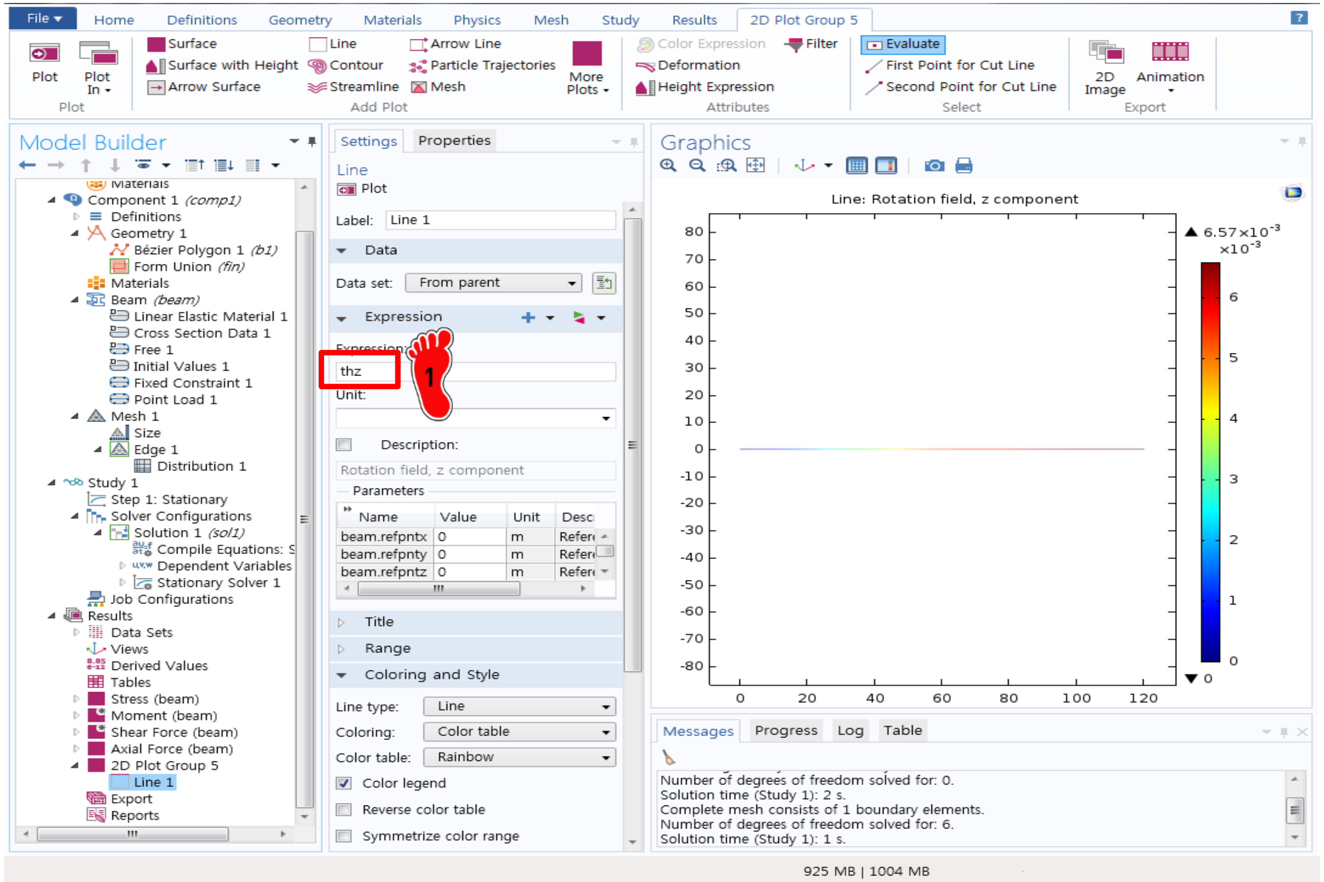
1 2D Plot Group 에서 Line 추가 후 beam.disp 를 확인

최대 변위는 0.5258

Analytical solution 0.5258

$$\begin{cases} \delta_B = \dfrac{1}{EI}\left[M_B\left(\dfrac{L^2}{2}\right)+P_B\left(\dfrac{L^3}{3}\right)\right] = 0.0730+0.5258 = 0.5988 \text{ in} \\ \theta_B = \dfrac{1}{EI}\left[M_B\left(L\right)+P_B\left(\dfrac{L^2}{2}\right)\right] = 0.00122+0.00657 = 0.00779 \text{ rad} \end{cases}$$

POST-PROCESSING



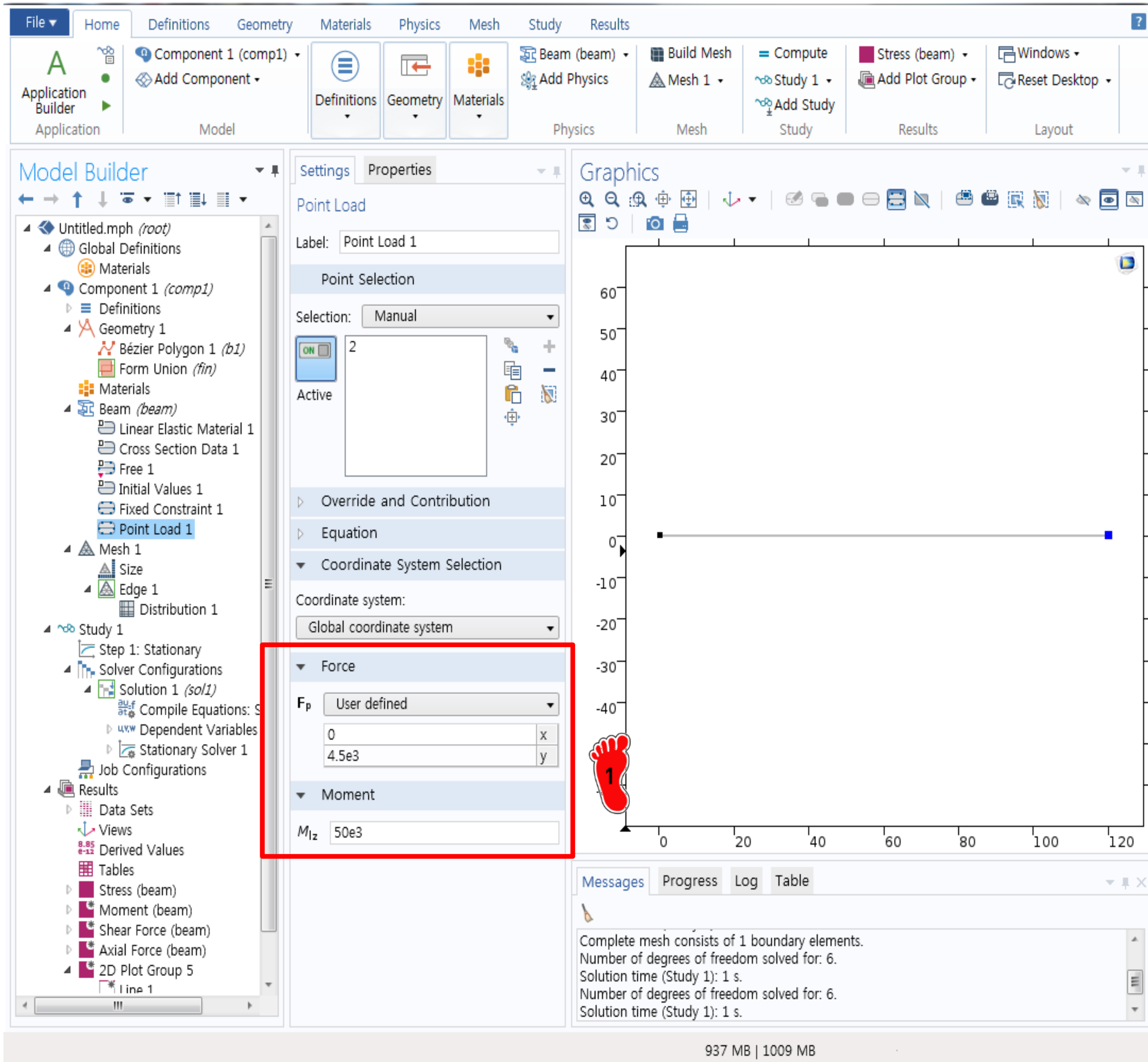
1 Expression 을 thz 로 변경

최대 각도는 6.57e-3

Analytical solution
6.57e-3

$$\begin{cases} \delta_B = \dfrac{1}{EI}\left[M_B\left(\dfrac{L^2}{2}\right) + P_B\left(\dfrac{L^3}{3}\right)\right] = 0.0730 + 0.5258 = 0.5988 \text{ in} \\ \theta_B = \dfrac{1}{EI}\left[M_B(L) + P_B\left(\dfrac{L^2}{2}\right)\right] = 0.00122 + \boxed{0.00657} = 0.00779 \text{ rad} \end{cases}$$

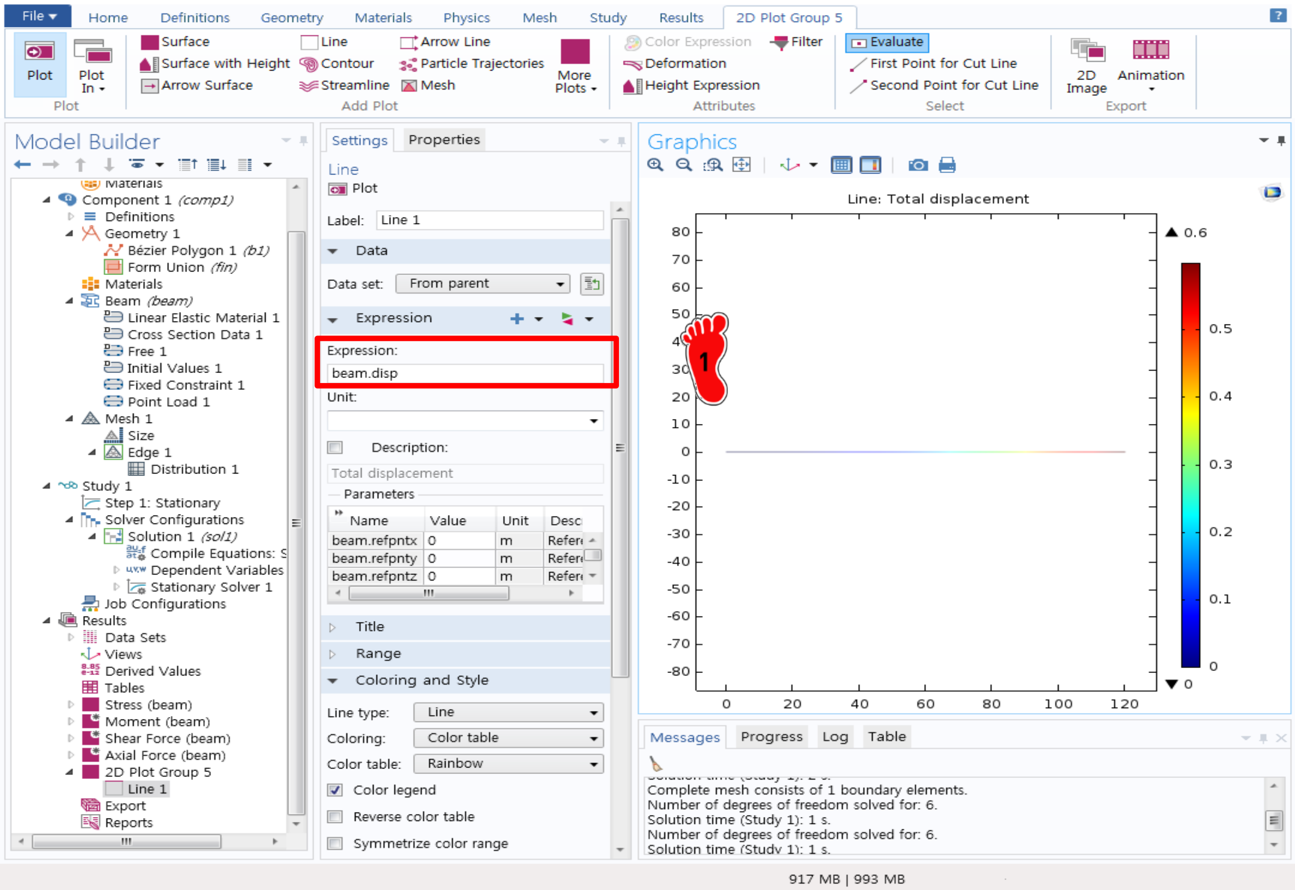
LOADING CONDITION



1 Point load 설정에서 모멘트 추가

50e3 입력

POST-PROCESSING



1 beam.disp 를 확인

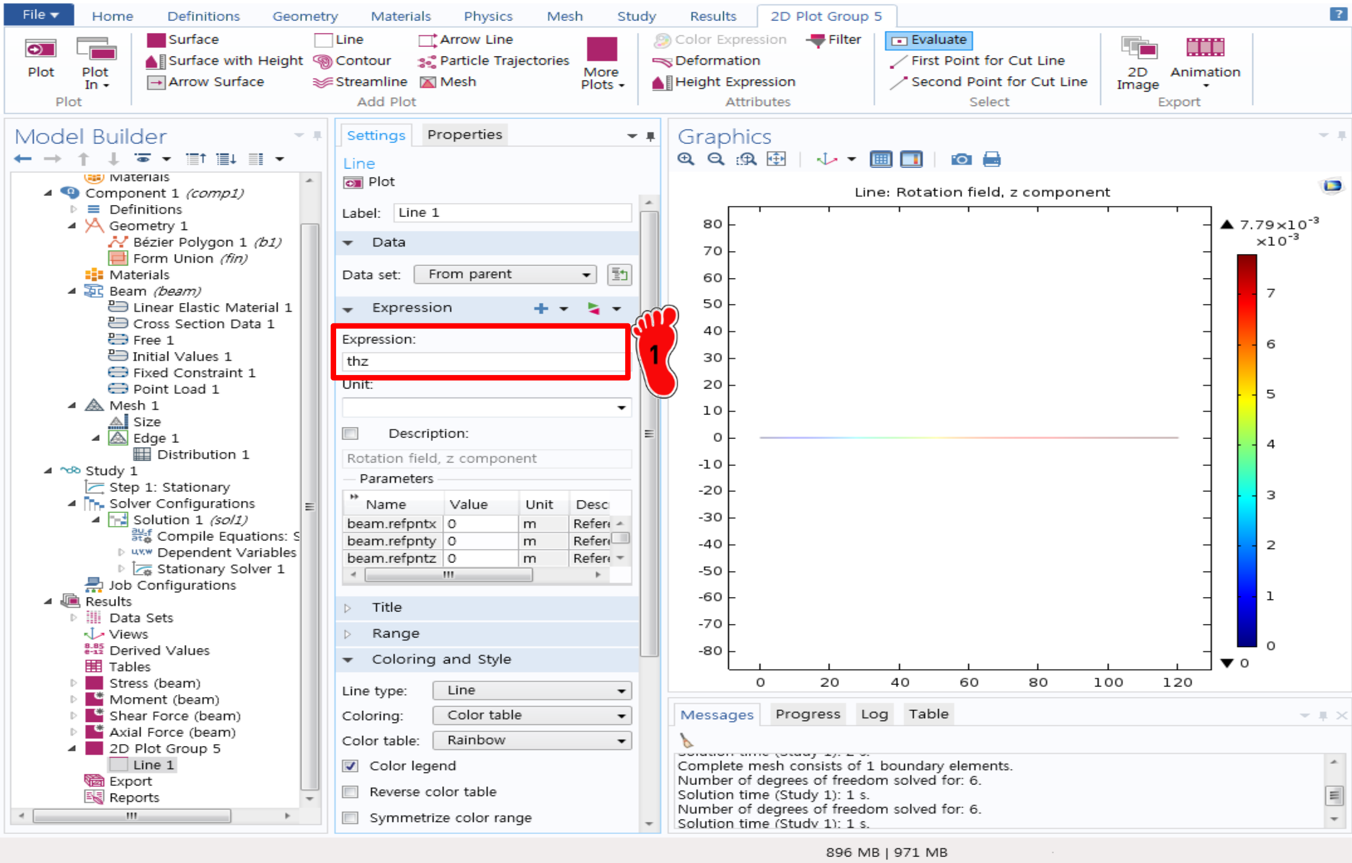
최대 변위는 0.599

Analytical solution 0.599

$$\delta_B = \frac{1}{EI}\left[M_B\left(\frac{L^2}{2}\right) + P_B\left(\frac{L^3}{3}\right)\right] = 0.0730 + 0.5258 = \boxed{0.599}\ \text{in}$$

$$\theta_B = \frac{1}{EI}\left[M_B(L) + P_B\left(\frac{L^2}{2}\right)\right] = 0.00122 + 0.00657 = 0.00779\ \text{rad}$$

POST-PROCESSING



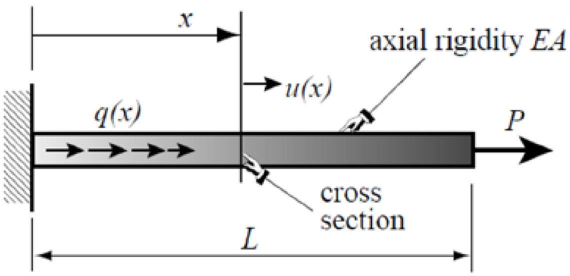
Expression 을 thz 로 변경

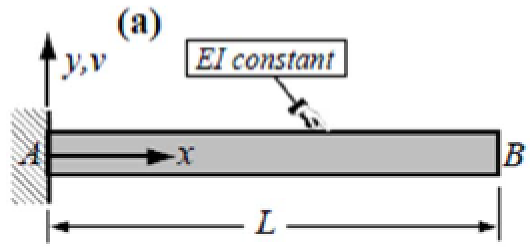
최대 각도는 7.79e-3 rad

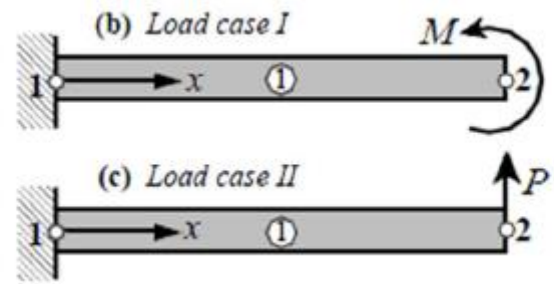
Analytical solution
7.79e-3 rad

$$\begin{cases} \delta_B = \dfrac{1}{EI}\left[M_B\left(\dfrac{L^2}{2}\right) + P_B\left(\dfrac{L^3}{3}\right)\right] = 0.0730 + 0.5258 = 0.599 \text{ in} \\ \theta_B = \dfrac{1}{EI}\left[M_B(L) + P_B\left(\dfrac{L^2}{2}\right)\right] = 0.00122 + 0.00657 = \boxed{0.00779} \text{ rad} \end{cases}$$

SYSTEM EQUATION BY HAND







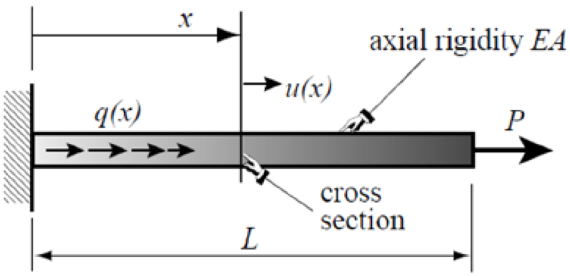
$$\frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ P \end{bmatrix}$$

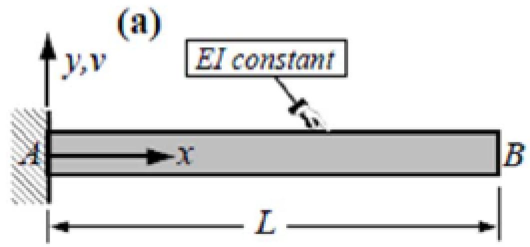
$$\rightarrow u_2 = \frac{PL}{EA}$$

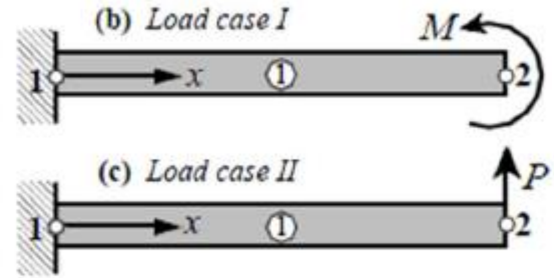
$$\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ M \end{bmatrix} \rightarrow \begin{bmatrix} v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \frac{ML^2}{2EI} \\ \frac{ML}{EI} \end{bmatrix}$$

$$\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ P \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \frac{PL^3}{3EI} \\ \frac{PL^2}{2EI} \end{bmatrix}$$

SYSTEM EQUATION BY HAND

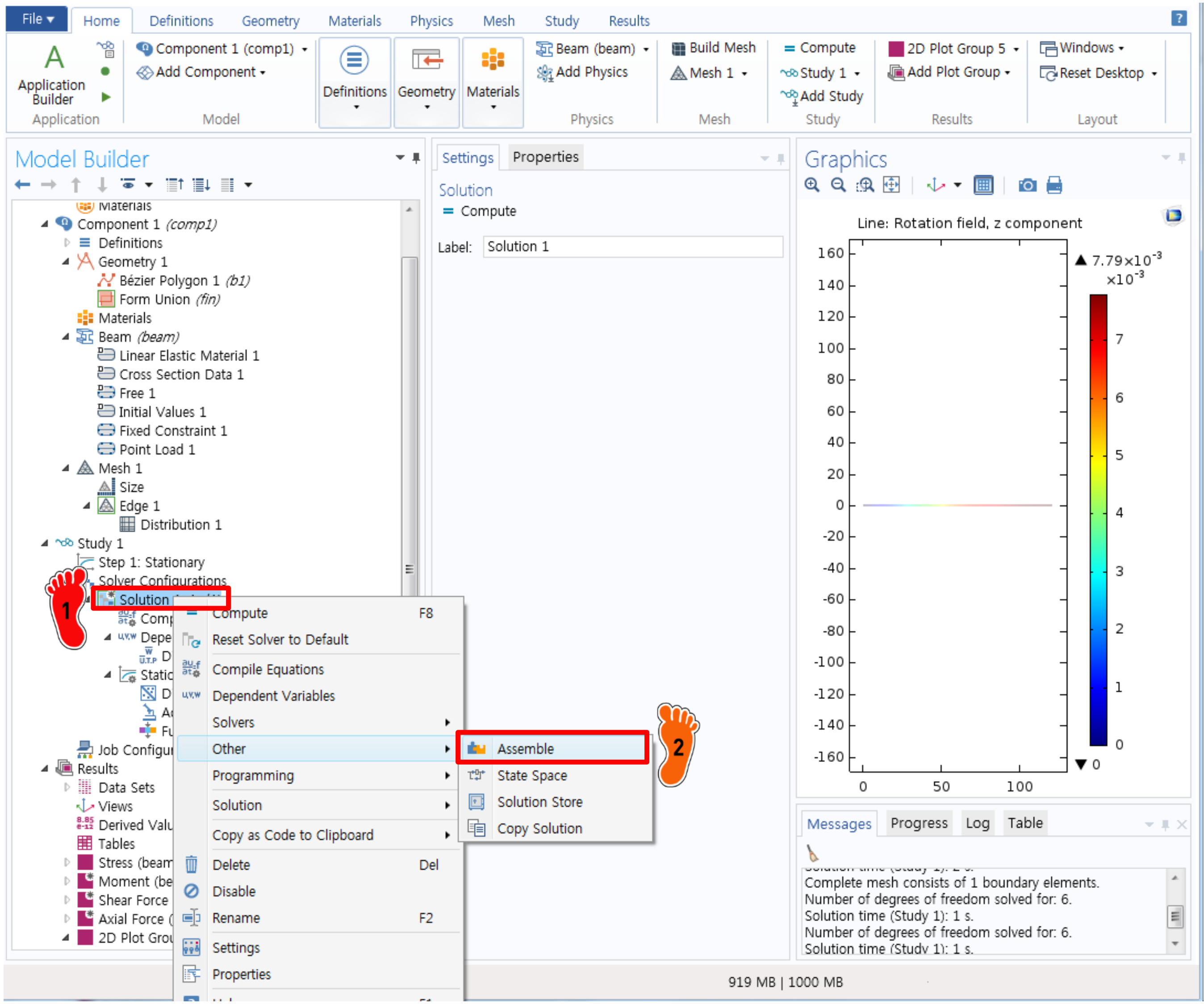






$$\begin{bmatrix} 1.64e^{8} & 0 & -2.05e^{6} & 8.22e^{7} & 0 & 2.05e^{6} \\ 0 & 2.14e^{6} & 0 & 0 & -2.14e^{6} & 0 \\ -2.05e^{6} & 0 & 3.42e^{4} & -2.05e^{6} & 0 & -3.42e^{4} \\ 8.22e^{7} & 0 & -2.05e^{6} & 1.64e^{8} & 0 & 2.05e^{6} \\ 0 & -2.14e^{6} & 0 & 0 & 2.14e^{6} & 0 \\ 2.05e^{6} & 0 & -3.42e^{4} & 2.05e^{6} & 0 & 3.42e^{4} \end{bmatrix} \begin{bmatrix} \theta_2 \\ u_2 \\ v_2 \\ \theta_1 \\ u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} M \\ P_x \\ P_y \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

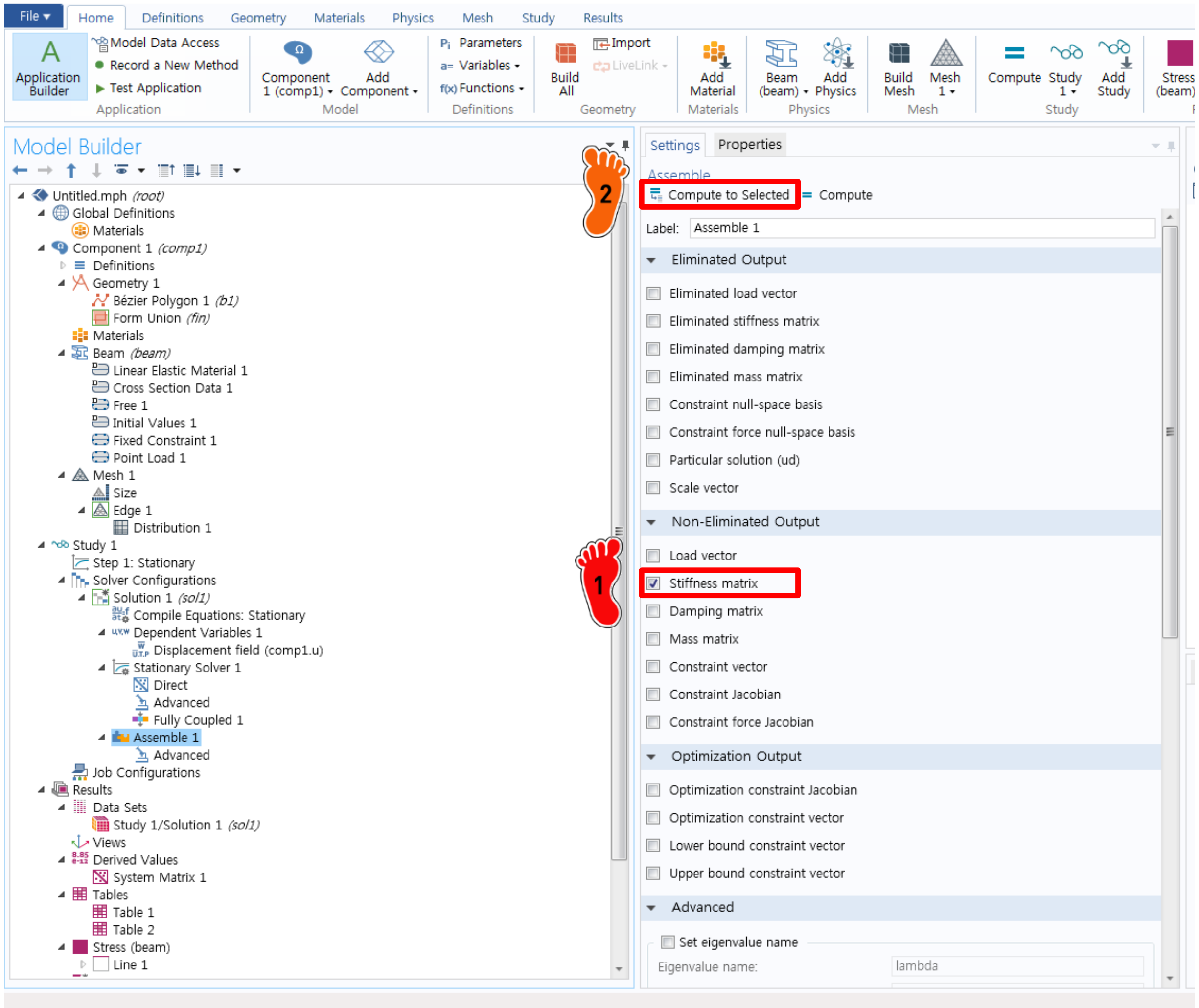
ASSEMBLE OPTION



1 Solution 마우스 우클릭

2 Assemble 클릭

ASSEMBLE OPTION



1 Stiffness matrix 체크박스에 체크

2 Compute to Selected 클릭

COMPARISON

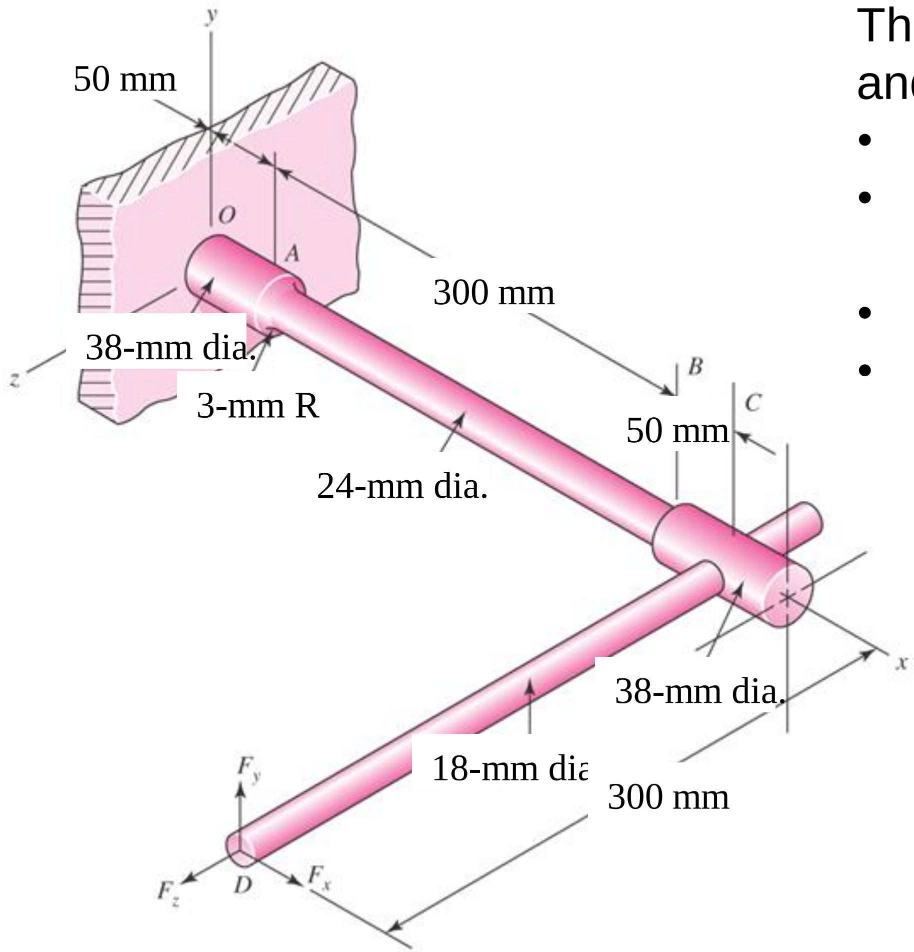
$$\mathbf{K}_{by\ hand} = \begin{bmatrix} 1.64e^{8} & 0 & -2.05e^{6} & 8.22e^{7} & 0 & 2.05e^{6} \\ 0 & 2.14e^{6} & 0 & 0 & -2.14e^{6} & 0 \\ -2.05e^{6} & 0 & 3.42e^{4} & -2.05e^{6} & 0 & -3.42e^{4} \\ 8.22e^{7} & 0 & -2.05e^{6} & 1.64e^{8} & 0 & 2.05e^{6} \\ 0 & -2.14e^{6} & 0 & 0 & 2.14e^{6} & 0 \\ 2.05e^{6} & 0 & -3.42e^{4} & 2.05e^{6} & 0 & 3.42e^{4} \end{bmatrix}$$

1.6433E8	0.0000	-2.0542E6	8.2167E7	0.0000	2.0542E6
0.0000	2.1363E6	0.0000	0.0000	-2.1363E6	0.0000
-2.0542E6	0.0000	34236	-2.0542E6	0.0000	-34236
8.2167E7	0.0000	-2.0542E6	1.6433E8	0.0000	2.0542E6
0.0000	-2.1363E6	0.0000	0.0000	2.1363E6	0.0000
2.0542E6	0.0000	-34236	2.0542E6	0.0000	34236

Stiffness matrix by COMSOL

- **Beam model: 2D**
  - ✓ **Textbook of "Solid Mechanics"**
- **Beam model: 3D**
- **Assignment**

CANTILEVERED HANDLE



The cantilevered bar is made from mild steel and is statically loaded. At cross section A,

- Determine the location of the critical stress.
- Determine the principal stresses and the maximum shear stress.
- Determine the angle of twist in bar OC.
- Determine the vertical deflection at the tip.

$E = 207\,\text{GPa}$

$G = 79.3\,\text{GPa}$

$F_{y@D} = 1000\,\text{N}$

$\sigma_1 = 298.8\,\text{MPa}$

$\sigma_2 = \text{-}40.9\,\text{MPa}$

$\tau_{max} = 169.8\,\text{MPa}$

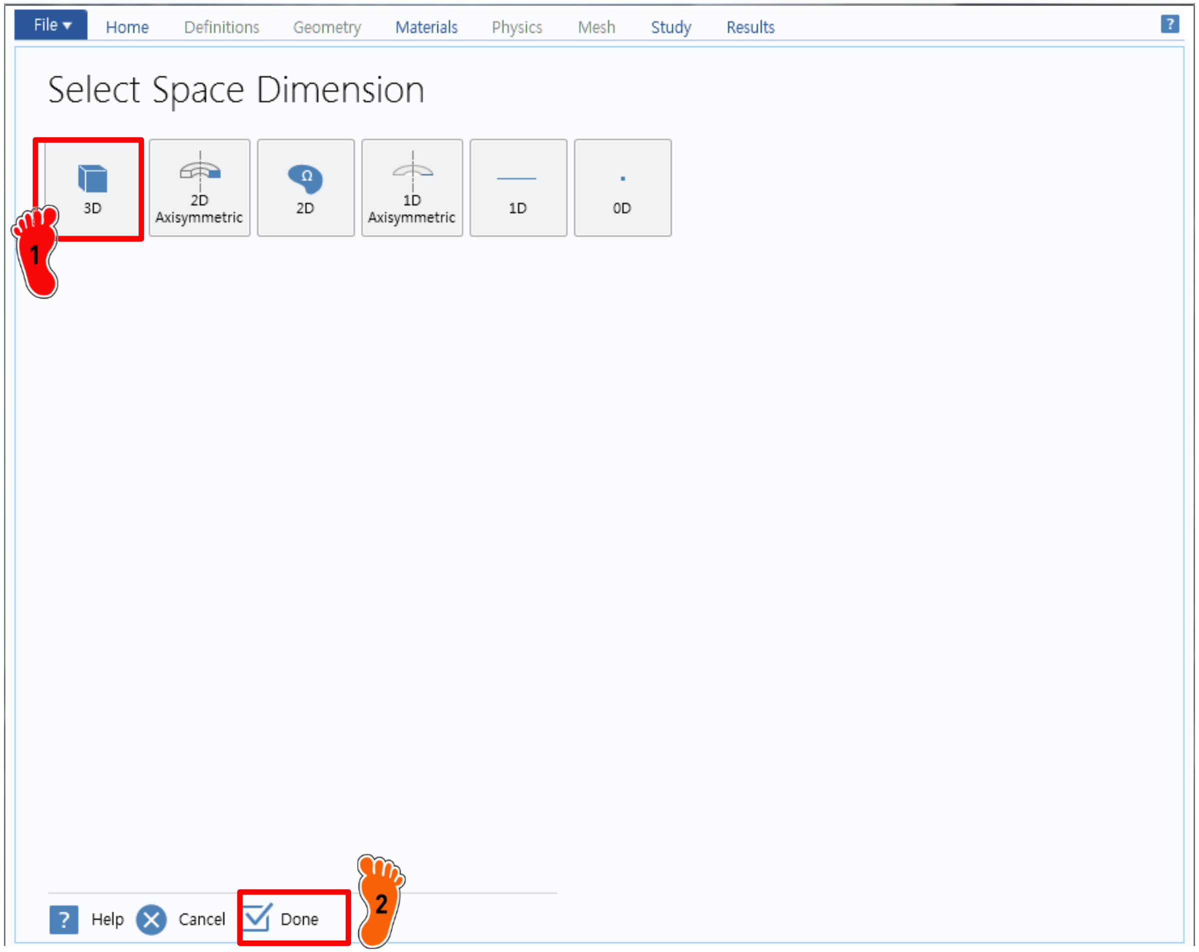
$\theta_C = 0.0367\,\text{rad}$

$y_D = 24.005\,\text{mm}$

The shear and normal stresses can be determined as functions of x and y directly from these functions, as well as such parameters as the principal stress. Since σ_y is zero everywhere, the principal stress is

$$\sigma_{p1} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

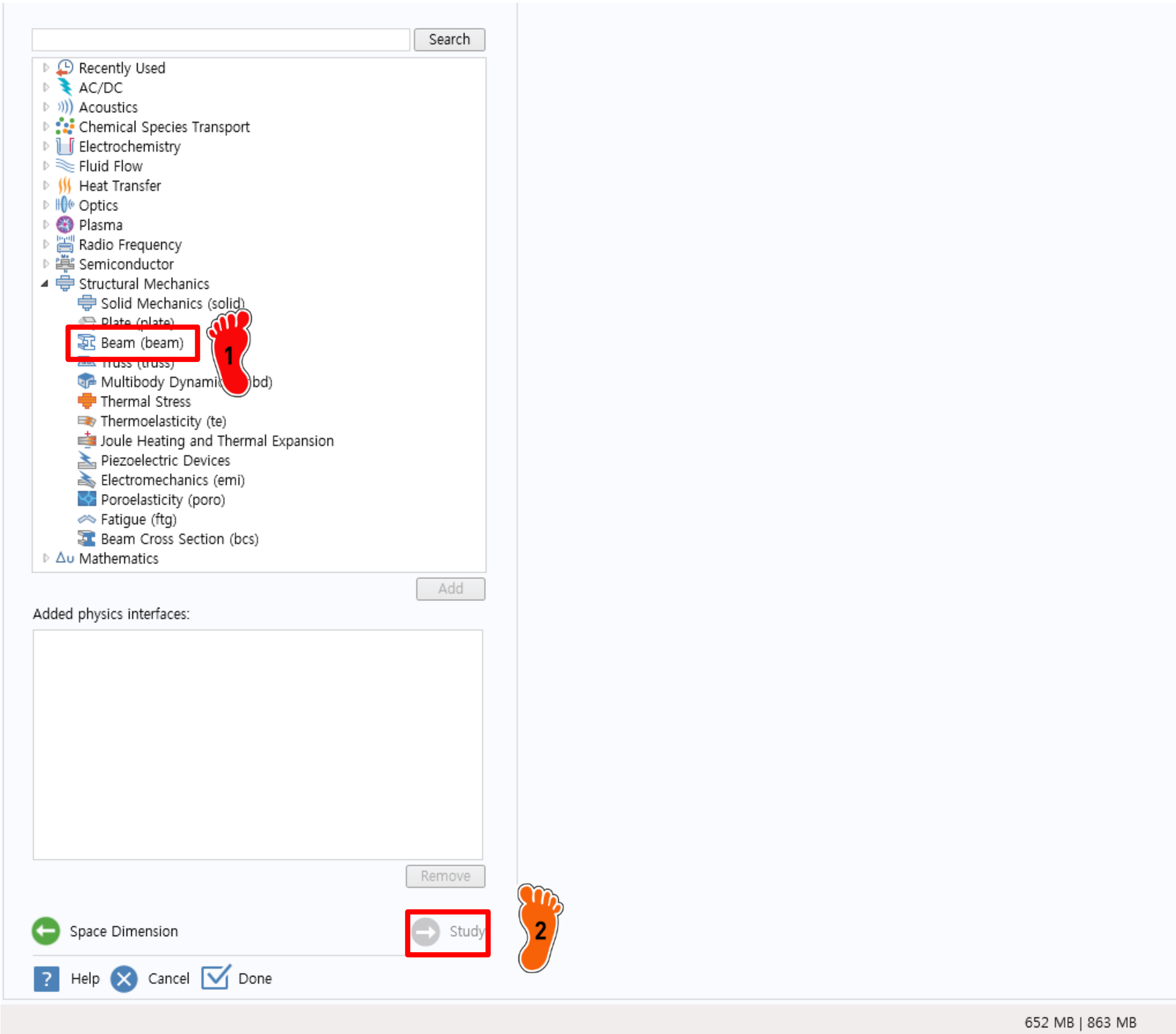
DIMENSION SELECTION



1 3D 선택

2 Done 클릭

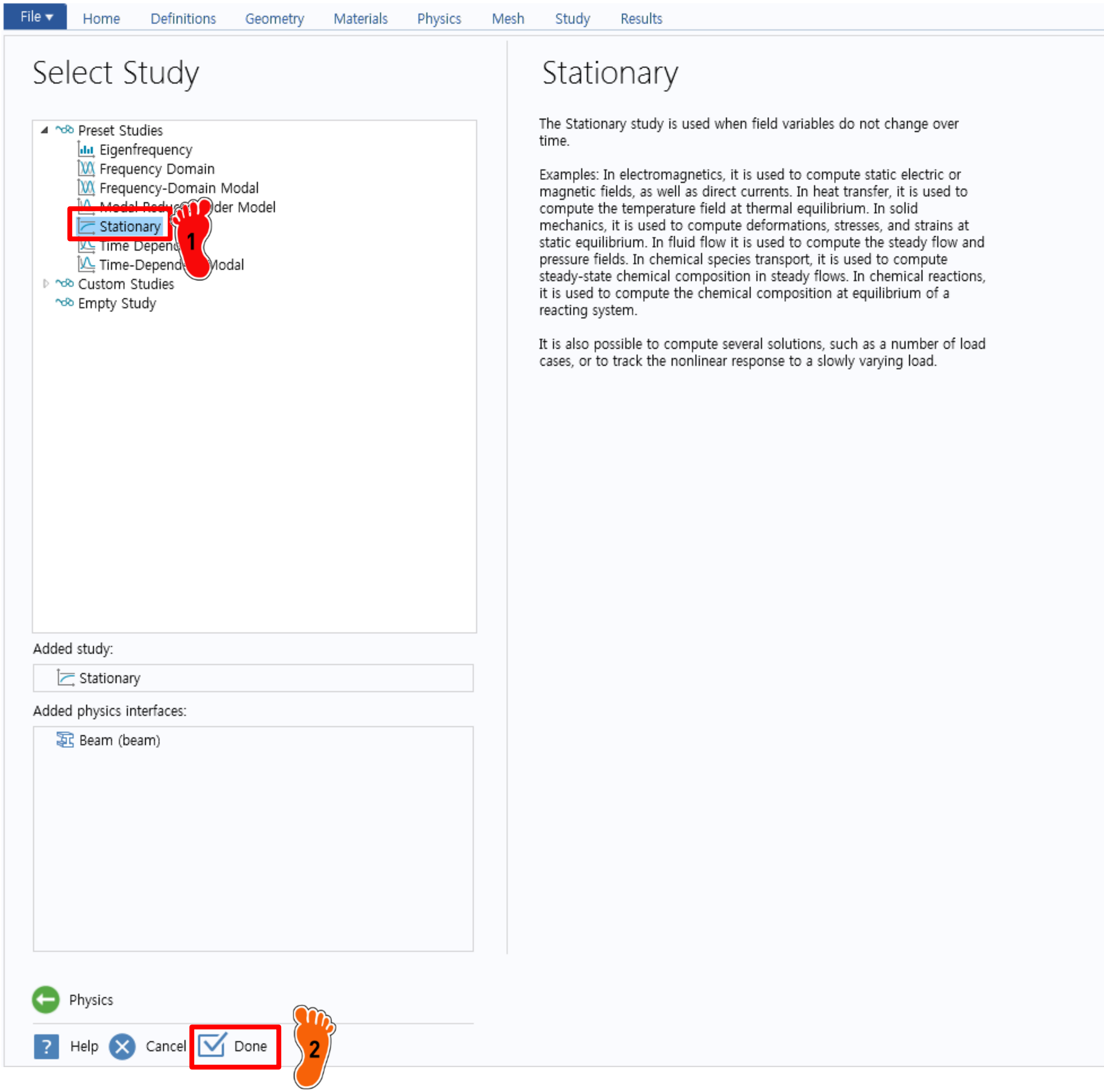
PHYSICS SELECTION



1 Structural Mechanics 의 Beam 추가

2 Study 클릭

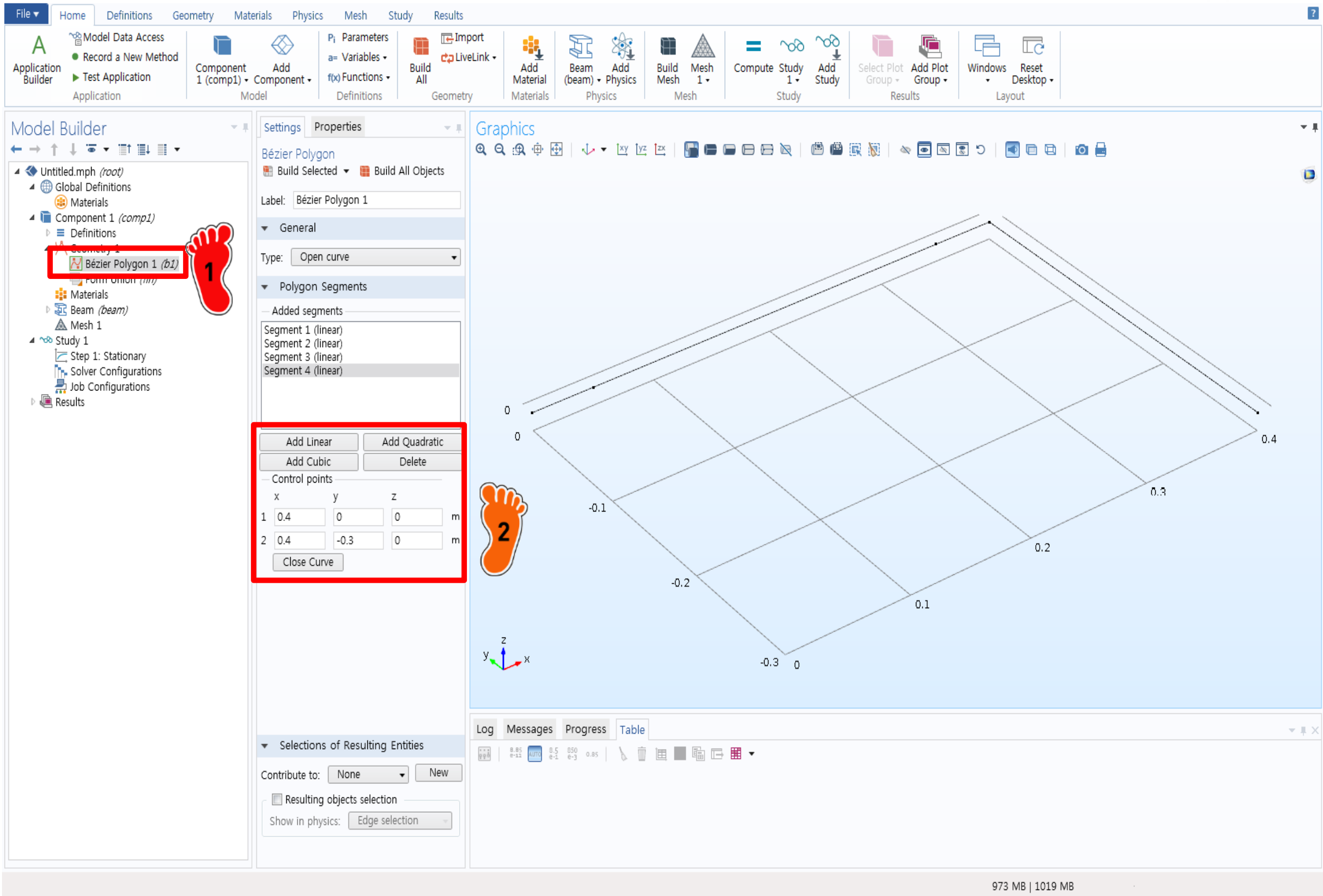
STUDY TYPE SELECTION



1 Stationary 선택

2 Done 클릭

GEOMETRY CREATION



1. Bezier Polygon 생성

2. Segment1
(0,0,0)~(0.05,0,0)

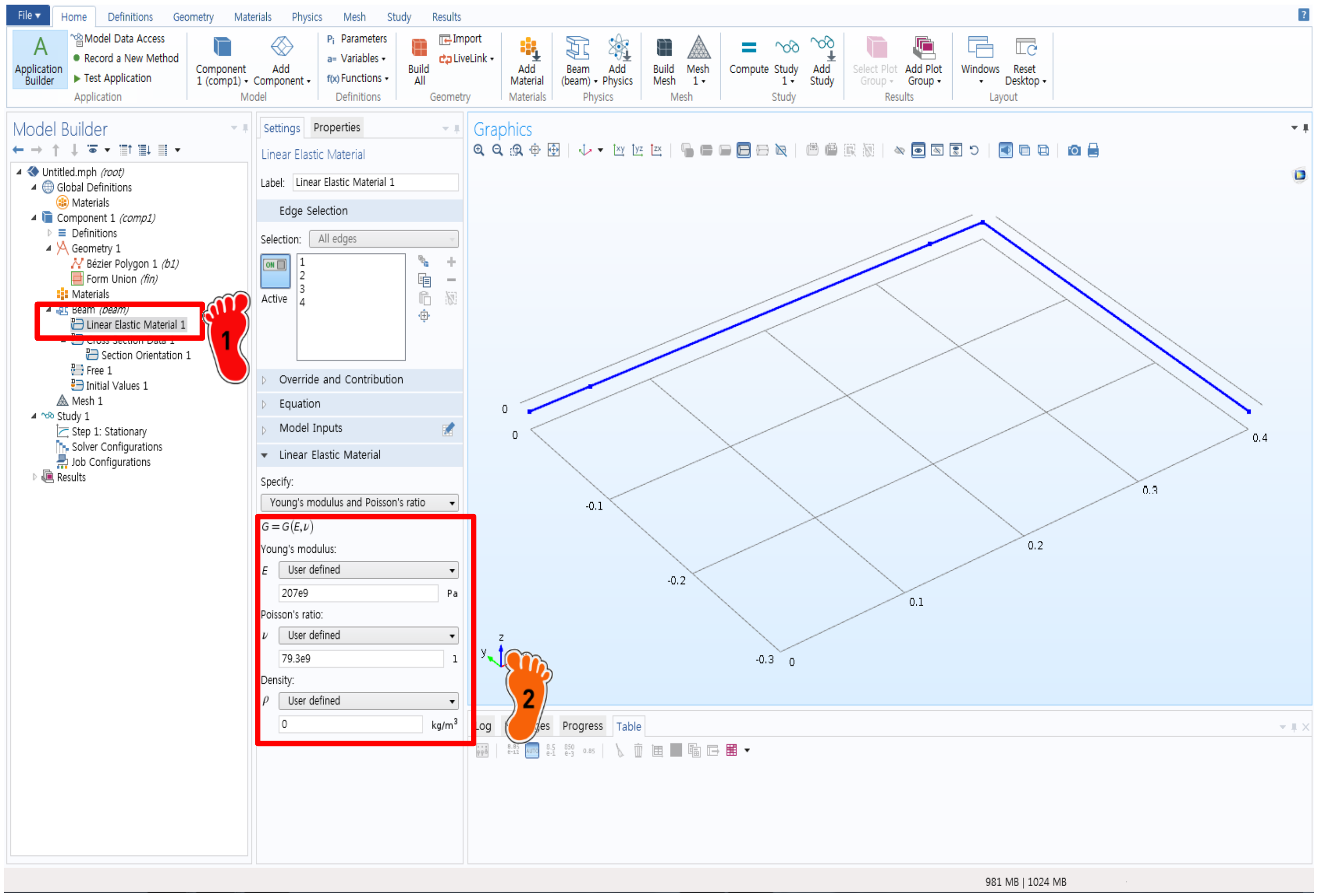
Segment2
(0.05,0,0)~(0.35,0,0)

Segment3
(0.35,0,0)~(0.4,0,0)

Segment4
(0.4,0,0)~(0.4,-0.3,0)

기하형상 생성

MATERIAL PROPERTY



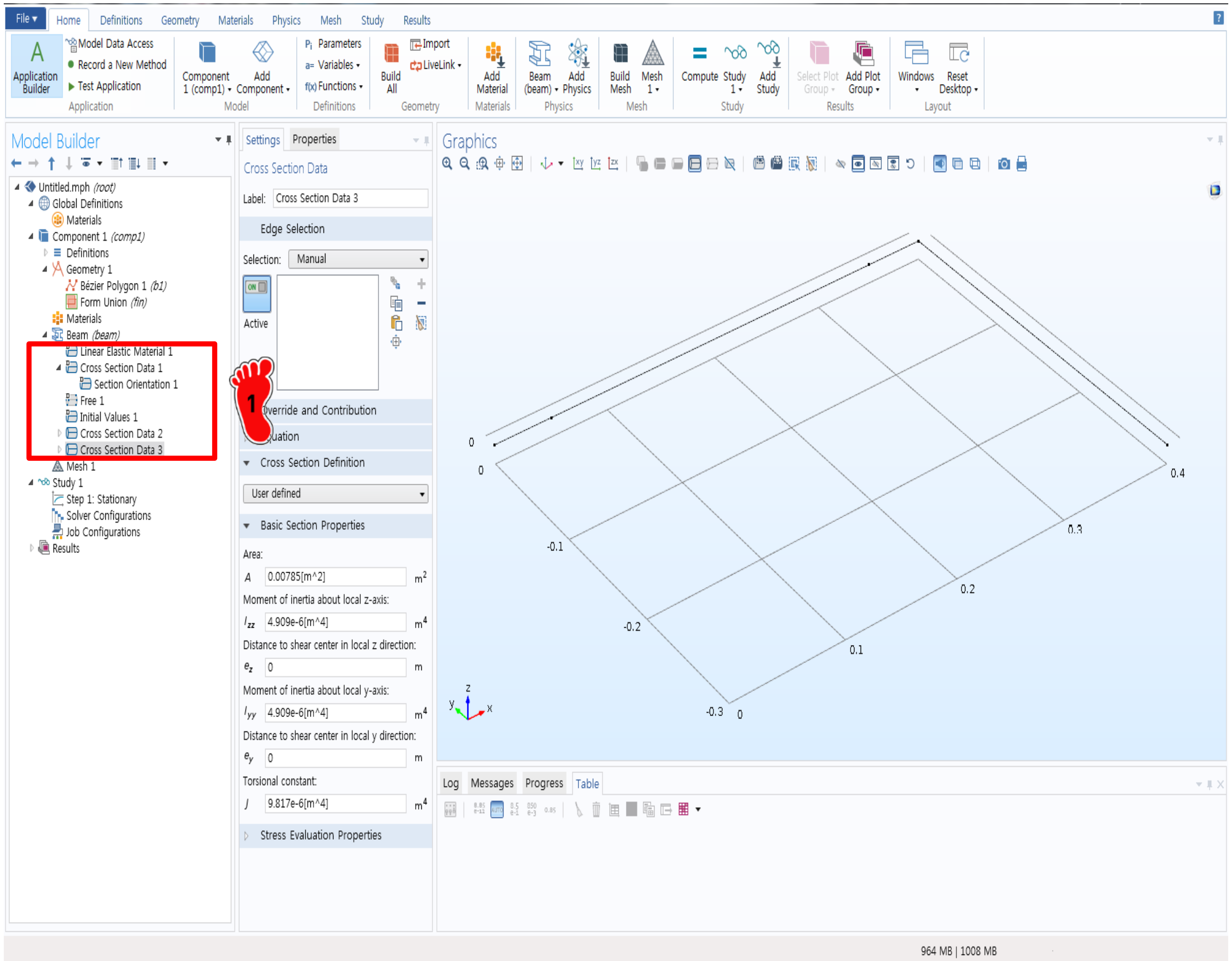
1 Linear Elastic Material 클릭

2 Specify 를 Young`s modulus and shear modulus 로 변경 후

E: 207e9
G: 79.3e9
rho : 0

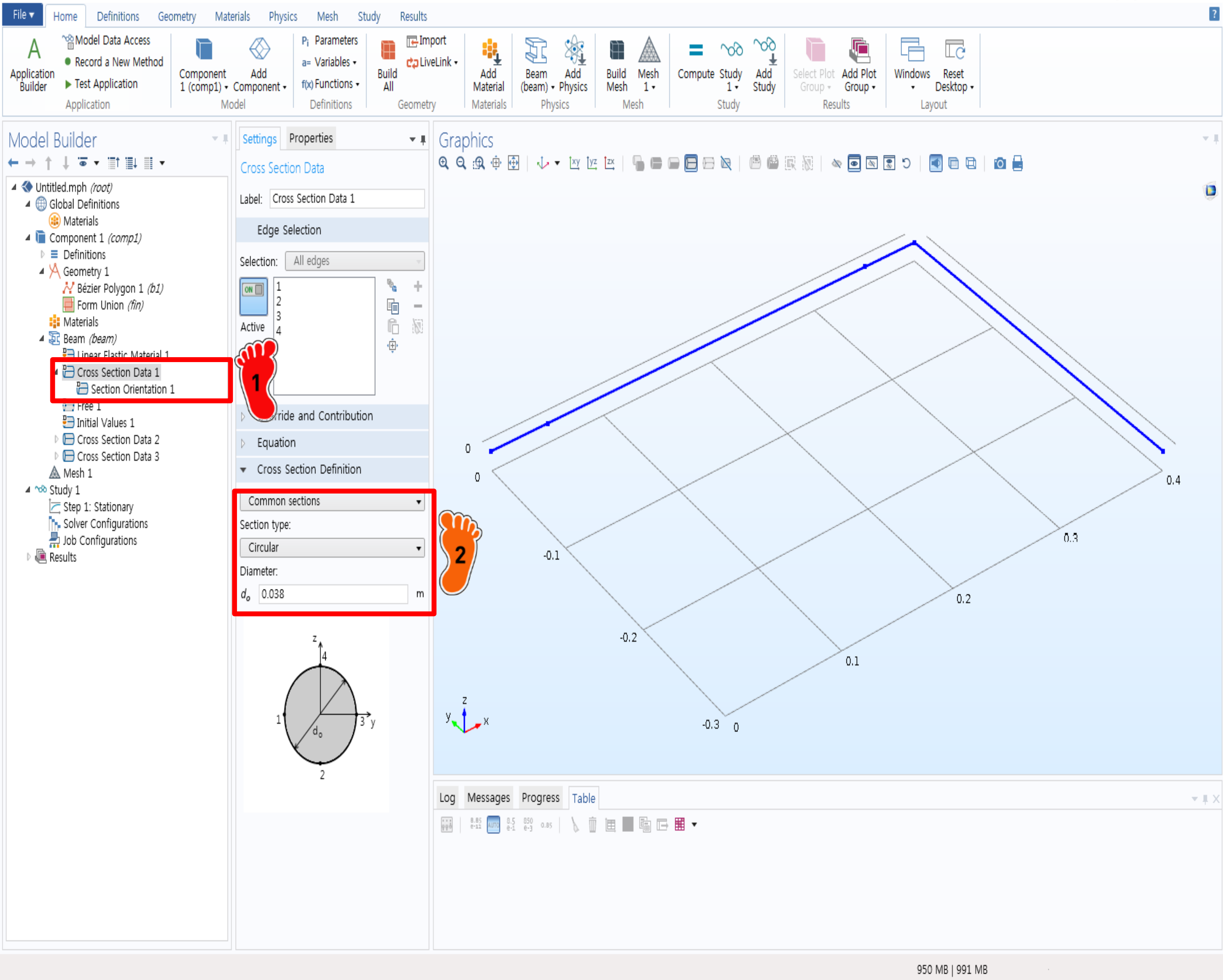
입력

CROSS SECTION



1 Cross Section Data 를 Duplicate 로 2 개 더 복사

CROSS SECTION



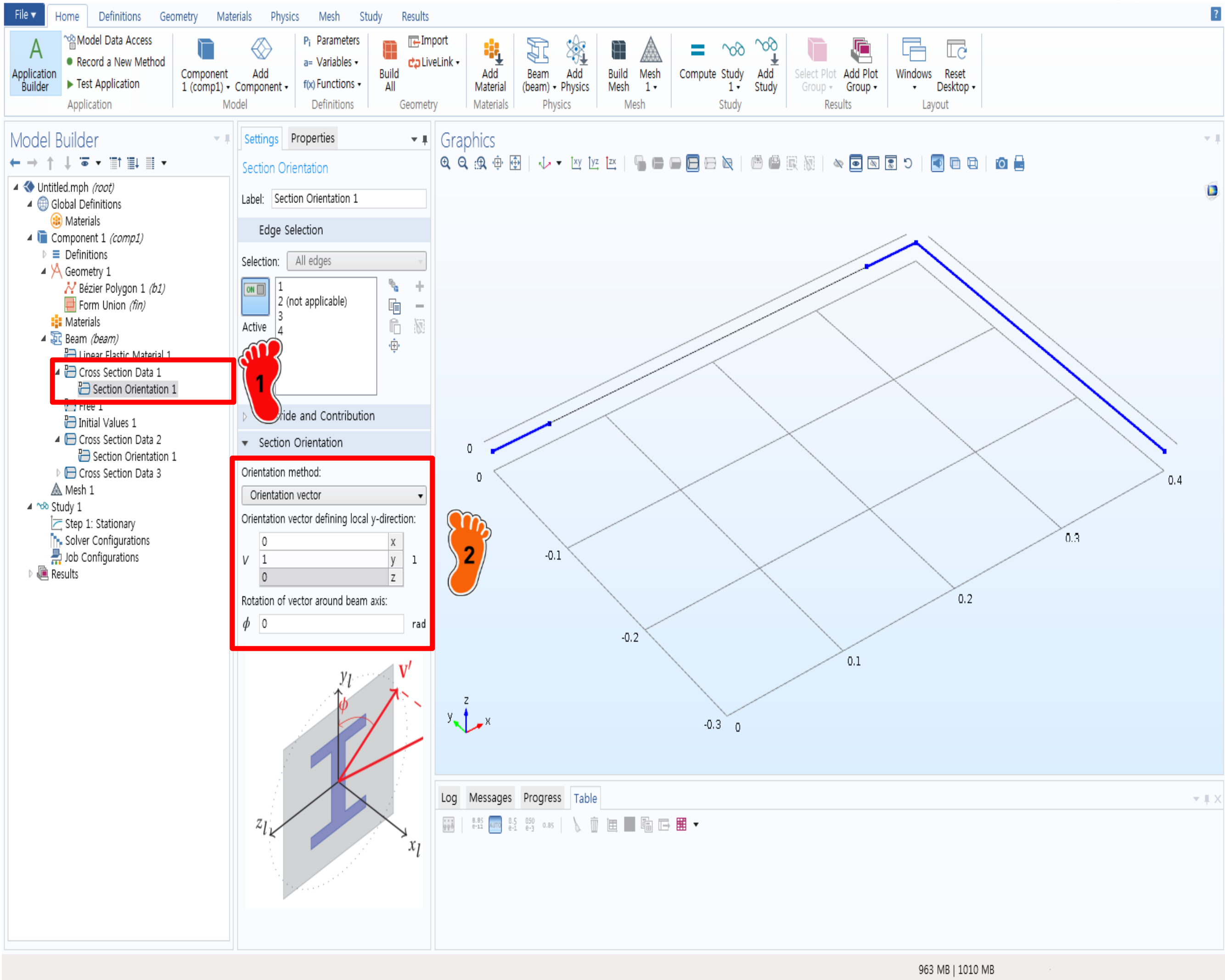
1 Cross Section Data 1 클릭

2 Common sections 변경

Circular 변경

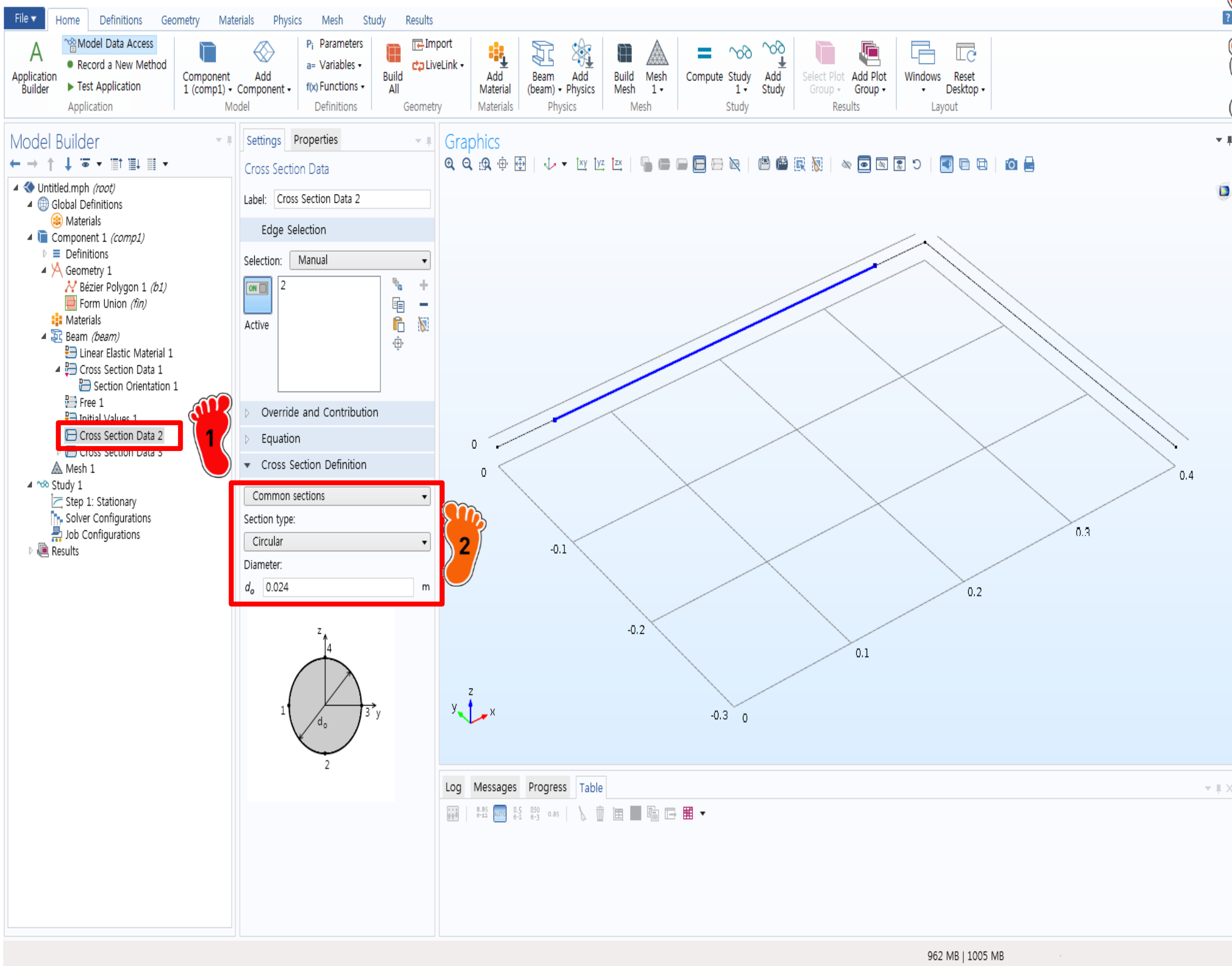
지름 0.038 입력

CROSS SECTION



1. Cross Section Data 1 하위 메뉴인 Section Orientation 1 클릭
2. Orientation vector 변경 후 (0,1,0) 입력

CROSS SECTION



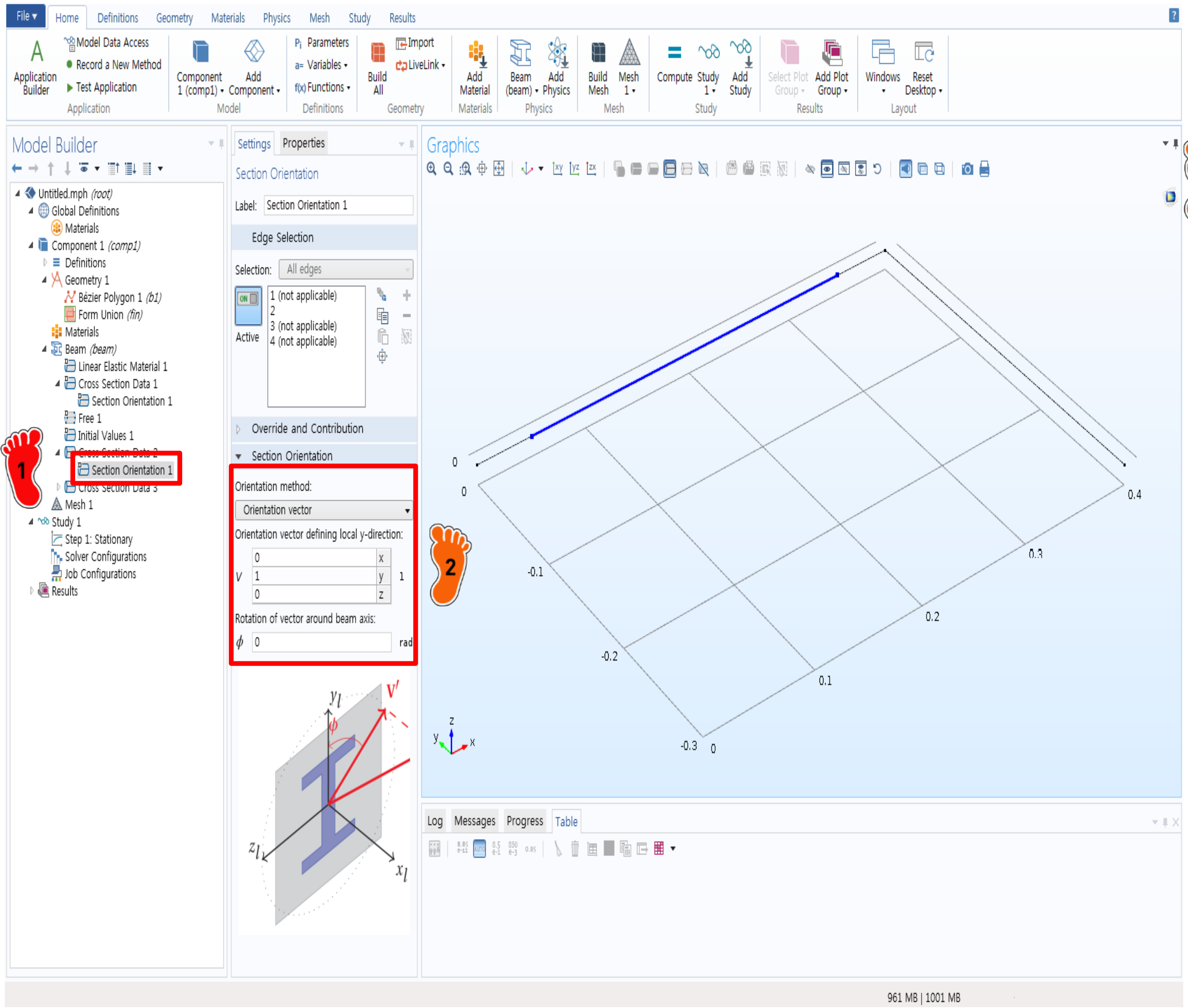
1. Cross Section Data 2 클릭
2. AB 부재 선택 후

Common sections 변경

Circular 변경

지름 0.024 입력

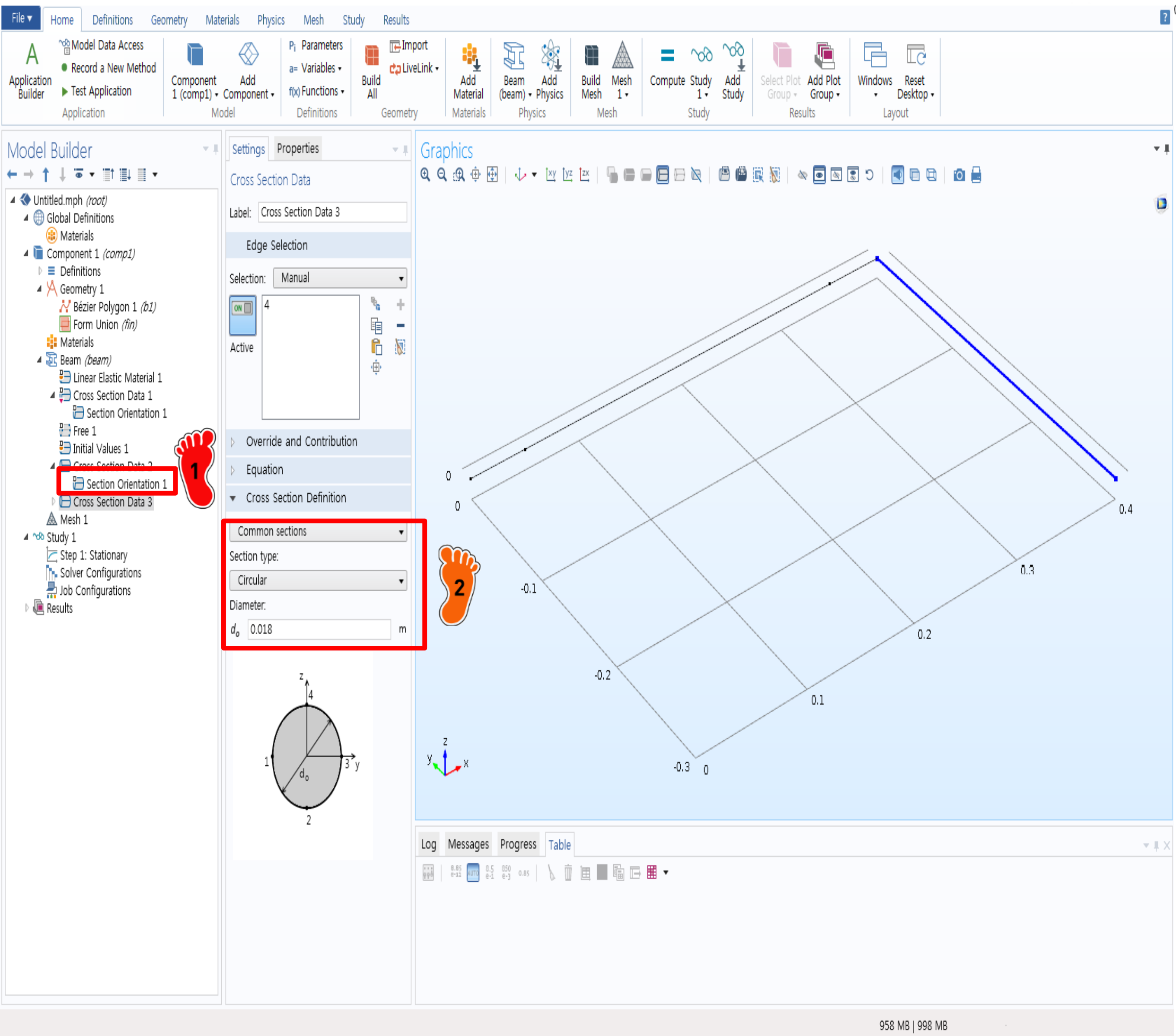
CROSS SECTION



1 Cross Section Data 2 하위 메뉴인 Section Orientation 1 클릭

2 Orientation vector 변경 후 (0,1,0) 입력

CROSS SECTION



1. Cross Section Data 3 클릭

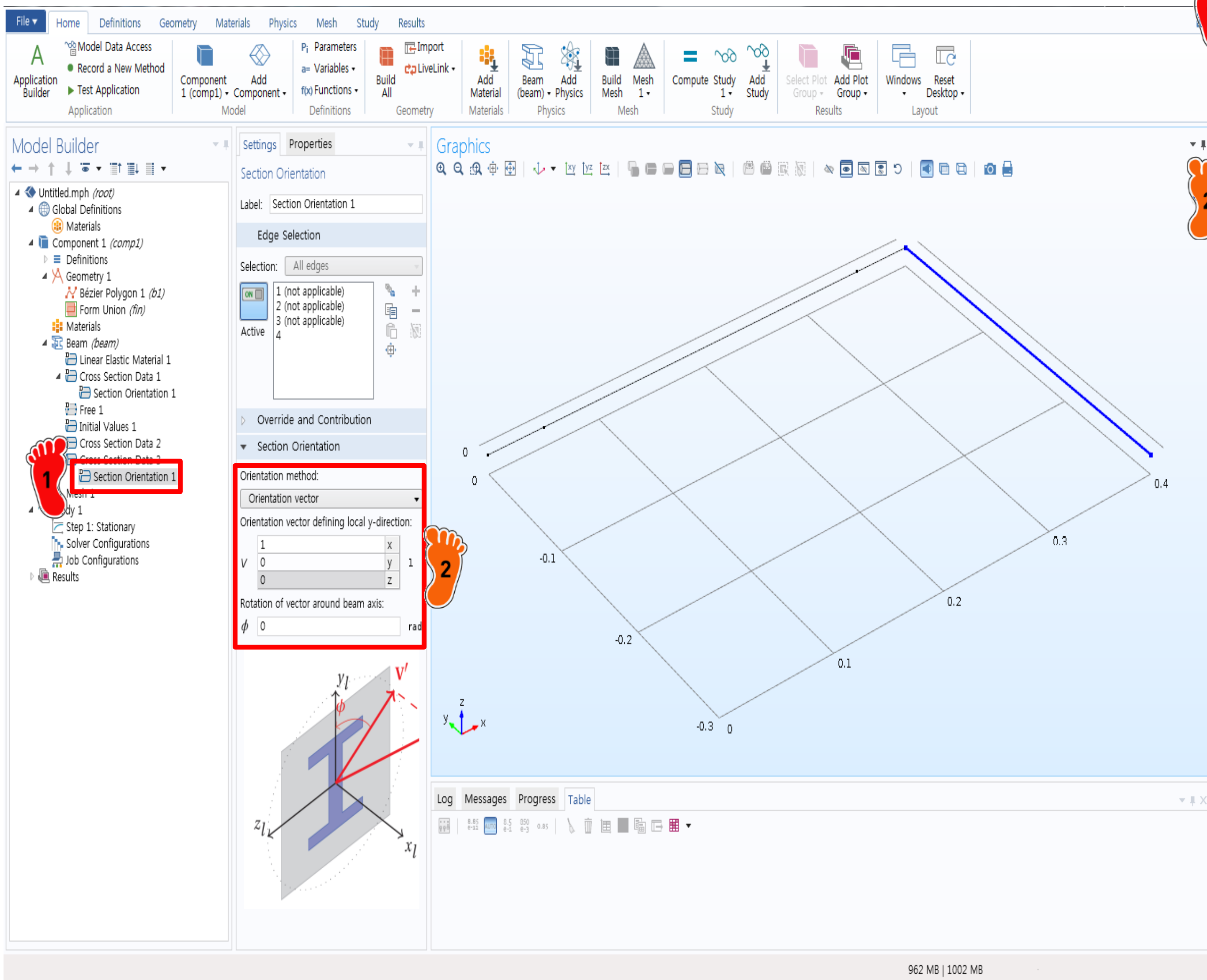
2. CD 부재 선택 후

   Common sections 변경

   Circular 변경

   지름 0.018 입력

CROSS SECTION

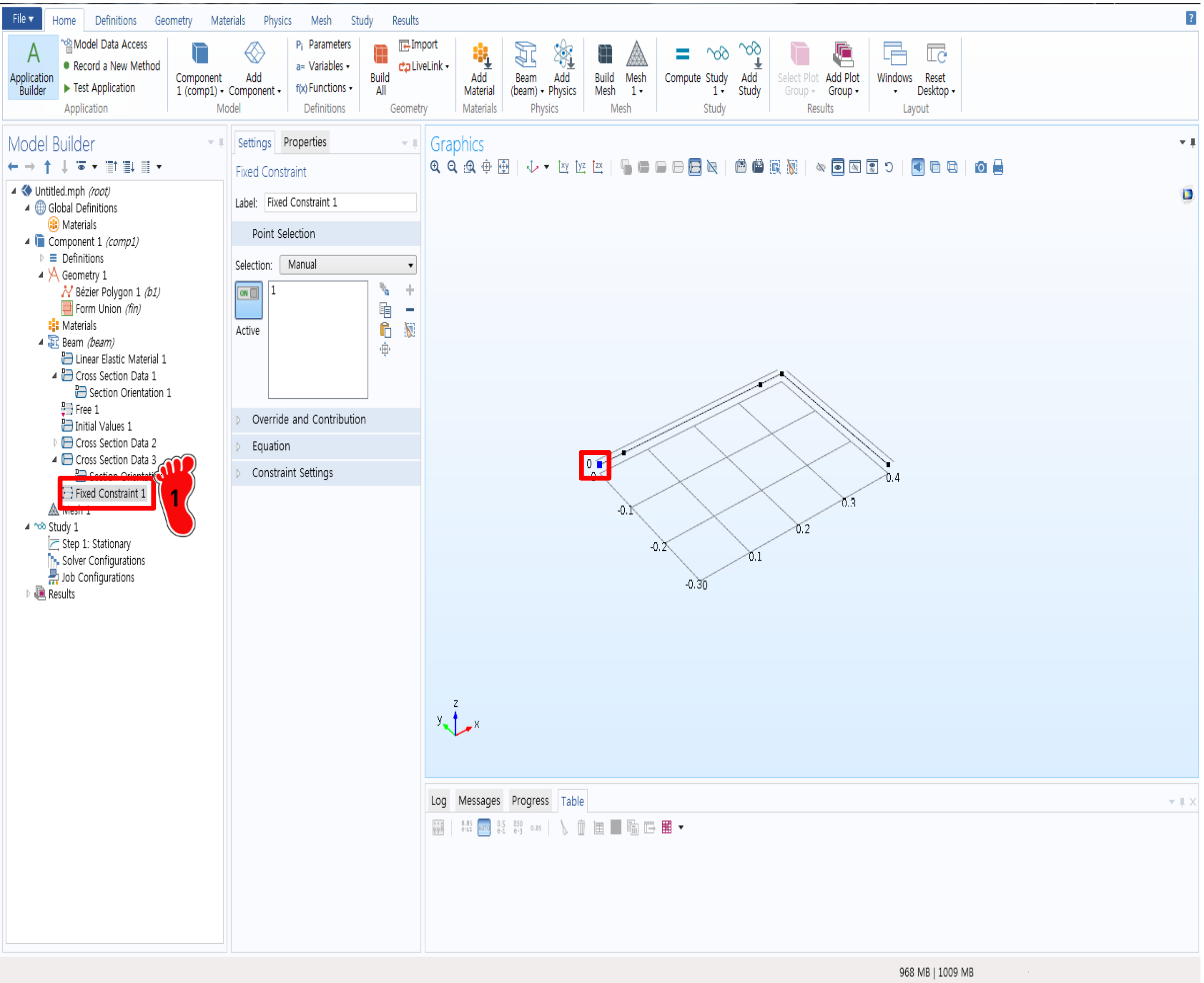


1. Cross Section Data 3 하위 메뉴인 Section Orientation 1 클릭

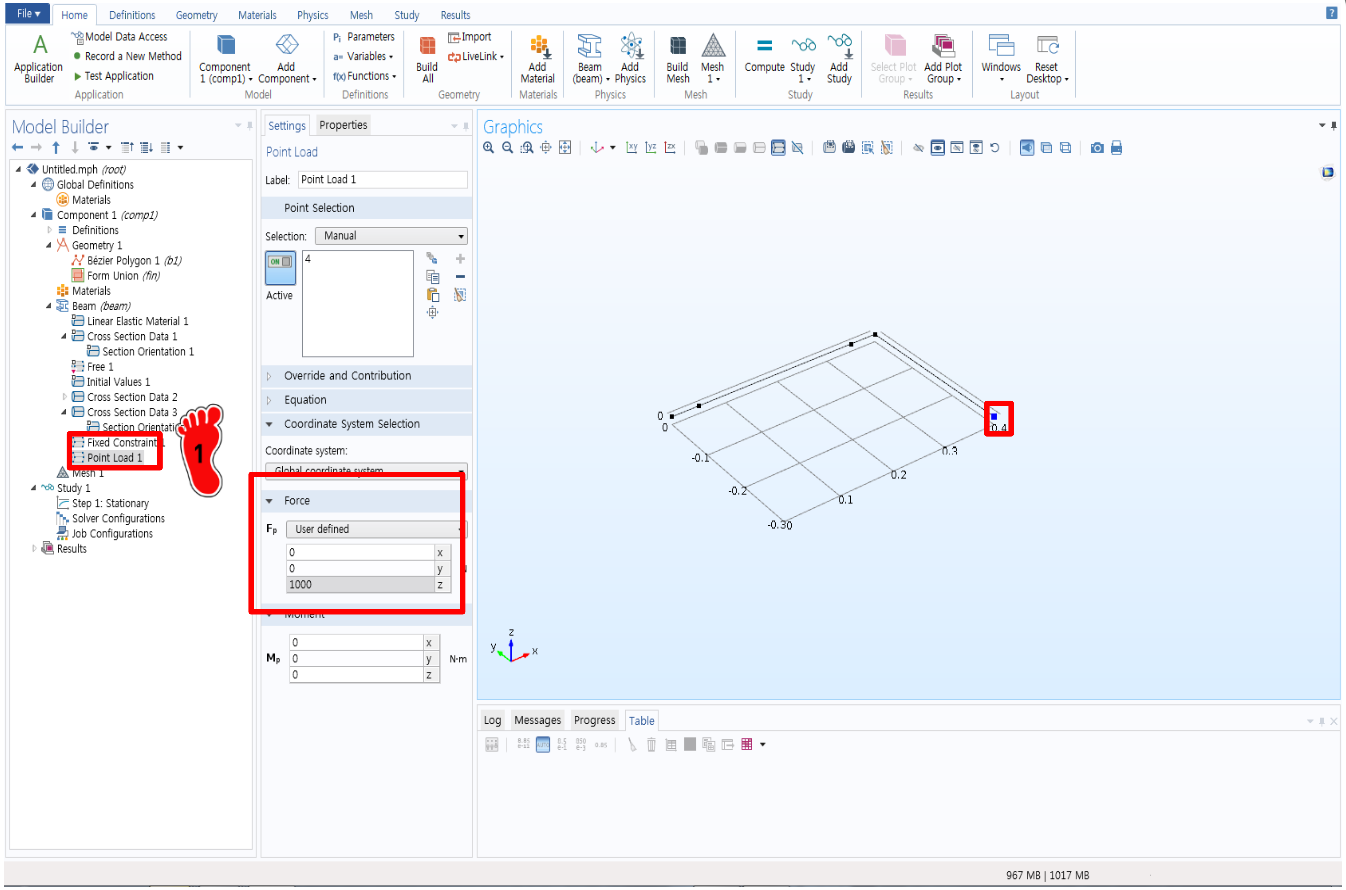
2. Orientation vector 변경 후 (1,0,0) 입력

BOUNDARY CONDITION

1 Fixed Constraint 를 그림에 표기된 절점에 입력

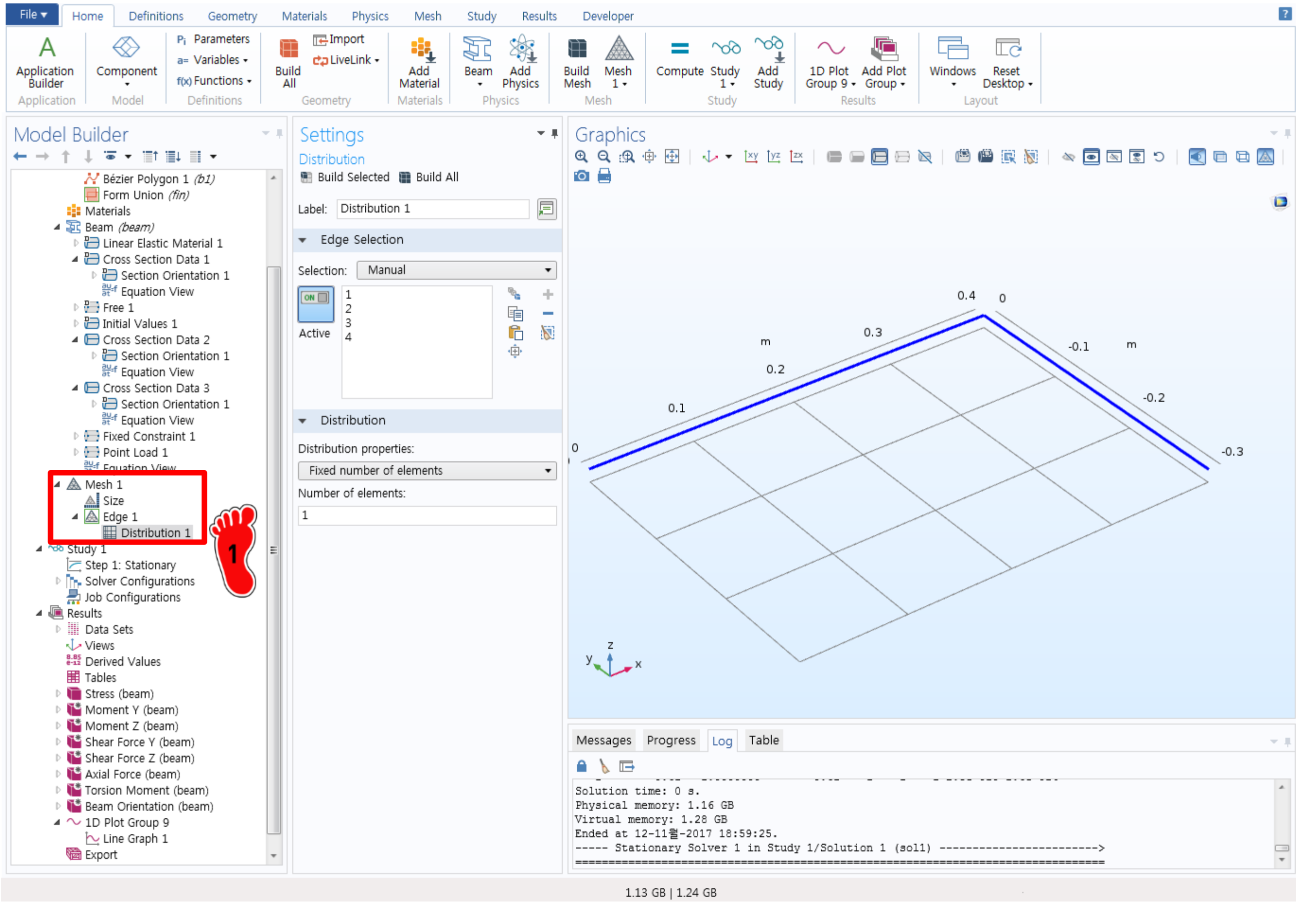


LOADING CONDITION



1 Point Load 조건을 그림에 표기된 절점에 z 방향으로 1000 N 입력

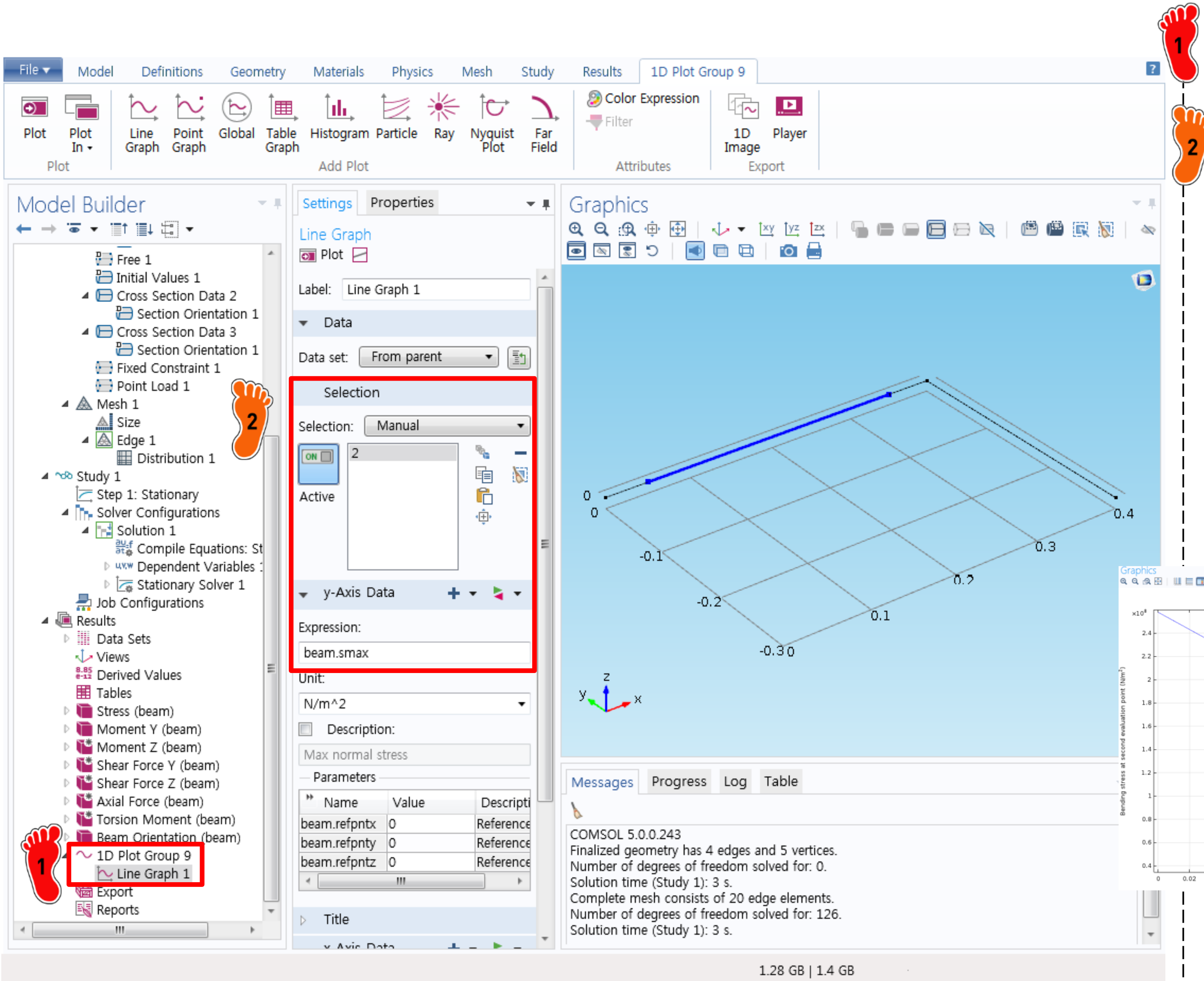
MESH



1 Distribution 1 를 갖는 메시 생성

확인 후 유한요소 해석 실행

POST-PROCESSING

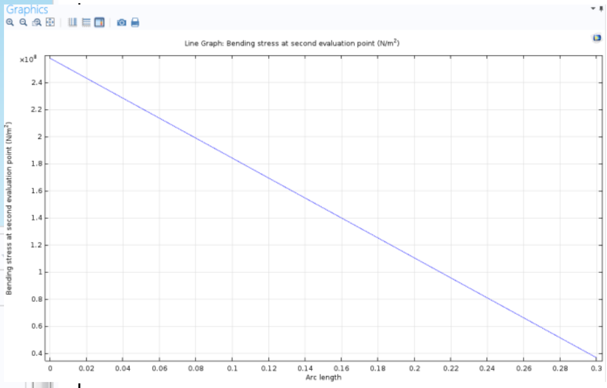


1D Plot Group 에서 Line 추가

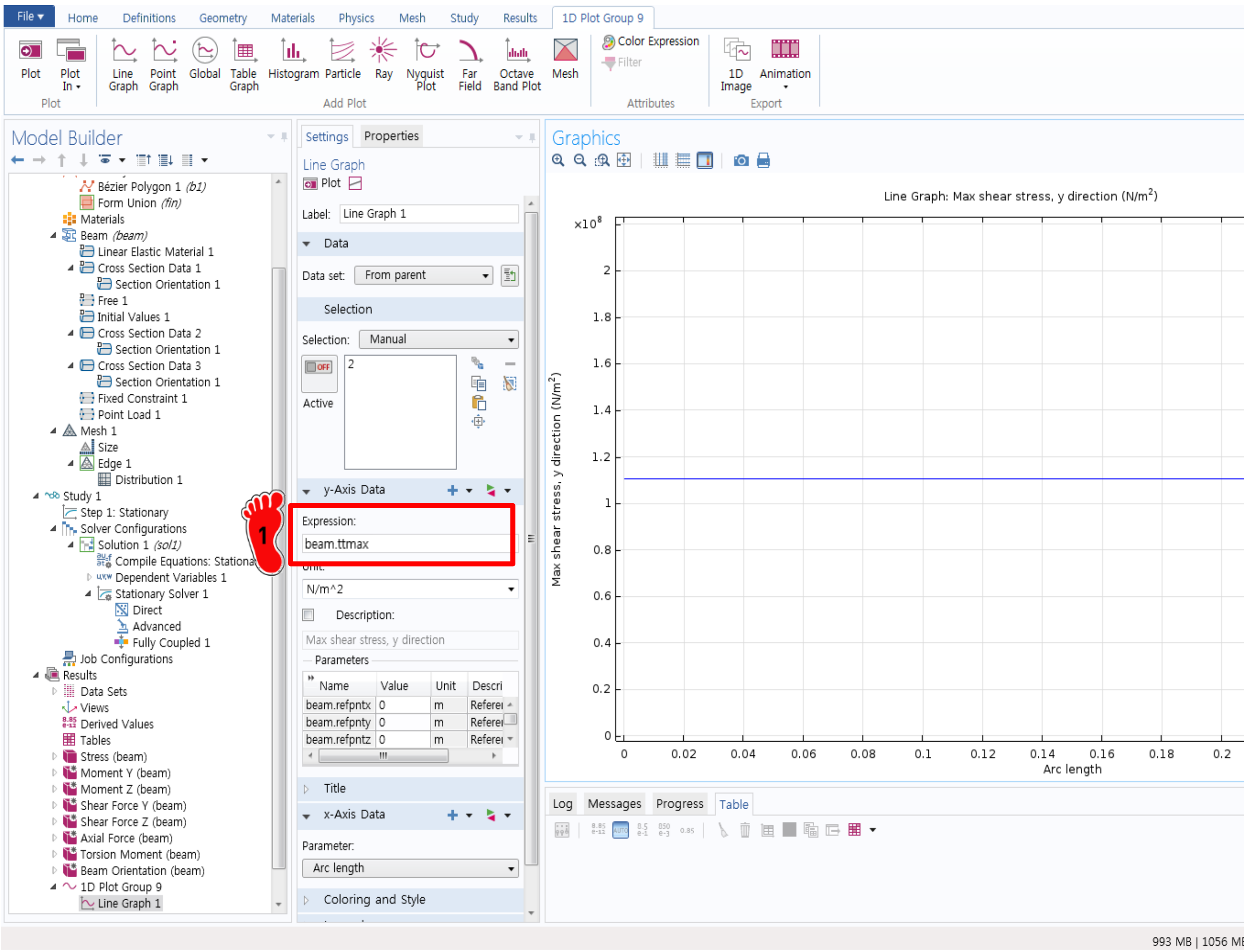
2번 부재 선택 후 beam.smax 입력

Plot 클릭

A 지점에서 σ_x = 258 MPa



POST-PROCESSING



1 beam.ttmax 입력 후 결과 확인

A 지점에서 τ_{max} = 110.5 MPa

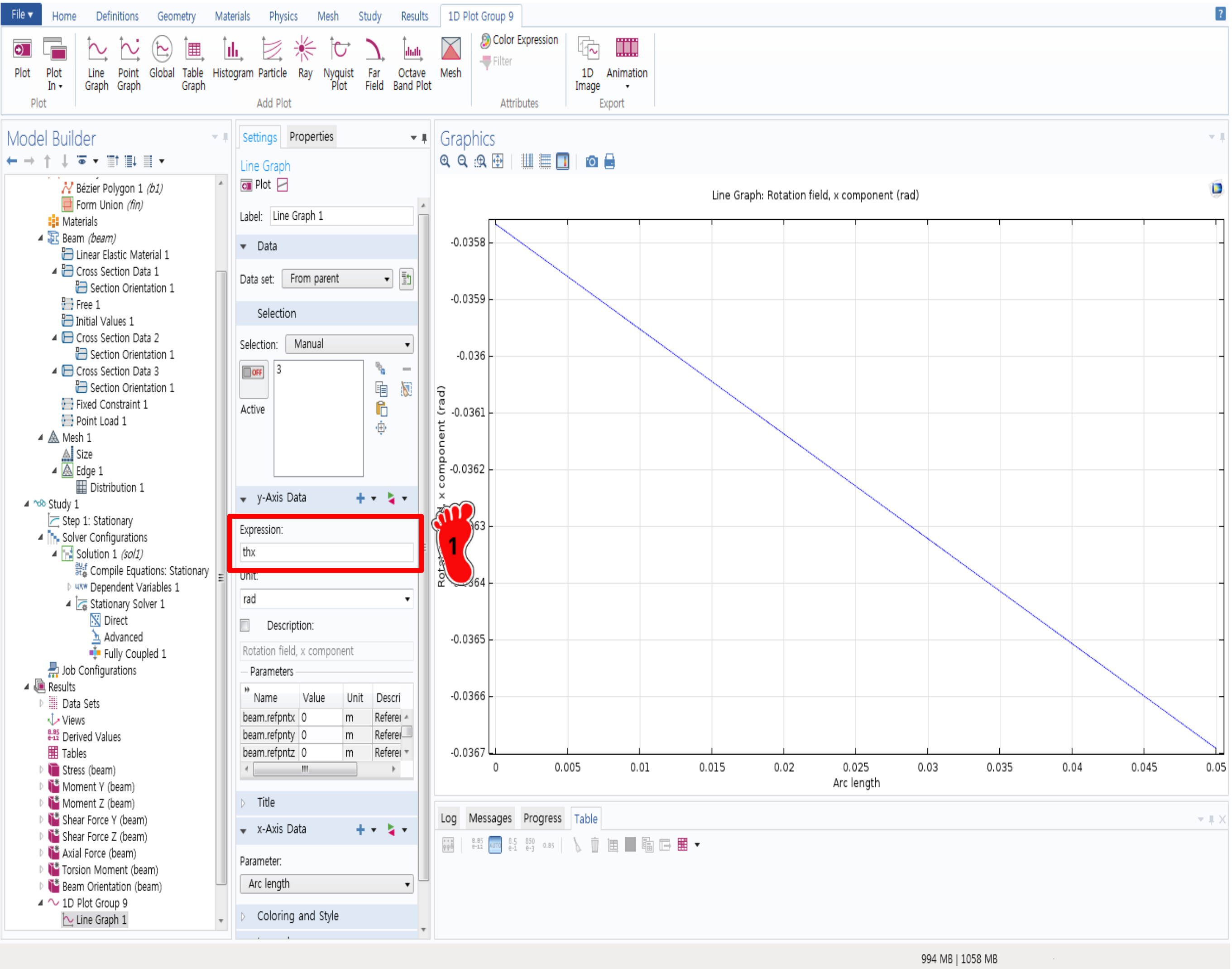
따라서 principal stress 를 계산하면

σ_1 = 298.9 MPa
σ_2 = -40.9 MPa
τ_{max} = 169.9 MPa

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

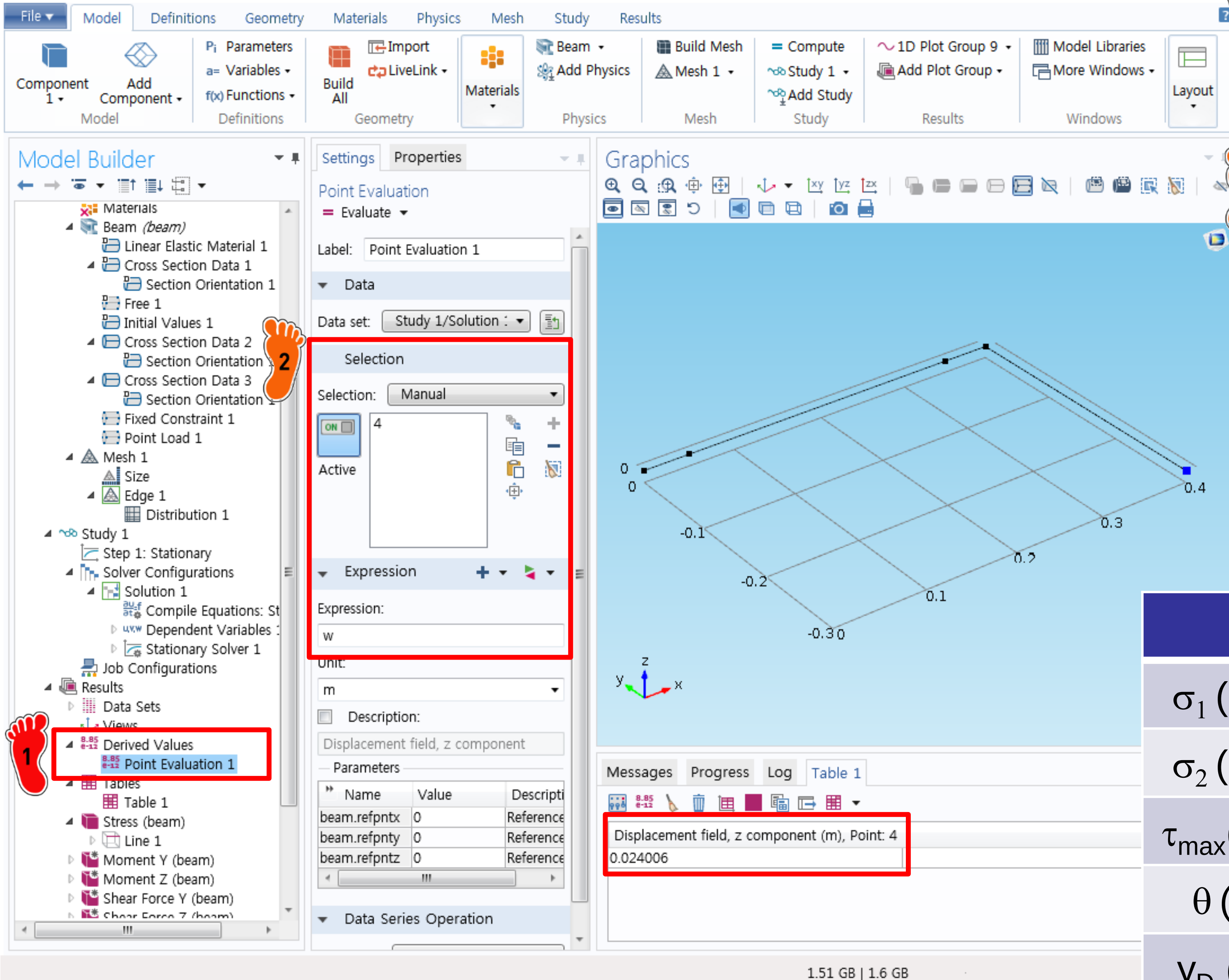
POST-PROCESSING



1 3번 부재의 회전 각도는 thx Expression 으로 확인

0.0367 rad 회전

POST-PROCESSING



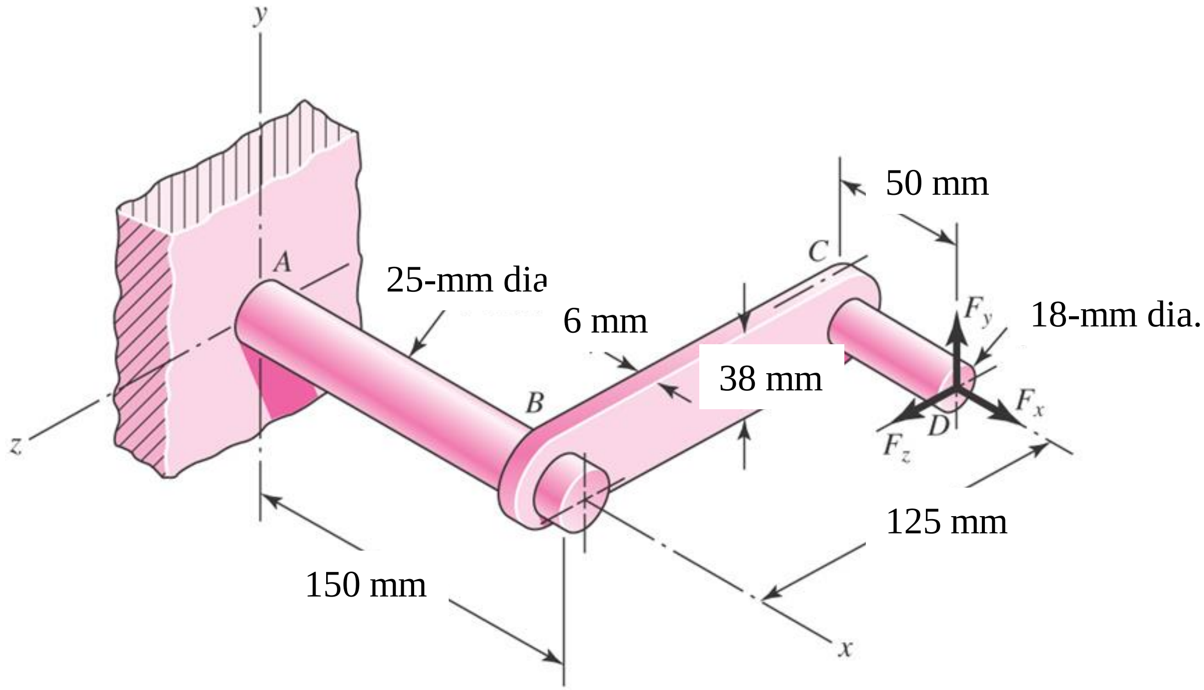
1. Derived Values에서 Point Evaluation 생성

2. 4번 Point 선택 후 w 입력

z 방향 변위는 24.006 mm

	Anal.	FEM
σ_1 (MPa)	298.8	298.9
σ_2 (MPa)	-40.9	-40.9
τ_{max}(MPa)	169.8	169.9
θ (rad)	0.0367	0.0367
y_D (mm)	24.005	24.006

ASSIGNMENT



Loads
- F_y = -800 N
- $F_x = F_z$ = 0 N

[Obtain both analytical and numerical solutions]

The cantilevered handle is made from mild steel that has been welded at the joints.

- Determine the location of the critical stress.
- Determine the principal stresses and the maximum shear stress.
- Determine the vertical deflection at the tip. (Use superposition for the analytical solution.)

[solutions]

$\sigma_1 = 113.7\text{MPa},\ \sigma_2 = -9.4\text{MPa},\ \tau_{max} = 61.6\text{MPa},\ y_D = -2.55\text{mm}$