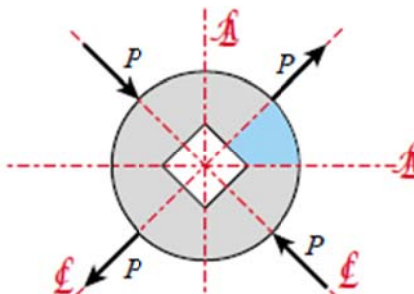
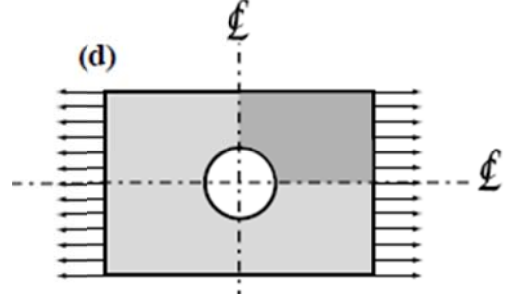
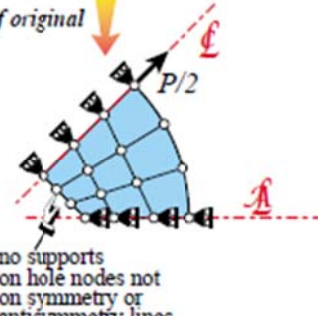
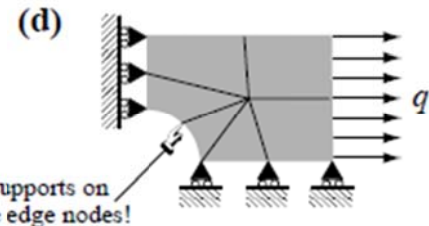


1. (2 pts each) (1) displacement (2) heat flux (3) pressure (4) charge density (electric intensity) (5) magnetic potential

2. 요소절점에 경계조건과 하중조건이 작용되어야 함

Symmetry and antisymmetry lines (marked in red)  Symmetry Line 2개, Anti-symmetry Line 2개 (각1점)	(d)  Symmetry Line 2개 (각2점)
Support conditions and applied loads  요소망 1점, 하중조건 1점, roller조건 각 2점	(d)  요소망 1점, 하중 1점, roller대칭/비대칭 각 2점

3.

$$\mathbf{N} = \begin{bmatrix} 1 - \frac{\bar{x}}{l} & \frac{\bar{x}}{l} \end{bmatrix} = \begin{bmatrix} 1 - \zeta & \zeta \end{bmatrix} \quad (5 \text{ pts})$$

$$\mathbf{K}^e = \int_0^1 EAB^T \mathbf{B} d\zeta = \int_0^1 \frac{EA}{l^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} l d\zeta = \frac{E}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \int_0^1 A(\zeta) d\zeta \quad (5 \text{ pts})$$

$$= \frac{E}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \int_0^1 [A_i(1-\zeta) + A_j\zeta] d\zeta = \frac{E(A_i + A_j)}{2l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \leftrightarrow \mathbf{K}^e = \frac{E\bar{A}}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (5 \text{ pts})$$

(형상함수 설명없이 유도 시 최대 7점)

4.

(1) master-slave method: u_1 (master) = u_3 (5 pts)

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \xrightarrow{\mathbf{u} = \mathbf{T}\bar{\mathbf{u}}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 4 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \rightarrow u_1 = u_2 = \frac{3}{2} = u_3$$

(2) penalty function method (5 pts)

$$\mathbf{w} \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_3 \end{bmatrix} = 0 \xrightarrow{\text{premultiply } \begin{bmatrix} 1 & -1 \end{bmatrix}^T} \mathbf{w} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \xrightarrow{\text{add}} \begin{bmatrix} 2+\mathbf{w} & -1 & 0-\mathbf{w} \\ -1 & 2 & -1 \\ 0-\mathbf{w} & -1 & 2+\mathbf{w} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

(3) Lagrange multiplier method (5 pts)

$$u_1 = u_3 \xrightarrow{\text{add}} \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1-\lambda \\ 0 \\ 2+\lambda \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -1 & 0 & 1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \lambda \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

5. (10 pts)

$$\left. \begin{aligned} N_1^e &= \frac{1}{4}(1-\xi)(1-\eta) \\ N_2^e &= \frac{1}{4}(1+\xi)(1-\eta) \\ N_3^e &= \frac{1}{4}(1+\xi)(1+\eta) \\ N_4^e &= \frac{1}{4}(1-\xi)(1+\eta) \end{aligned} \right\} \xrightarrow{\xi=0.8, \eta=0.9} \left\{ \begin{aligned} N_1^e &= \frac{1}{4}(1-0.8)(1-0.9) = 0.005 \\ N_2^e &= \frac{1}{4}(1+0.8)(1-0.9) = 0.045 \\ N_3^e &= \frac{1}{4}(1+0.8)(1+0.9) = 0.855 \\ N_4^e &= \frac{1}{4}(1-0.8)(1+0.9) = 0.095 \end{aligned} \right.$$

$$x = \sum_{i=1}^4 N_i^e x_i = 0.005(1.0) + 0.045(3.0) + 0.855(3.5) + 0.095(1.5) = 3.275$$

$$y = \sum_{i=1}^4 N_i^e y_i = 0.005(1.0) + 0.045(1.5) + 0.855(4.0) + 0.095(2.5) = 3.73$$

6. (5 pts each)

$$(1) N_1 = c_1(\eta-1)(\xi-1)\xi \rightarrow N_1(-1,-1) = -4c_1 = 1 \rightarrow c_1 = -\frac{1}{4} \rightarrow N_1 = -\frac{1}{4}(\eta-1)(\xi-1)\xi$$

$$(2) \text{ compatibility } \begin{cases} \text{edge1-4: } N_1(\xi=-1) = -\frac{1}{2}(\eta-1) \rightarrow \text{linear} \xleftrightarrow{OK} \text{two points} \\ \text{edge1-5-2: } N_1(\eta=-1) = -\frac{1}{2}(\xi-1)\xi \rightarrow \text{quadratic} \xleftrightarrow{OK} \text{three points} \end{cases}$$

$$(3) \text{ completeness: } \sum_{i=1}^6 N_i = 1$$

$$(4) \begin{cases} \text{edge1-4: } N_1 \text{ varies linearly over 6-node element side} \\ \xleftrightarrow{OK} \text{shape functions of 4-node bilinear quadrilateral vary linearly over any side} \\ \text{edge1-5-2: } N_1 \text{ varies quadratically over 6-node element side} \\ \xleftrightarrow{OK} \text{shape functions of 9-node bilinear quadrilateral vary quadratically over any side} \end{cases}$$

$$(5) r = \min(n_F - n_R, 3n_G) = \min(12 - 3, 3n_G) \rightarrow n_G \geq 3$$

$$(6) n_G \geq 3 \rightarrow p = 2$$