1. (1) 1.618, golden ratio (2) 2.618δ (3) penalty parameter (4) stochastic (5) simulated annealing (6) reproduction (7) mutation (8) linprog/quadprog (9) ExitFlag (10) iteration

2. (1) (5 pts.) 면적 아니고 framework 길이의 합 임

$$\begin{aligned} &Minimize\ \ f = 80w + 120d + 200h \\ &\text{subject to}\ \ g_1 = 1 - wdh/600 \le 0 \\ &\qquad g_2 = -w \le 0 \\ &\qquad g_3 = -d \le 0 \\ &\qquad g_4 = -h \le 0 \end{aligned}$$

(2) (5 pts.) LP subproblem @ (w, d, h)=(10, 10, 4)

$$g_1(10,10,4) = \frac{1}{3}$$

$$\nabla g_1(10,10,4) = \left(-\frac{dh}{600}, -\frac{wh}{600}, -\frac{wd}{600}\right) = \left(-\frac{1}{15}, -\frac{1}{15}, -\frac{1}{6}\right)$$

$$\begin{aligned} &Minimize\ \ \bar{f} = 80d_1 + 120d_2 + 200d_3 \\ &\text{subject to}\ \ \bar{g}_1 = -\frac{1}{15}d_1 - \frac{1}{15}d_2 - \frac{1}{6}d_3 \le -\frac{1}{3} \\ &\qquad \bar{g}_2 = -d_1 \le 10 \\ &\qquad \bar{g}_3 = -d_2 \le 10 \\ &\qquad \bar{g}_4 = -d_3 \le 4 \\ &\qquad -5 \le d_1 \le 5 \\ &\qquad -5 \le d_2 \le 5 \\ &\qquad -2 \le d_3 \le 2 \end{aligned}$$

(3) (5 pts.) QP subproblem @ (w, d, h)=(10, 10, 4)

$$\begin{aligned} &Minimize\ \ \bar{f} = 80d_1 + 120d_2 + 200d_3 + 0.5\left(d_1^2 + d_2^2 + d_3^2\right) \\ &\text{subject to}\ \ \bar{g}_1 = -\frac{1}{15}d_1 - \frac{1}{15}d_2 - \frac{1}{6}d_3 \le -\frac{1}{3} \\ &\qquad \bar{g}_2 = -d_1 \le 10 \\ &\qquad \bar{g}_3 = -d_2 \le 10 \\ &\qquad \bar{g}_4 = -d_3 \le 4 \end{aligned}$$

3. $f(\boldsymbol{x}) = 100\left(x_2 - x_1^2\right)^2 + \left(1 - x_1\right)^2$ and $\boldsymbol{x}^{(0)} = (0,\ 1)^T$, $\boldsymbol{x}^{(1)} = (1,\ 0)^T$

$$\nabla f(\boldsymbol{x}) = \begin{bmatrix} -400x_1x_2 + 400x_1^3 - 2 + 2x_1 \\ 200\left(x_2 - x_1^2\right) \end{bmatrix}, \nabla^2 f(\boldsymbol{x}) = \begin{bmatrix} -400x_2 + 1200x_1^2 + 2 & -400x_1 \\ -400x_1 & 200 \end{bmatrix}$$

(1) (5 pts.) steepest descent (Cauchy) method

$$\boldsymbol{d}^{(1)} = -\nabla f\left(\boldsymbol{x}^{(1)}\right) = \begin{bmatrix} -400 & 200 \end{bmatrix}^T$$

(2) (5 pts.) conjugate gradient method

$$\boldsymbol{d}^{(0)} = -\nabla f\left(\boldsymbol{x}^{(0)}\right) = \begin{bmatrix} 2 \\ -200 \end{bmatrix}, \beta_1 = \left(\frac{\left\|\nabla f\left(\boldsymbol{x}^{(1)}\right)\right\|}{\left\|\nabla f\left(\boldsymbol{x}^{(0)}\right)\right\|}\right)^2$$

$$\boldsymbol{d}^{(1)} = -\nabla f\left(\boldsymbol{x}^{(1)}\right) + \beta_1 \boldsymbol{d}^{(0)} = \begin{bmatrix} -400 \\ 200 \end{bmatrix} + \underbrace{\frac{\left(400^2 + 200^2\right)}{\left(2^2 + 200^2\right)}}_{\approx 5} \begin{bmatrix} 2 \\ -200 \end{bmatrix} \approx \begin{bmatrix} -390 \\ -800 \end{bmatrix}$$

(3) (5 pts.) Newton's method

$$\boldsymbol{d}^{(1)} = -\left[\nabla^2 f\left(\boldsymbol{x}^{(1)}\right)\right]^{-1} \nabla f\left(\boldsymbol{x}^{(1)}\right) = -\begin{bmatrix} 1202 & -400 \\ -400 & 200 \end{bmatrix}^{-1} \begin{bmatrix} 400 \\ -200 \end{bmatrix} = \frac{-1}{80400} \begin{bmatrix} 200 & 400 \\ 400 & 1202 \end{bmatrix} \begin{bmatrix} 400 \\ -200 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

4.

(1) (10 pts.) augmented Lagrange multiplier method

$$\left.\begin{aligned} &\textit{Minimize} \;\; f(\boldsymbol{x}) = x_1^2 + x_2^2 \\ &\text{subject to} \;\; h(\boldsymbol{x}) = x_1 + x_2 - 1 = 0 \end{aligned}\right\} \rightarrow A(\boldsymbol{x}, \lambda, r) = x_1^2 + x_2^2 + \lambda(x_1 + x_2 - 1) + r(x_1 + x_2 - 1)^2$$

$$\left.\begin{aligned} \frac{\partial A}{\partial x_1} &= 2x_1 + \lambda + 2r(x_1 + x_2 - 1) = 0 \\ \frac{\partial A}{\partial x_2} &= 2x_2 + \lambda + 2r(x_1 + x_2 - 1) = 0 \end{aligned}\right\} \rightarrow x_1 = x_2 = \frac{2r - \lambda}{2 + 4r} \xrightarrow{r=1} x_1 = x_2 = \frac{2 - \lambda}{6}$$

(2) (5 pts.) $\lambda^{k+1} = \lambda^k + 2r\, h(\boldsymbol{x}^k)$

iteration	λ	x1 = x2	h($\boldsymbol{x}$)
1	0	1/3	-1/3
2	-2/3	4/9	-1/9
3	-8/9	13/27	-1/27
exact	-1	1/2	0

Method	MV-OPT problem type solved	Can find feasible discrete solution?	Can find global minimum for convex problem?	Need gradients?
BBM	1–5	Yes	Yes	No/Yes
SA	1–5	Yes	Yes	No
Genetic algorithm	1–5	Yes	Yes	No
Sequential linearization	1	Yes	Yes	Yes
Dynamic round-off	1	Yes	No guarantee	Yes
Neighborhood search	1	Yes	Yes	Yes

Method	Can solve discrete problems?	General constraints?	Tries to find all x*?	Phases	Needs gradients?
Covering (D)	No	No	Yes	G	1
Zooming (D)	Yes[1]	Yes	No	L	1
Generalized descent (D)	No	No	No	G	Yes
Tunneling (D)	No	Yes	No	L + G	1
Multistart (S)	Yes[1]	Yes	Yes	L + G	1
Clustering (S)	Yes[1]	Yes	Yes	L + G	1
Controlled random search (S)	Yes	No	No	L + G	No
Acceptance–rejection (S)	Yes[1]	Yes	No	G	No
Stochastic integration (S)	No	No	No	G	No
Genetic (S)	Yes	No	No	G	No
Stochastic zooming (S)	Yes[1]	Yes	No	L + G	1
Domain elimination (S)	Yes[1]	Yes	Yes	L + G	1

Method	Always yields Pareto optimal point?	Can yield all Pareto optimal points?	Involves weights?	Depends on function continuity?	Uses utopia point?
Genetic	Yes	Yes	No	No	No
Weighted sum	Yes	No	Yes	Problem type and optimization engine determines this	Utopia point or its approximation is needed for function normalization or in the formulation of the method
Weighted min–max	Yes[a]	Yes	Yes	Same as above	Same as above
Weighted global criterion	Yes	No	Yes	Same as above	Same as above
Lexicographic	Yes[b]	No	No	Same as above	No
Bounded objective function	Yes[c]	No	No	Same as above	No
Goal programming	No	No	No[d]	Same as above	No