Contents

- Genetic Algorithms (GAa)
- Differential Evolution Algorithm (DEA)
- Ant Colony Optimization (ACO) Algorithm
- Particle Swarm Optimization (PSO) Algorithm

Basic Concept (1)

- optimization algorithms inspired by natural phenomena
  - stochastic programming, evolutionary algorithms, genetic programming, swarm intelligence, evolutionary computation, nature-inspired metaheuristics methods
- general class of direct search methods
  - do not require the continuity or differentiability of problem functions
  - evaluate functions at any point within the allowable ranges for the design variables
- use stochastic ideas and random numbers in their calculations to search for the optimum point
  - executed at different times, the algorithms can lead to a different sequence of designs and a different solution even with the same initial conditions
  - tend to converge to a global minimum point for the function, but there is no guarantee of convergence or global optimality

$\text{Minimize } f(\mathbf{x}) \text{ for } \mathbf{x} \in S$

Basic Concept (2)

- can overcome some of the challenges that are due to
  - multiple objectives, mixed design variables, irregular/noisy problem functions, implicit problem functions, expensive and/or unreliable function gradients, and uncertainty in the model and the environment
- very general and can be applied to all kinds of problems— discrete, continuous, and nondifferentiable
- relatively easy to use and program since they do not require the use of gradients of cost or constraint functions
- drawbacks of these algorithms
  - require a large amount of function evaluations → use of massively parallel computers
  - no absolute guarantee that a global solution has been obtained → execute the algorithm several times and allow it to run longer

Genetic Algorithm (GA)

- Search algorithm based on the mechanics of natural selection and natural genetics subject to Darwin's theory of "survival of the fittest" among string structures
  - Basic operations of natural genetics: reproduction, crossover, mutation
  - Mixed continuous-discrete variables, discontinuous and nonconvex design spaces (practical optimum design problems)
  - Global optimum solution with a high probability
    - J. H. Holland, Adaptation in Natural and Artificial Systems, University of Michigan Press, Ann Arbor, Mich., 1975
    - D. E. Goldberg, Genetic Algorithms in Search, Optimization, and Machine Learning, Addison-Wesley, 1983

$$
\begin{aligned}
&\text{maximize} \quad f(\boldsymbol{x}) \\
&\text{subject to} \quad l_i \le x_i \le u_i, \quad i = 1, \ldots, n
\end{aligned}
$$

GA: Terminology

- Gene: each design variable (x)
- Chromosome: group of design variables
- Individual: each design point
- Fitness: how good is the individual?
- Population: group of individuals

Individual
Chromosome [0.00,4.00,2.80,19.0] or [0000010011101101] Fitness 25.8

- Genetic operators: drive the search
  - Selection: select the high fitness individuals, exploit the info
  - Crossover: parents create children, explore the design space
  - Mutation: sudden random changes in chromosomes
  - Inversion: reverse gene sequence (sometimes improve diversity)
- Generation: each cycle of genetic operations

GA: Characteristics

- A population of points is used for starting the procedure instead of a single design point.
  - Size of the population: 2n to 4n (n: # of DVs)
  - Less likely to get trapped at a local optimum
- GAs use only the values of the objective function.
  - No derivatives used in the search procedure
- Design variables are represented as strings of binary variables that correspond to the chromosomes in natural genetics.
  - Naturally applicable for solving discrete and integer programming problems
- The objective function value corresponding to a design vector plays the role of fitness in natural genetics.
- In every new generation, a new set of strings is produced by using randomized parents selection and crossover from the old generation.
  - Efficiently explore the new combinations with the available knowledge

Design Representation: Schema (1)

- it needs to be encoded (ie, defined)
  - binary encoding, real-number coding, integer encoding
- V–string for a binary string
  - represents the value of a variable
  - the component of a design vector (a gene)
- D–string for a binary string
  - represents a design of the system
  - particular combination of n V–strings (n: number of design variables)
  - genetic string (or a chromosome)

Design Representation: Schema (2)

V-string $\Leftrightarrow$ discrete value of a variable having N_c allowable discrete values

let m be the smallest integer satisfying $2^m > N_c$

$$j = \sum_{i=1}^{m} ICH(i)\, 2^{(i-1)} + 1 \text{ where } ICH(i): \text{value of the } i\text{-th digit (either 0 or 1)}$$

$$\text{when } j > N_c,\ j = INT\left(\frac{N_c}{2^m - N_c}\right)(j - N_c)$$

$n = 3,\ N_c = 10 \rightarrow m = 4$

j value for three V-strings $\rightarrow \{7, 16, 14\} \rightarrow \{7, 6, 4\}$

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ |0110| & |1111| & |1101| \end{bmatrix} \qquad 3467\ 0254\ 7932\ 7612 \xrightarrow[5\sim9\rightarrow1]{0\sim4\rightarrow0} 0011\ 0010\ 1100\ 1100$$

Fitness Function: defines the relative importance of a design

$$F_i = (1+\varepsilon)\, f_{\max} - f_i$$

f_i : cost function (penalty function value for a constrained problems) for the i-th design

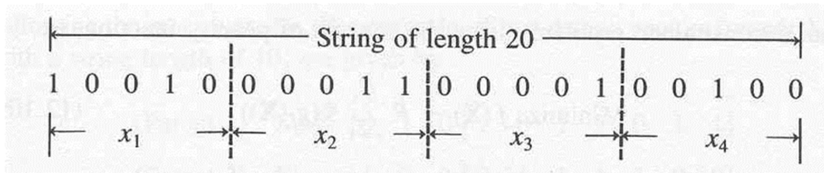
$f_{\max}$: largest recorded cost (penalty) function value

Genetic Operations

- Coding and decoding of design variables

$$\text{binary number}: b_q b_{q-1} \cdots b_2 b_1 b_0 \quad where\ \ b_k = 0 \text{ or } 1$$

$$x = x_l + \frac{x_u - x_l}{2^q - 1} \underbrace{\sum_{k=0}^{q} 2^k b_k}_{decimal}$$

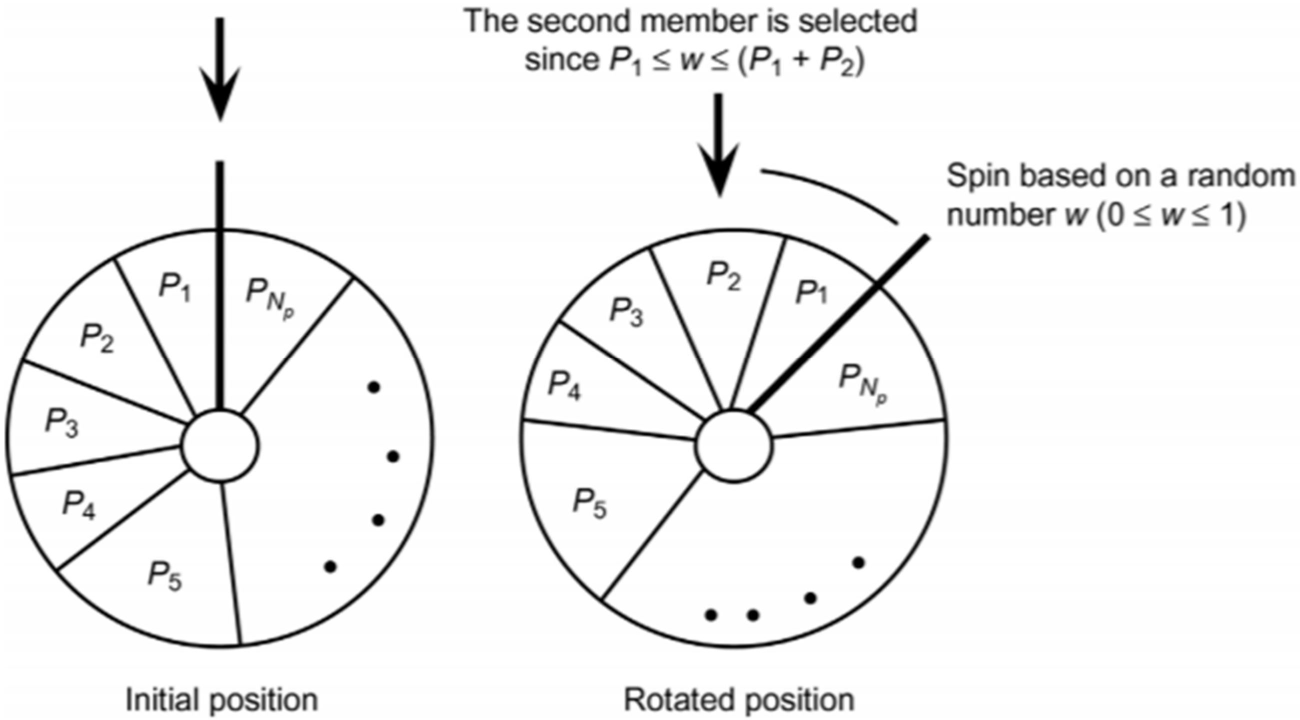


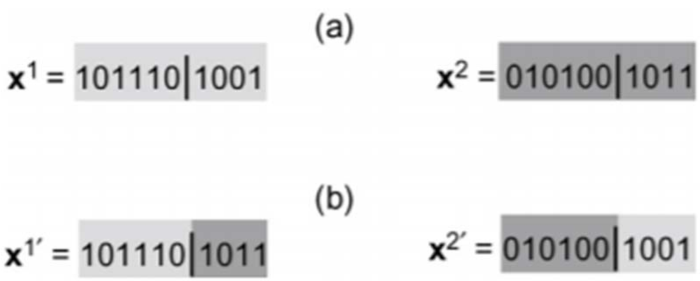
$$2^q \geq \frac{x_u - x_l}{\Delta x} + 1 \quad (\Delta x : accuracy)$$

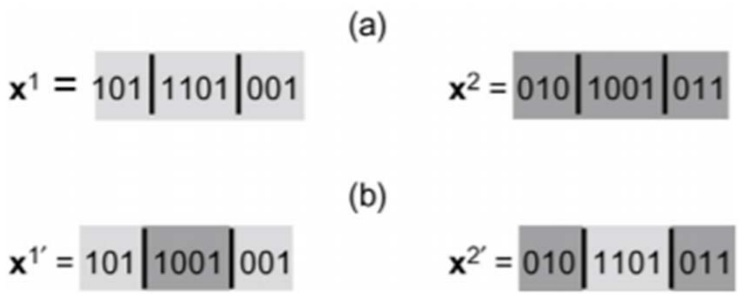
- Creation of a mating pool (selection)
  - The weaker members are replaced by stronger ones based on the fitness values.
  - Eg., Roulette wheel selection

$$f_i' = \frac{f_i + C}{D} \quad where\ \ C = 0.1 f_h - 1.1 f_l,\ \ D = \max(1,\ f_h + C)$$

$$S = \sum_{i=1}^{z} f_i' \ (z : \text{population size}),\ \ \sum_{i=1}^{j-1} f_i' \leq rS \leq \sum_{i=1}^{j} f_i' \ (r : \text{random number}, z \text{ times})$$


The second member is selected
since $P_1 \le w \le (P_1 + P_2)$
Spin based on a random
number w $(0 \le w \le 1)$
P_1 P_{N_p} P_2 P_3 P_4 P_5
Initial position
P_2 P_1 P_3 P_4 P_{N_p} P_5
Rotated position


(a)
$\mathbf{x}^1$ = 101110|1001
$\mathbf{x}^2$ = 010100|1011
(b)
$\mathbf{x}^{1'}$ = 101110|1011
$\mathbf{x}^{2'}$ = 010100|1001


(a)
$\mathbf{x}^1$ = 101|1101|001
$\mathbf{x}^2$ = 010|1001|011
(b)
$\mathbf{x}^{1'}$ = 101|1001|001
$\mathbf{x}^{2'}$ = 010|1101|011

Example

Maximize $f(x_1, x_2) = 21.5 + x_1 \sin(4\pi x_1) + x_2 \sin(20\pi x_2)$

subject to $-3.0 \le x_1 \le 12.1$

$4.1 \le x_2 \le 5.8$

binary coding with 5 decimal digits

$x_1 : 2^{17} < [12.1 - (-3.0)] \times 10^5 = 151{,}000 \le 2^{18} \rightarrow q_1 = 18$

$x_2 : 2^{14} < [5.8 - 4.1] \times 10^5 = 17{,}000 \le 2^{15} \rightarrow q_2 = 15$

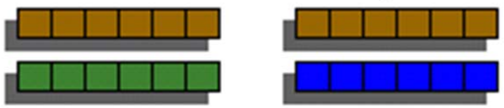
chromosome: $\underbrace{000001010100101001}_{x_1:18bit \rightarrow 5417}\underbrace{101110111111110}_{x_2:15bit \rightarrow 24318}$

$$\rightarrow \begin{cases} x_1 = -3.0 + 5417 \times \dfrac{12.1 - (-3.0)}{2^{18} - 1} = -2.68797 \\ x_1 = 4.1 + 24318 \times \dfrac{5.8 - 4.1}{2^{15} - 1} = 5.36165 \end{cases}$$

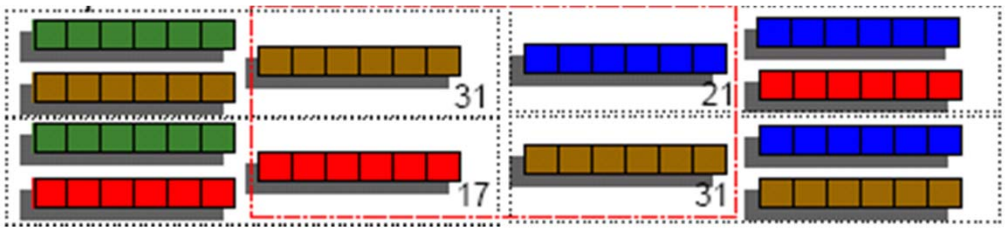
Reproduction/Selection

- Exploit the available information – drive to high fitness region
- Better individuals go to mating pool
- Population of high fitness individuals increased
- Remove low fitness individuals
- Improve the mean fitness
- Roulette wheel
  - Select individuals probabilistically

Individual	Fitness	Probability
	8	0.10
	21	0.27
	31	0.40
	17	0.22
	77	



- Tournament selection
  - Compare two individuals at a time



Crossover

- Chromosomes are spliced – genes are shared

- Diversify the population – explore the design space
- Select parents randomly and mate them probabilistically(c_p)
- Binary crossover

$x_1^1 = 10$, $x_2^1 = 11$, 010|101011 → 010|000111 y_1^1 =08, y_2^1 =07

$x_1^2 = 20$, $x_2^2 = 07$, 101|000111 → 101|101011 y_1^2 =22, y_2^2 =11

- Real crossover

$$y_i^1 = \alpha_i x_i^1 + \beta_i x_i^2; \qquad y_i^2 = \beta_i x_i^1 + \alpha_i x_i^2$$

$$x_1^1 = 10, x_2^1 = 11; \qquad x_1^2 = 20, x_2^2 = 07$$

$$\alpha_1 = 0.5, \beta_1 = 1.25; \qquad \alpha_2 = 1.5, \beta_2 = -0.75$$

$$y_1^1 = 0.5x_1^1 + 1.25x_1^2 = 30; \quad y_2^1 = 1.5x_2^1 - 0.75x_2^2 = 11.25$$

$$y_1^2 = 1.25x_1^1 + 0.5x_1^2 = 22.5; \quad y_2^2 = -0.75x_2^1 + 1.5x_2^2 = 2.25$$

Mutation / Inversion

- Some genes change randomly
- Sometimes these changes are favorable
- Select genes(bits) probabilistically for mutation(m_p:0.005~0.1)
- Binary mutation

10110111 [x_1=11,x_2=7] → 00110011 [x_1=11,x_2=3]

- Real mutation

$$y_i = x_i + \delta \Delta x_i \quad x_2 = 11;\ \Delta x = 1, \delta = 0.5;\ y_i = 11 + 0.5 = 11.5$$

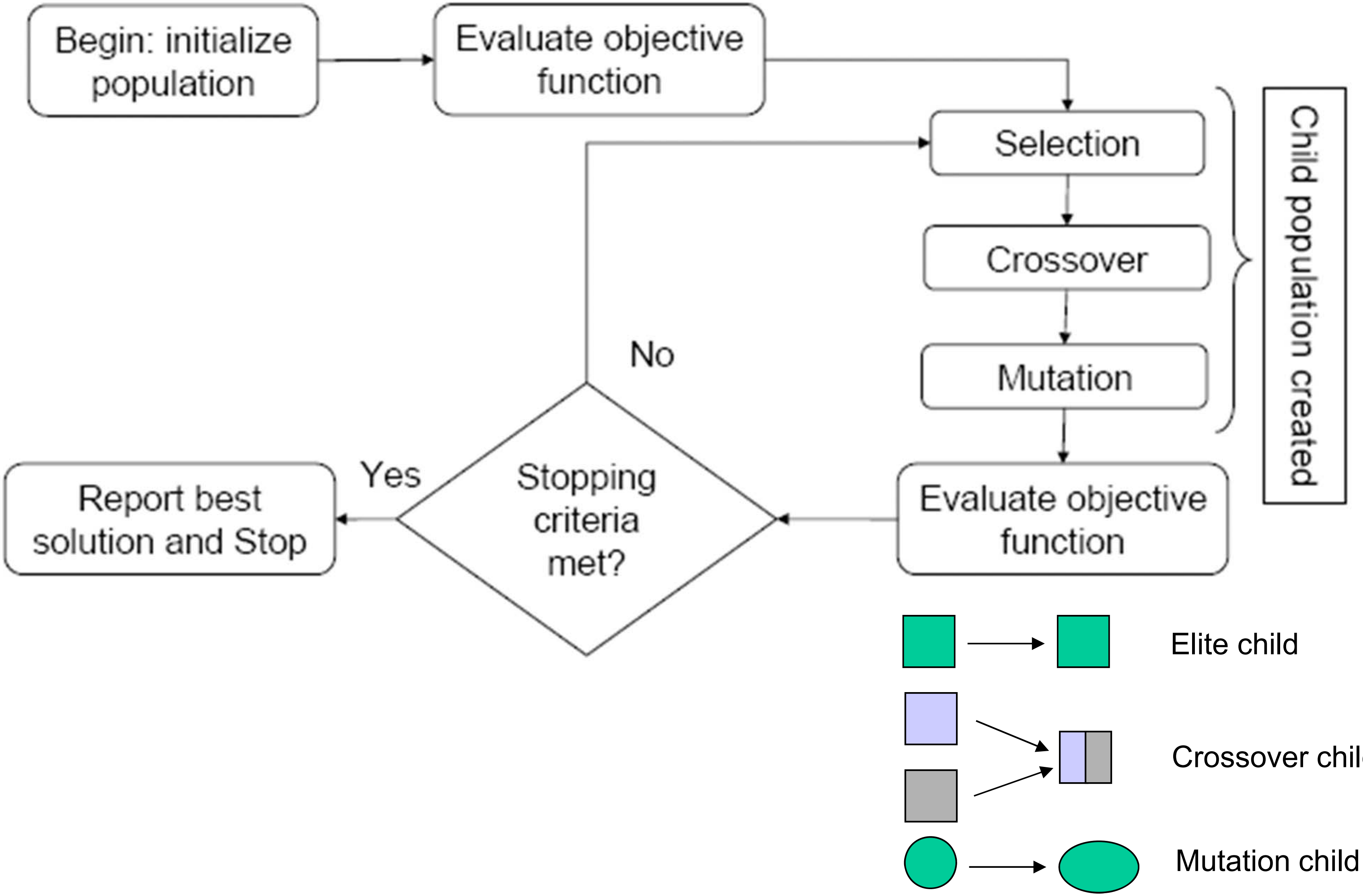
- Inversion: seldom used

10110101 [x_1=11,x_2=5] 10011101 [x_1=9,x_2=13]

Stopping Criteria

- Number of generations
- Number of function evaluations
- No improvement for a certain number of generations

Flowchart of Simple Genetic Algorithm



Observations

- Advantages
  - Gaining ground as practical tools
  - Relatively simple and elegant because of their analogy with nature
  - Public domain and commercial codes exist
- Disadvantages
  - A number of control parameters which affect the efficiency of solution
  - Large number of fitness function evaluations
  - Not strictly guarantee the optimality of the solution
  - Unconstrained minimization algorithm$\rightarrow$penalty function?

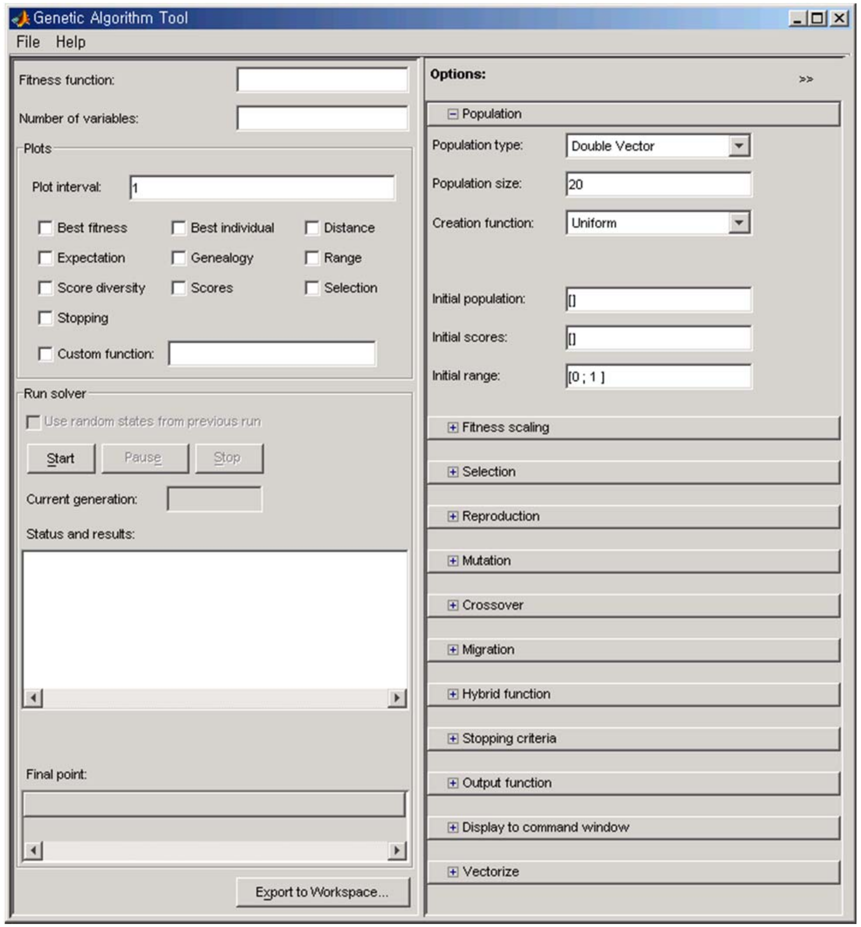
$$\text{minimize } f(\boldsymbol{x}) + R\sum_{j=1}^{m}\langle g_j(\boldsymbol{x})\rangle^2$$

$$\text{subject to } x_l \le x_i \le x_u, \quad i = 1, \ldots, n$$

$$\Rightarrow \text{maximize } F(\boldsymbol{x}) = F_{\max} - \left[f(\boldsymbol{x}) + R\sum_{j=1}^{m}\langle g_j(\boldsymbol{x})\rangle^2 \right]$$

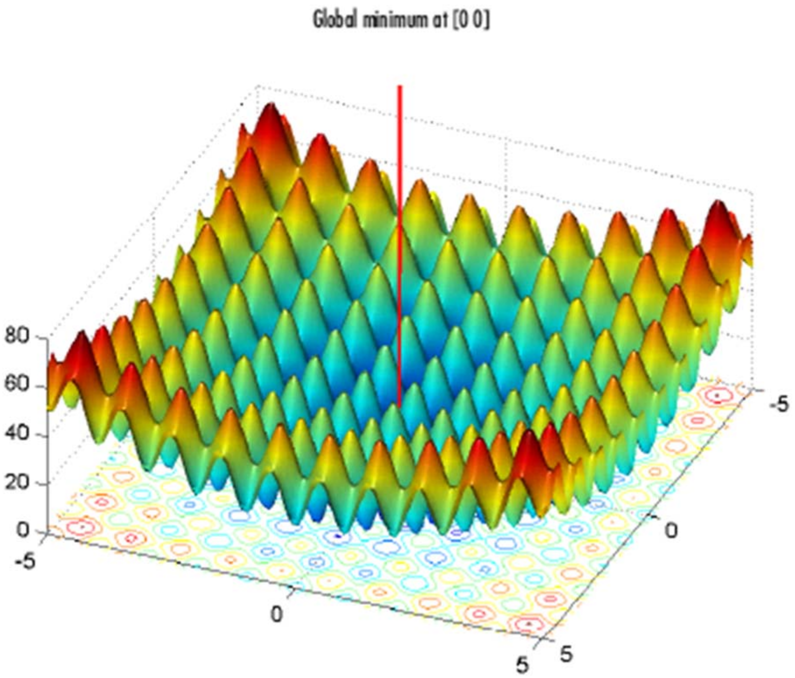
MATLAB: Genetic Algorithm Toolbox

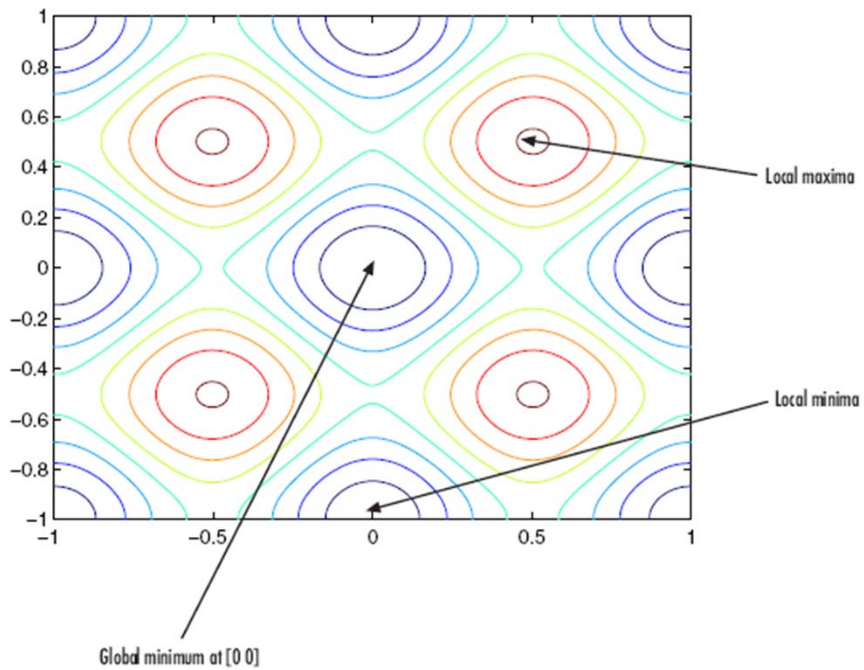
- [x fval] = ga(@fitnessfun, nvars, options)
- gatool



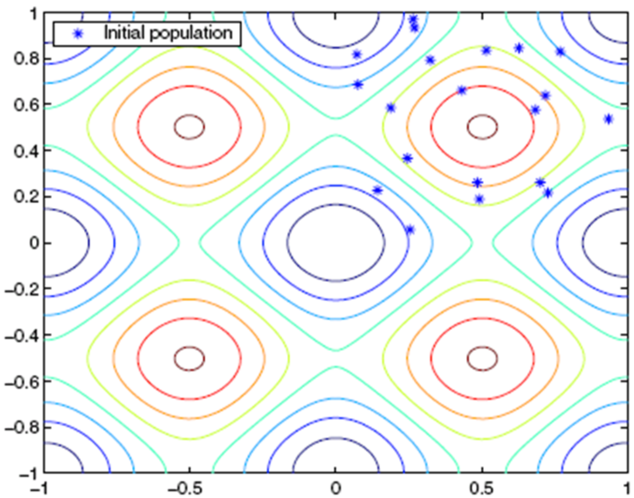
Rastrigin's Function

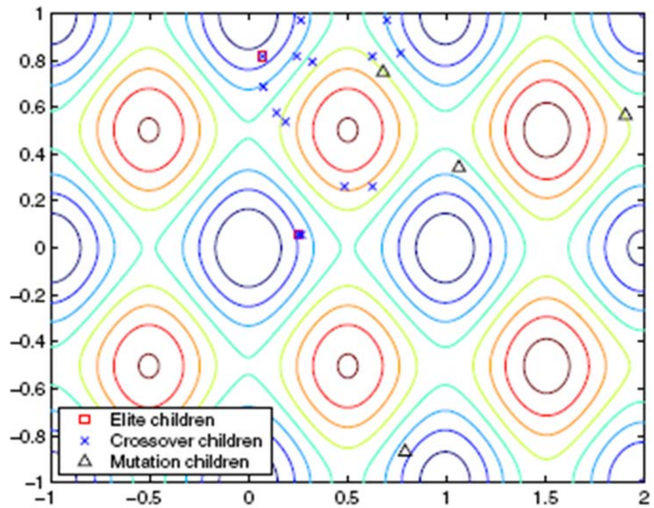
$$Ras(\boldsymbol{x}) = 20 + x_1^{\,2} + x_2^{\,2} - 10(\cos 2\pi x_1 + \cos 2\pi x_2)$$

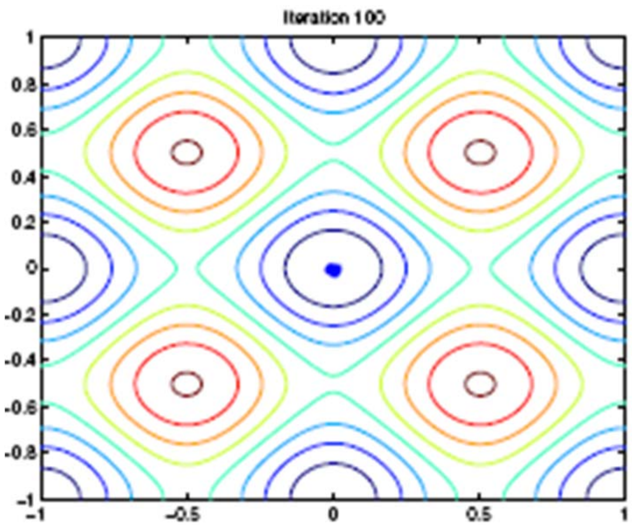


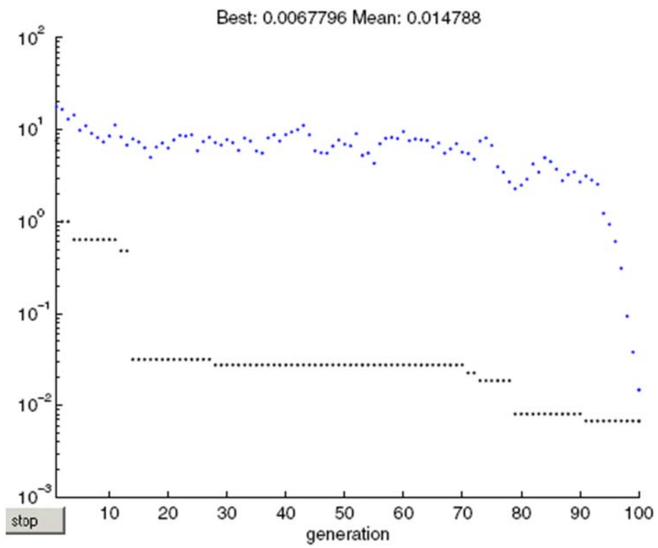


Iteration History









Options

- options = gaoptimset('option-item', value)

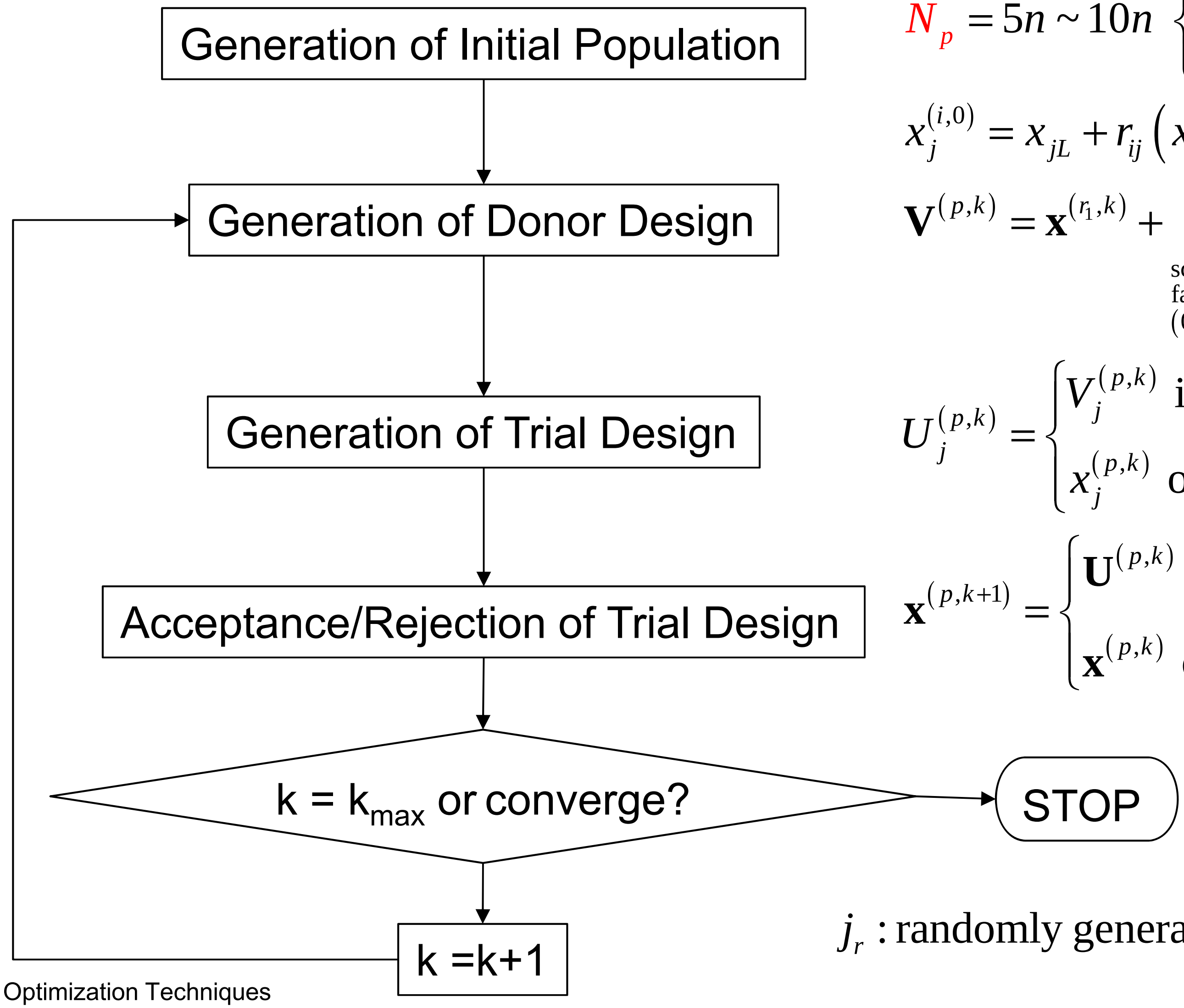
```
options =

        PopulationType: 'doubleVector'
          PopInitRange: [2x1 double]
        PopulationSize: 20
            EliteCount: 2
     CrossoverFraction: 0.8000
    MigrationDirection: 'forward'
     MigrationInterval: 20
     MigrationFraction: 0.2000
           Generations: 100
             TimeLimit: Inf
          FitnessLimit: -Inf
         StallGenLimit: 50
        StallTimeLimit: 20
                TolFun: 1.0000e-006
                TolCon: 1.0000e-006
     InitialPopulation: []
         InitialScores: []
        InitialPenalty: 10
         PenaltyFactor: 100
          PlotInterval: 1
           CreationFcn: @gacreationuniform
     FitnessScalingFcn: @fitscalingrank
```

Differential Evolution Algorithm (DEA)

  - Compared to GAs, DEAs are easier to implement on the computer
  - Unlike GAs, they do not require binary number coding and encoding
- Four steps in executing the basic DEA
  - Step 1: Generation of the initial population of designs
  - Step 2: Mutation with difference of vectors to generate a so-called donor design vector
  - Step 3: Crossover/recombination to generate a so-called trial design vector
  - Step 4: Selection, that is, acceptance or rejection of the trial design vector using the fitness function, which is usually the cost function

Differential Evolution Algorithm (DEA)



$$N_p = 5n \sim 10n \quad \begin{cases} i\text{-th member of poulation} \\ j = 1, \ldots, n \end{cases}$$

$$x_j^{(i,0)} = x_{jL} + r_{ij}\left(x_{jU} - x_{jL}\right)$$

$$\mathbf{V}^{(p,k)} = \mathbf{x}^{(r_1,k)} + \underbrace{F}_{\substack{\text{scale}\\ \text{factor}\\ (0.4\sim1)}} \underbrace{\left(\mathbf{x}^{(r_2,k)} - \mathbf{x}^{(r_3,k)}\right)}_{\text{difference vector}}$$

$$U_j^{(p,k)} = \begin{cases} V_j^{(p,k)} & \text{if } \left(r_{pj} \le C_r\right) \text{ or } \left(j = j_r\right) \\ x_j^{(p,k)} & \text{otherwise} \end{cases}$$

$$\mathbf{x}^{(p,k+1)} = \begin{cases} \mathbf{U}^{(p,k)} & \text{if } f\left(\mathbf{U}^{(p,k)}\right) \le f\left(\mathbf{x}^{(p,k)}\right) \\ \mathbf{x}^{(p,k)} & \text{otherwise} \end{cases}$$

j_r : randomly generated index between 1 and n

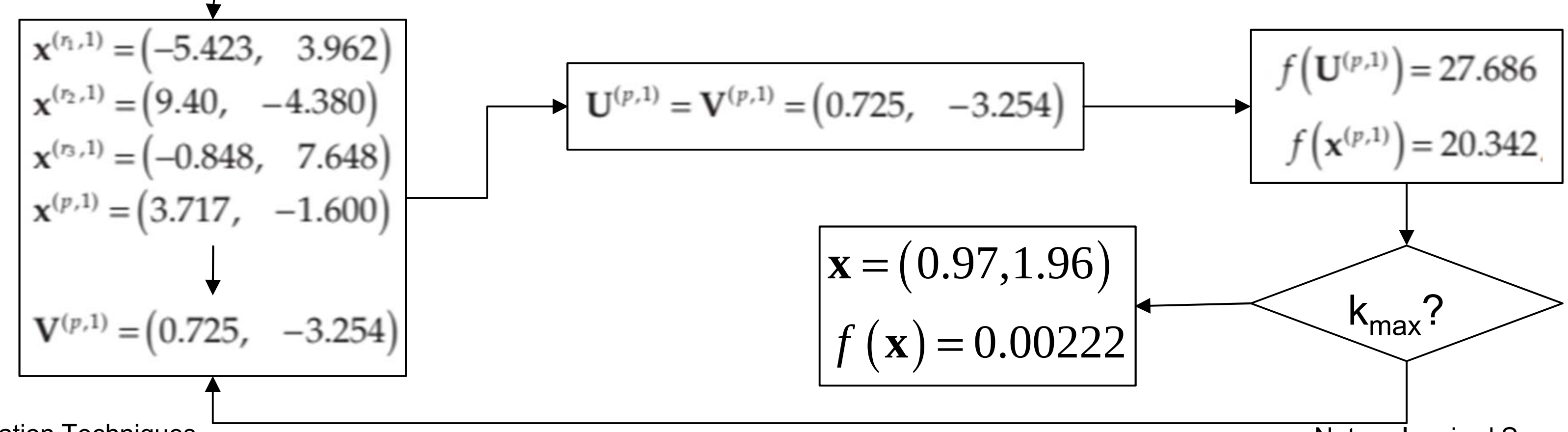
Example 17.3 DEA

Minimize $f(\mathbf{x}) = (x_1 - 1)^2 + (x_2 - 2)^2$

subject to $-10 \le x_1 \le 10, \ -10 \le x_2 \le 10$

$$\begin{cases} n = 2 \\ N_p = 5n = 10 \\ k_{\max} = 10{,}000 \\ C_r = 0.8 \\ F = 0.6 \end{cases}$$

x_i number	x_1	x_2
1	3.717	−1.600
2	9.400	−4.380
3	9.048	−8.659
4	−2.935	−2.920
5	−5.423	3.962
6	−4.442	2.470
7	−0.848	7.648
8	−8.394	−5.238
9	2.678	−2.884
10	7.059	−1.567



Ant Colony Optimization (ACO)

- emulates the food searching behavior of ants developed by Dorigo (1992)
- search for an optimal path for a problem represented by a graph based on the behavior of ants seeking the shortest path between their colony and a food source
- class of metaheuristics and swarm intelligence methods
- originally for discrete variable combinatorial optimization problems

ACO Terminology

- Pheromone: pherin (to transport) + hormone (to stimulate)
  - a secreted or excreted chemical factor that triggers a social response in members of the same species
- Pheromone trail
  - ants deposit pheromones wherever they go
  - other ants can smell the pheromones and are likely to follow an existing trail
- Pheromone density
  - when ants travel on the same path again and again, they continuously deposit pheromones on it
  - In this way the amount of pheromones increases and ants are likely to follow paths having higher pheromone densities
- Pheromone evaporation
  - pheromones have the property of evaporation over time
  - if a path is not being traveled by the ants, the pheromones evaporate, and the path disappears over time

Ant Behavior

- Initially ants move from their nest randomly to search for food
- Upon finding it, they return to their colony following the path they took to it while laying down pheromone trails
- If other ants find such a path, they are likely to follow it instead of moving randomly
- The path is thus reinforced, since ants deposit more pheromone on it
- However, the pheromone evaporates over time

- pheromone density is higher on shorter paths than on the longer ones → eventually all the ants follow the shortest path

Virtual Ants: Simple Model

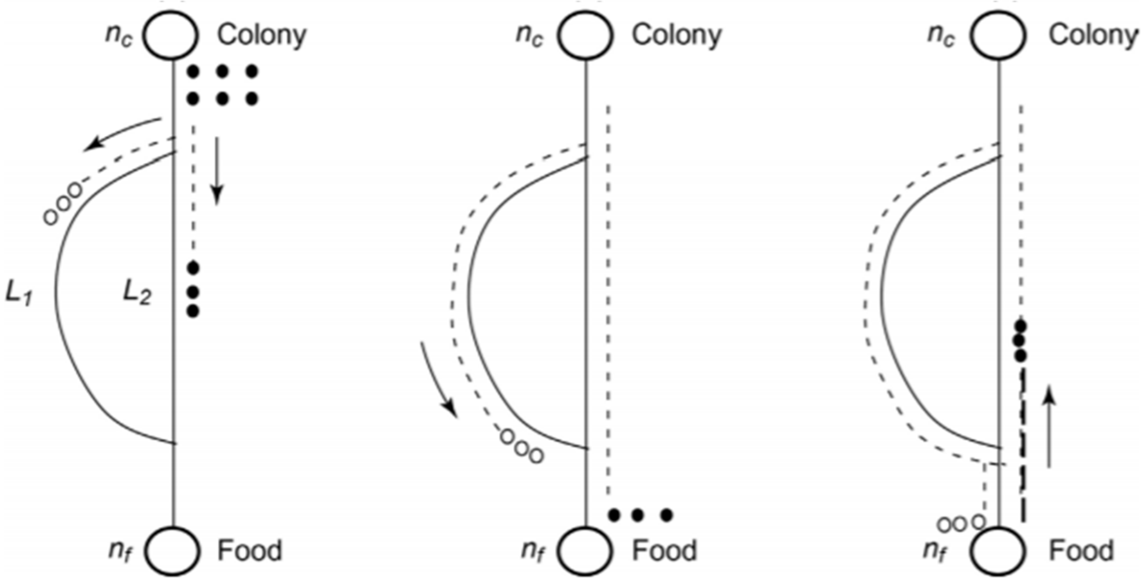
$$\underbrace{G}_{\text{graph}} = (N, L) \text{ where } N = \text{node}\begin{cases} n_c: \text{ ant colony} \\ n_f: \text{food source} \end{cases} \text{ and } L = \text{link}\begin{cases} L_1: \text{ length of } d_1 \\ L_2: \text{ length of } d_2 \end{cases} (d_1 > d_2)$$

virtual pheromone value (τ_i) for ith-path (initially set as one): strength of the pheromone trail

$n_c \rightarrow n_f$ (finding): selection of the path based on the probability $p_i = \dfrac{\tau_i}{\tau_1 + \tau_2}$, $i = 1, 2$

$n_f \rightarrow n_c$ (returning): pheromone reinforcement $\tau_i \leftarrow \tau_i + \dfrac{Q}{d_i}$ where Q: positive constant

evaporation: $\tau_i \leftarrow (1-\rho)\tau_i$ where ρ: pheromone evaporation rate, $\rho \in (0,1]$



Traveling Salesman Problem (1)

- classical combinatorial optimization problem
  - traveling salesman is required to visit a specified number of cities (called a tour)
  - The goal is to visit a city only once while minimizing the total distance traveled
- Assumptions
  - While a real ant can take a return path to the colony that is different from the original path depending on the pheromone values, a virtual ant takes the return path that is the same as the original path
  - The virtual ant always finds a feasible solution and deposits pheromone only on its way back to the nest
  - While real ants evaluate a solution based on the length of the path from their nest to the food source, virtual ants evaluate their solution based on a cost function value

Traveling Salesman Problem (2)

- "finding a path from the nest to the food source" to "finding a feasible solution to the TS problem."

x_j : j-th component of the design variable vector $\mathbf{x}$ (link selected from the j-th city)

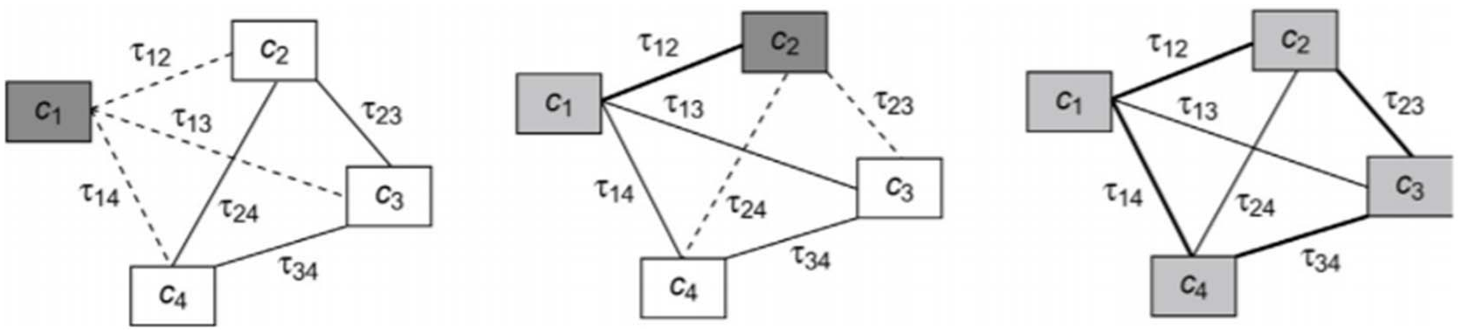
x_{ij} : link between the i-th city and the j-th city (distance between them)

D_i : list of integers corresponding to the cities that can be visited from the i-th city

$$D_1 = \{2,3,4\} \Leftrightarrow \text{feasible links} \{x_{12}, x_{13}, x_{14}\} \rightarrow p_{1j} = \frac{\tau_{1j}}{\tau_{12} + \tau_{13} + \tau_{14}}$$

$$D_2 = \{3,4\} \Leftrightarrow \text{feasible links} \{x_{23}, x_{24}\} \rightarrow p_{2j} = \frac{\tau_{2j}}{\tau_{23} + \tau_{24}}$$

$$c_1 \rightarrow c_2 \rightarrow c_3 \rightarrow c_4 \rightarrow c_1 \Leftrightarrow \mathbf{x} = [x_{12} \quad x_{23} \quad x_{34} \quad x_{41}]$$



Design Optimization (1)

- Problem definition
  - unconstrained discrete variable design optimization problem

$$\text{Minimize } f(\mathbf{x})$$

$$x_i \in D_i = \left(d_{i1}, \ldots, d_{iq_i}\right) \quad i = 1, \ldots n$$

- Finding feasible solutions
  - Selection of an initial link
  - Selection of a link from layer R
  - Obtaining feasible solutions for all ants

$$p_{1j}^{(00)} = \frac{\tau_{1j}^{(00)}}{\sum_{r=1}^{q_i} \tau_{1r}^{(00)}}; \; j = 1, \ldots, q_i$$

$$p_{ij}^{(rs)} = \frac{\tau_{ij}^{(rs)}}{\sum_{l=1}^{q_i} \tau_{il}^{(rs)}}; \; \begin{cases} j = 1, \ldots, q_i \\ i = r + 1 \end{cases}$$

$$\mathbf{x}^{(k)}, \; f\left(\mathbf{x}^{(k)}\right); \; k = 1, \ldots, \underbrace{N_a}_{5n\sim 10n}$$

$\tau_{ij}^{(rs)}$: pheromone value for the link from node rs to node ij

$p_{ij}^{(rs)}$: probability of selection of the link from node rs to node ij

$$\begin{cases} r : \text{layer number (design variable number)} \\ s : \text{allowable value number for the design variable number } r \end{cases}$$

Design Optimization (2)

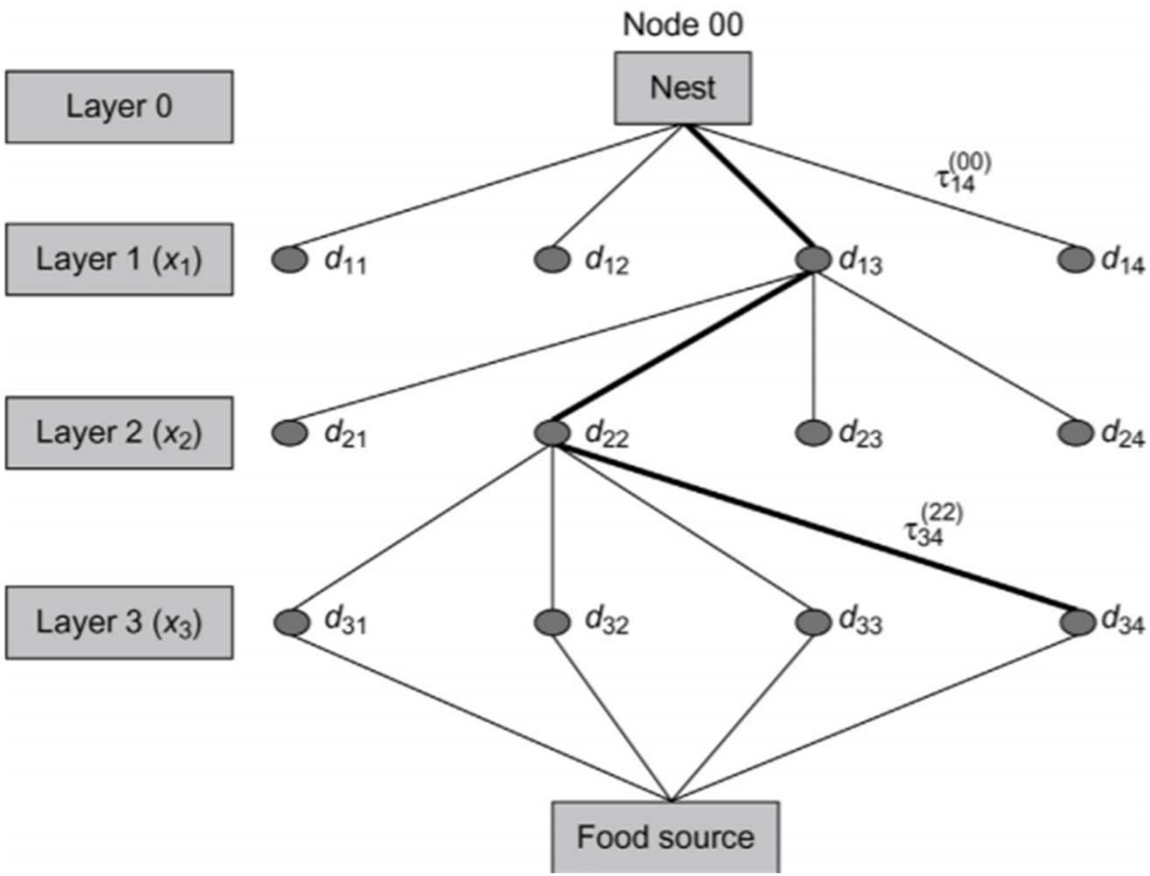
- Pheromone Evaporation
  - Once all of the ants have reached their destination (all of them have found solutions), pheromone evaporation (ie, reduction in the pheromone level) is performed for all links

$$\tau_{ij}^{(rs)} \leftarrow \left(1 - \underbrace{\rho}_{0.4\sim0.8}\right)\tau_{ij}^{(rs)} \text{ for all } r, s, i, j$$

- Pheromone Deposit
  - After pheromone evaporation, the ants start their journey back to their nest, which means that they will deposit pheromone on the return trail

$$\tau_{ij}^{(rs)} \leftarrow \tau_{ij}^{(rs)} + \frac{Q}{f\left(\mathbf{x}^{(k)}\right)} \text{ for all } r, s, i, j \text{ belonging to } k\text{-th ant's solution}$$

Example 17.4 ACO



Particle Swarm Optimization (PSO)

- population-based stochastic optimization technique, introduced by Kennedy and Eberhart (1995)
- mimics the social behavior of bird flocking or fish schooling
- class of metaheuristics and swarm intelligence methods
- many similarities with evolutionary computation techniques such as GA and DE
  - starts with a randomly generated set of solutions (initial population)
  - An optimum solution is then searched by updating generations
  - fewer algorithmic parameters to specify compared to GAs
  - not use any of the GAs’ evolutionary operators (crossover, mutation)
  - not use any of the GAs’ evolutionary operators such as crossover and mutation→easier to implement

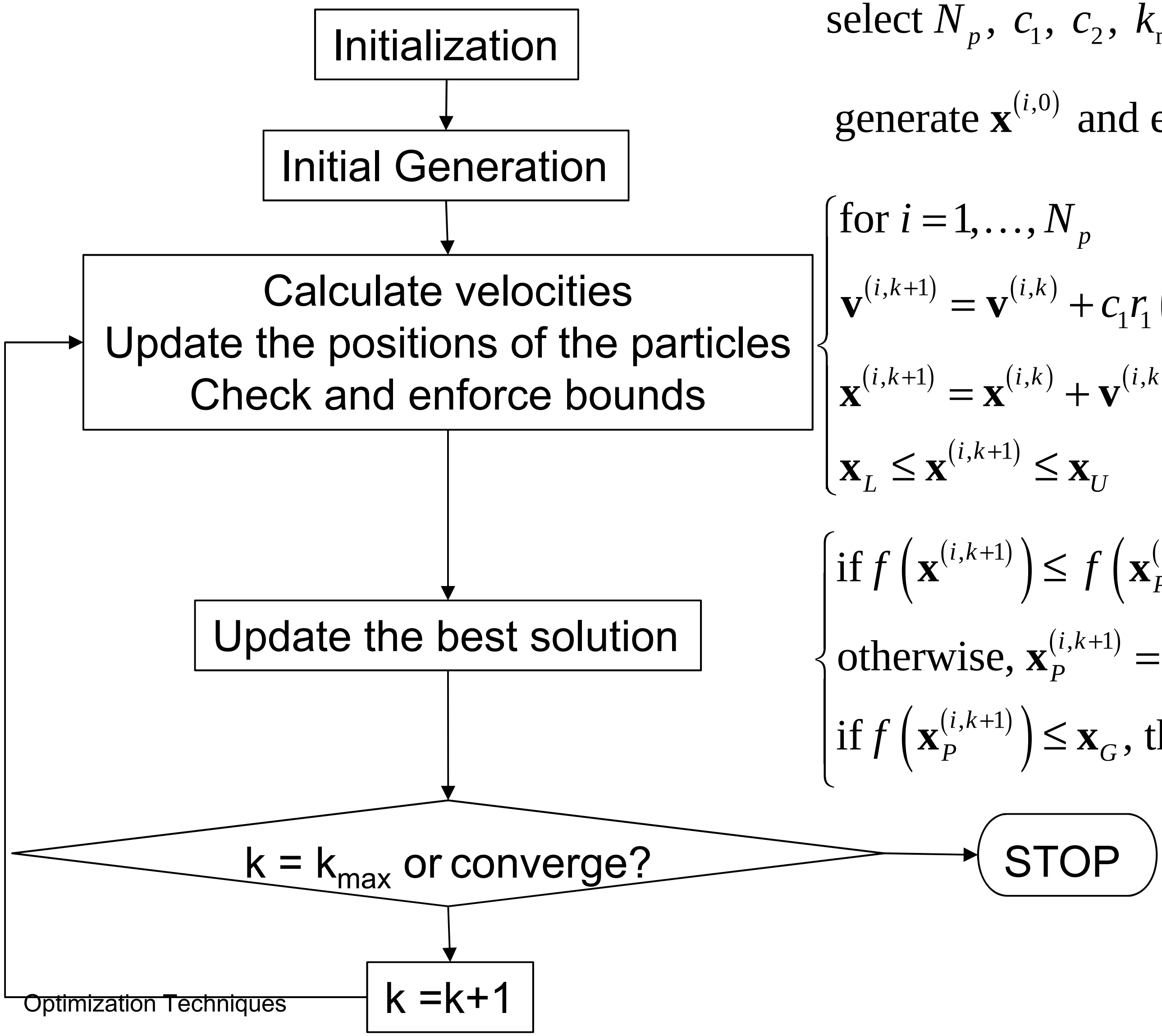
Swarm Behavior

- emulate the social behavior of a swarm of animals, such as a flock of birds or a school of fish (moving in search for food)
- an individual behaves according to its limited intelligence as well as to the intelligence of the group
- Each individual observes the behavior of its neighbors and adjusts its own behavior accordingly
- If an individual member discovers a good path to food, other members follow this path no matter where they are situated in the swarm

PSO Terminology

- Particle: identify an individual in the swarm (eg, a bird in the flock or a fish in the school)
- Particle position: refers to the coordinates of the particle ↔ design point
- Particle velocity: The term refers to the rate at which the particles are moving in space ↔ design change
- Swarm leader: particle having the best position ↔ design point having the smallest value for the cost function

Particle Swarm Optimization Algorithm



$$\text{select } N_p,\ c_1,\ c_2,\ k_{\max}.\ \text{set } \mathbf{v}^{(i,0)} = 0,\ k = 1.$$

$$\text{generate } \mathbf{x}^{(i,0)} \text{ and evaluate } f\left(\mathbf{x}^{(i,0)}\right) \to \mathbf{x}_G^{(k)}$$

$$\begin{cases} \text{for } i = 1,\ldots,N_p \\ \mathbf{v}^{(i,k+1)} = \mathbf{v}^{(i,k)} + c_1 r_1 \left(\mathbf{x}_P^{(i,k)} - \mathbf{x}^{(i,k)}\right) + c_2 r_2 \left(\mathbf{x}_G^{(i,k)} - \mathbf{x}^{(i,k)}\right) \\ \mathbf{x}^{(i,k+1)} = \mathbf{x}^{(i,k)} + \mathbf{v}^{(i,k+1)} \\ \mathbf{x}_L \le \mathbf{x}^{(i,k+1)} \le \mathbf{x}_U \end{cases}$$

$$\begin{cases} \text{if } f\left(\mathbf{x}^{(i,k+1)}\right) \le f\left(\mathbf{x}_P^{(i,k)}\right), \text{ then } \mathbf{x}_P^{(i,k+1)} = \mathbf{x}^{(i,k+1)} \\ \text{otherwise, } \mathbf{x}_P^{(i,k+1)} = \mathbf{x}_P^{(i,k)} \\ \text{if } f\left(\mathbf{x}_P^{(i,k+1)}\right) \le \mathbf{x}_G, \text{ then } \mathbf{x}_G = \mathbf{x}_P^{(i,k+1)} \end{cases}$$