Problem Formulation Process (1)

- Step 1: Project/Problem Statement
  - Is the project goal clear?
  - descriptive statement for the project/ problem
  - overall *objectives* of the project and the *requirements* to be met
- Step 2: Data and Information Collection
  - Is all the information available to solve the problem?
  - Performance requirements, resource limits, cost of raw materials
  - Identification of analysis procedures and tools
  - project statement is vague, and assumptions about modeling of the problem need to be made in order to formulate and solve it

Problem Formulation Process (2)

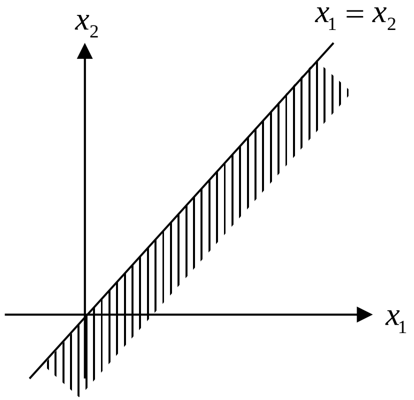
- Step 3: Identification/Definition of Design Variables
  - What are these variables? How do I identify them?
  - identify a set of variables that describe the system, called the *design variables*
  - should be independent of each other, minimum number
  - As many independent parameters as possible should be designated as design variables at the problem formulation phase
- Step 4: Optimization Criterion
  - How do I know that my design is the best?
  - must be a scalar function whose numerical value can be obtained once a design is specified (*function of the design variable vector*)
  - *maximized* or *minimized* depending on problem requirements
  - criterion that is to be minimized is usually called a *cost function* in engineering literature

Problem Formulation Process (3)

- Step 5: Formulation of Constraints
  - What restrictions do I have on my design?
  - All restrictions placed on the design
  - identify all constraints and develop expressions for them
  - must be designed and fabricated with the given *resources* and must meet *performance requirements*

Problem Formulation Steps

- Identification of *design variables*
  - Parameters chosen to describe the design
  - Independent of each other, minimum number
- Identification of an *objective (cost) functions*
  - Criterion to compare various designs
  - As a function of the design variables
  - Single/Multi-objective
- Identification of all *design constraints*
  - All restrictions placed on a design
  - Feasible/Infeasible
  - Explicit/Implicit, Linear/Nonlinear, Equality/Inequality



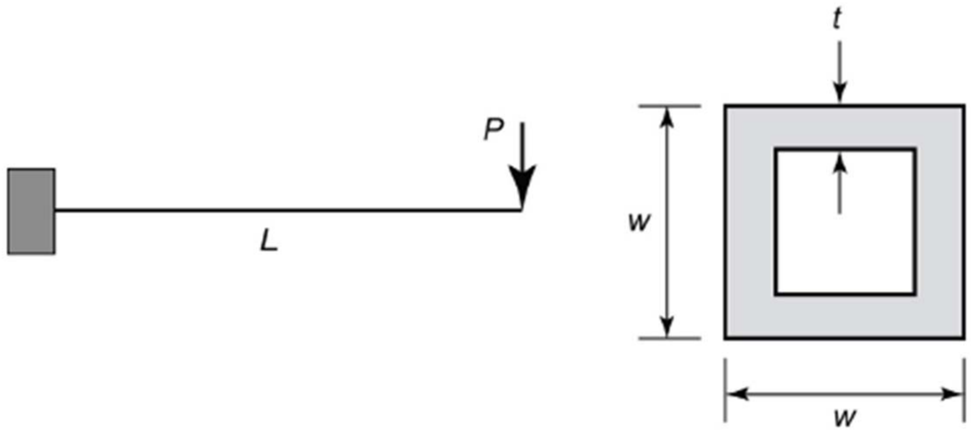
DESIGN OF A CANTILEVER BEAM

- Step 1: Problem Statement

Cantilever beams are used in many practical applications in civil, mechanical, and aerospace engineering. To illustrate the step of problem description, we consider the design of a hollow square-cross-section *cantilever beam* to support a load of 20 kN at its end. The beam, made of steel, is 2 m long, as shown in Fig. 2.1. The failure conditions for the beam are as follows: (1) the material should not fail under the action of the load, and (2) the deflection of the free end should be no more than 1 cm. The width-to-thickness ratio for the beam should be no more than 8 to avoid local buckling of the walls. A *minimum-mass* beam is desired. The width and thickness of the beam must be within the following limits:

$$60 \le \text{width} \le 300\ \text{mm} \tag{a}$$

$$3 \le \text{thickness} \le 15\ \text{mm} \tag{b}$$



DESIGN OF A CANTILEVER BEAM

- Step 2: Data and Information Collection

Notation	Data
A	Cross-sectional area, mm^2
E	Modulus of elasticity of steel, 21×10^4 N/mm^2
G	Shear modulus of steel, 8×10^4 N/mm^2
I	Moment of inertia of the cross-section, mm^4
L	Length of the member, 2000 mm
M	Bending moment, N/mm
P	Load at the free end, 20,000 N
Q	Moment about the neutral axis of the area above the neutral axis, mm^3
q	Vertical deflection of the free end, mm
q_a	Allowable vertical deflection of the free end, 10 mm
V	Shear force, N
w	Width (depth) of the section, mm
t	Wall thickness, mm
σ	Bending stress, N/mm^2
σ_a	Allowable bending stress, 165 N/mm^2
τ	Shear stress, N/mm^2
τ_a	Allowable shear stress, 90 N/mm^2

$$A = w^2 - (w-2t)^2 = 4t(w-t), \text{ mm}^2$$

$$I = \frac{1}{12} w \times w^3 - \frac{1}{12}(w-2t) \times (w-2t)^3 = \frac{1}{12} w^4 - \frac{1}{12}(w-2t)^4, \text{ mm}^4$$

$$Q = \frac{1}{2} w^2 \times \frac{w}{4} - \frac{1}{2}(w-2t)^2 \times \frac{(w-2t)}{4} = \frac{1}{8} w^3 - \frac{1}{8}(w-2t)^3, \text{ mm}^3$$

$$M = PL, \text{ N/mm}$$

$$V = P, \text{ N}$$

$$\sigma = \frac{Mw}{2I}, \text{ N/mm}^2$$

$$\tau = \frac{VQ}{2It}, \text{ N/mm}^2$$

$$q = \frac{PL^3}{3EI}, \text{ mm}$$

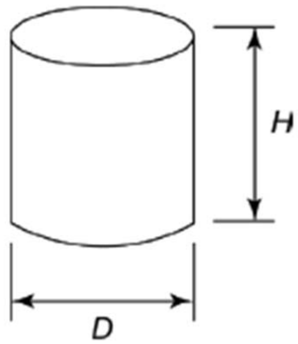
$$Q = \int_A y\,dA: \text{ first moment of area} \rightarrow \tau = \frac{V(x)Q(y)}{Ib(y)}$$

DESIGN OF A CANTILEVER BEAM

- Step 3: Definition of Design Variables
  - w = outside width (depth) of the section, mm
  - t = wall thickness, mm
- Step 4: Optimization Criterion
  - Design a minimum-mass cantilever beam
  - cross-sectional area of the beam:
- Step 5: Formulation of Constraints
  - Bending stress constraint
  - Shear stress constraint
  - Deflection constraint
  - Width-thickness restriction
  - Dimension restrictions

Design of a Can (1)

- Step 1: Problem Statement
  - Design a can to hold at least 400ml of liquid
  - Production in billions → Minimize the manufacturing cost
  - Cost directly related to the surface area of the sheet metal
  - Minimize the sheet metal required to fabricate the can
  - Diameter of the can should be no more than 8 cm. Also, it should not be less than 3.5 cm.
  - Height of the can should be no more than 18 cm and no less than 8 cm.



- Step 2: Data and Information Collection

Design of a Can (2)

- Step 3: Design variables
  - Diameter of the can (cm) / Height of the can (cm)
- Step 4: Cost function
  - Total surface area of the sheet metal

$$f(D,H) = \pi DH + 2\left(\frac{\pi D^2}{4}\right)$$

- Step 5: Constraints
  - Volume: $\left(\frac{\pi D^2}{4}\right)H \geq 400$
  - Size of the can: side/technological/sizing constraints, simple bounds, upper and lower limits

$$3.5 \leq D \leq 8; \quad 8 \leq H \leq 18$$

Insulated Spherical Tank Design (1)

- Step 1: Problem Statement
  - Choose insulation thickness to minimize the life-cycle cooling cost for a spherical tank
  - Cooling cost: installing and running the refrigeration equipment + installing the insulation
  - 10-yr life, 10% annual interest rate, no salvage value, tank radius: r
- Step 2: Data and Information Collection
  - Capacity of the refrigeration equipment (annual heat gain)

$$G = \frac{(365)(24)(\Delta T)A}{c_1 t} \ \mathrm{W \cdot hr}$$

$A = 4\pi r^2$: surface area of the spherical tank

ΔT : average difference between the internal and external temperatures (K)

c_1 : thermal resistivity per unit thickness (K · m/W)

t : insulation thickness (m)

Insulated Spherical Tank Design (2)

- Step 3: Design variable
  - Insulation thickness: t (m)
- Step 4: Cost function
  - Insulation, refrigeration equipment, operations for 10 yrs

$$f = c_2 A t + c_3 G + c_4 G \left[\underbrace{uspwf(0.1, 10)}_{=6.14457} \right] \qquad (\text{assuming } t << r)$$

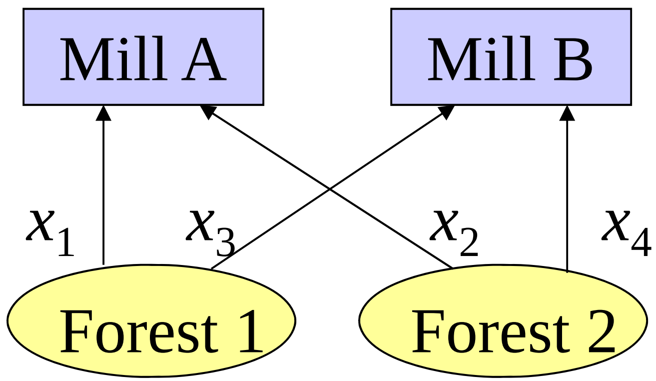
    - c_2: insulation cost per cubic meter ($/m^3)
    - c_3: cost of the refrigeration equipment per Wh of capacity ($/Wh)
    - c_4: annual cost of running the refrigeration equipment per Wh ($/Wh)
- Step 5: Constraints

$$t > 0 \rightarrow t \geq 0 \rightarrow t \geq t_{\min}$$

Saw Mill Operation (1)

- Step 1: Problem Statement
  - Each forest can yield up to 200 logs/day
  - Cost to transport the logs is estimated at 15 cents/km/log
  - At least 300 logs are needed each day
  - Minimize the cost of transportation of logs each day
- Step 2: Data and Information Collection

	Distance (km)		Capacity /day
Mill	Forest 1	Forest 2	
A	24.0	20.5	240 logs
B	17.2	18.0	300 logs



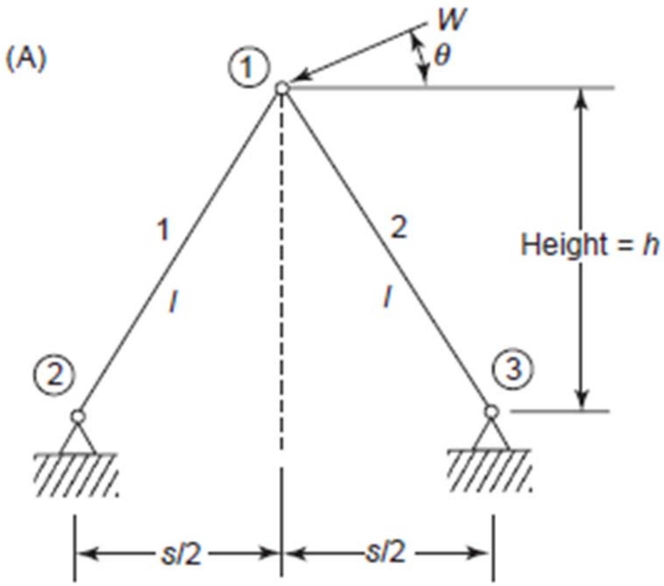
Saw Mill Operation (2)

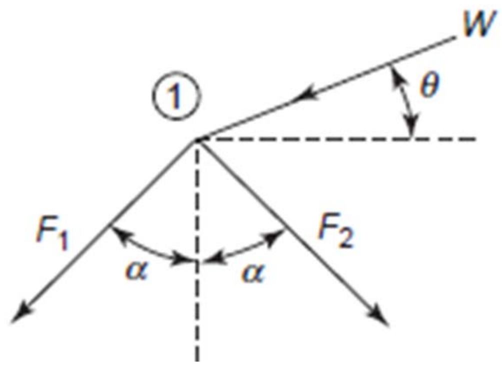
- Step 3: Design variables : x_1, x_2, x_3, x_4
- Step 4: Cost function
  - Cost of transportation of logs each day
- Step 5: Constraints
  - Mill capacities :
  - Yield of forests :

Linear Programming problem
→Integer Programming problem

Two-Bar Structure (1)

- Step 1: Problem Statement
  - Design a two-bar bracket to support a force W without failure
  - Cost directly related to the size of the two bars
  - To minimize the total mass of the bracket while satisfying performance, fabrication, and space limitations





$$\begin{cases} \sum F_x = -F_1 \sin\alpha + F_2 \sin\alpha - W\cos\theta = 0 \\ \sum F_y = -F_1 \cos\alpha - F_2 \cos\alpha - W\sin\theta = 0 \end{cases}$$

$$\sin\alpha = \frac{s}{2l}, \quad \cos\alpha = \frac{h}{l}, \quad l = \sqrt{h^2 + \left(\frac{s}{2}\right)^2}$$

Two-Bar Structure (2)

- Step 3: Design variables (hollow circular tubes)
  - x_1: height of the truss, x_2: span of the truss
  - x_3, x_4: outer/inner diameters of member 1
  - x_5, x_6: outer/inner diameters of member 2

$$A_1 = \frac{\pi}{4}\left(x_3^{\;2} - x_4^{\;2}\right), A_2 = \frac{\pi}{4}\left(x_5^{\;2} - x_6^{\;2}\right)$$

(d_0, r) where $r = \frac{d_i}{d_0}$

(d_0, d_i)

(d_0, d_i, r)?

- Step 4: Cost function
  - Minimize the mass: $m = \rho\left[l\left(A_1 + A_2\right)\right] = \rho\sqrt{x_1^{\;2} + \left(0.5x_2\right)^2}\,\frac{\pi}{4}\left(x_3^{\;2} - x_4^{\;2} + x_5^{\;2} - x_6^{\;2}\right)$
- Step 5: Constraints
  - stress in each member ≤ material allowable stress
  - Side constraints

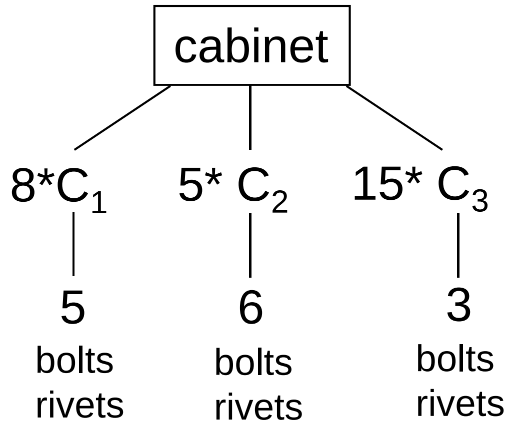
$$\left|\sigma_i = \frac{F_i}{A_i}\right| \le \sigma_a \quad \left(i = 1,2\right)$$

$$x_{il} \le x_i \le x_{iu} \quad \left(i = 1,\ldots,6\right)$$

Design of a Cabinet

- Determine the number of components to be bolted and riveted to minimize the cost
  - Each cabinet requires 8^*C_1, $5^*\ C_2$, $15^*\ C_3$ components
  - Assembly of C_1 needs either 5 bolts or 5 rivets; C_2 6 bolts or 6 rivets ; C_3 3 bolts or 3 rivets
  - A total of 100 cabinets must be assembly daily
  - Bolting and riveting capacities per day are 6000 and 8000, respectively

Cost ($)	C_1	C_2	C_3
bolt	0.7	1.0	0.6
rivet	0.6	0.8	1.0



Formulation 1 (component level)

- Design variables (for 100 cabinets)
  - $x_1/ x_3/ x_5$ = number of $C_1/ C_2/ C_3$ to be bolted
  - $x_2/ x_4/ x_6$ = number of $C_1/ C_2/ C_3$ to be riveted
- Cost function
- Constraints

Formulation 2 (bolt/rivet level)

- Design variables
  - x_1/ x_2/ x_3 = total number of bolts required for all C_1/ C_2/ C_3
  - x_4/ x_5/ x_6 = total number of rivets required for all C_1/ C_2/ C_3
- Cost function
- Constraints

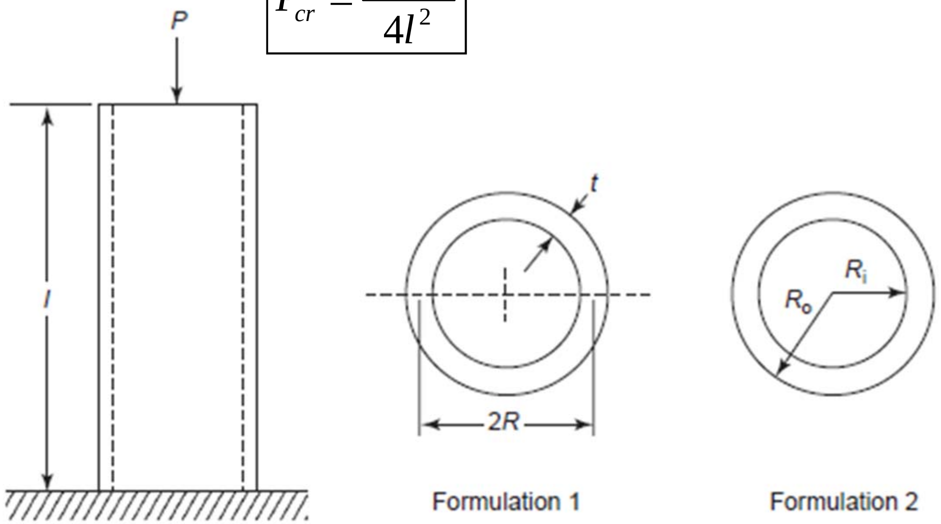
Formulation 3 (←Formulation 1)

- Design variables (for one cabinet)
  - x_1/ x_3/ x_5 = number of C_1/ C_3/ C_5 to be bolted on one cabinet
  - x_2/ x_4/ x_6 = number of C_2/ C_4/ C_6 to be riveted on one cabinet
- Cost function
- Constraints

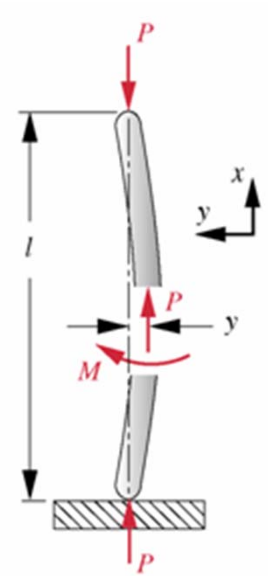
Minimum Weight Tubular Column Design

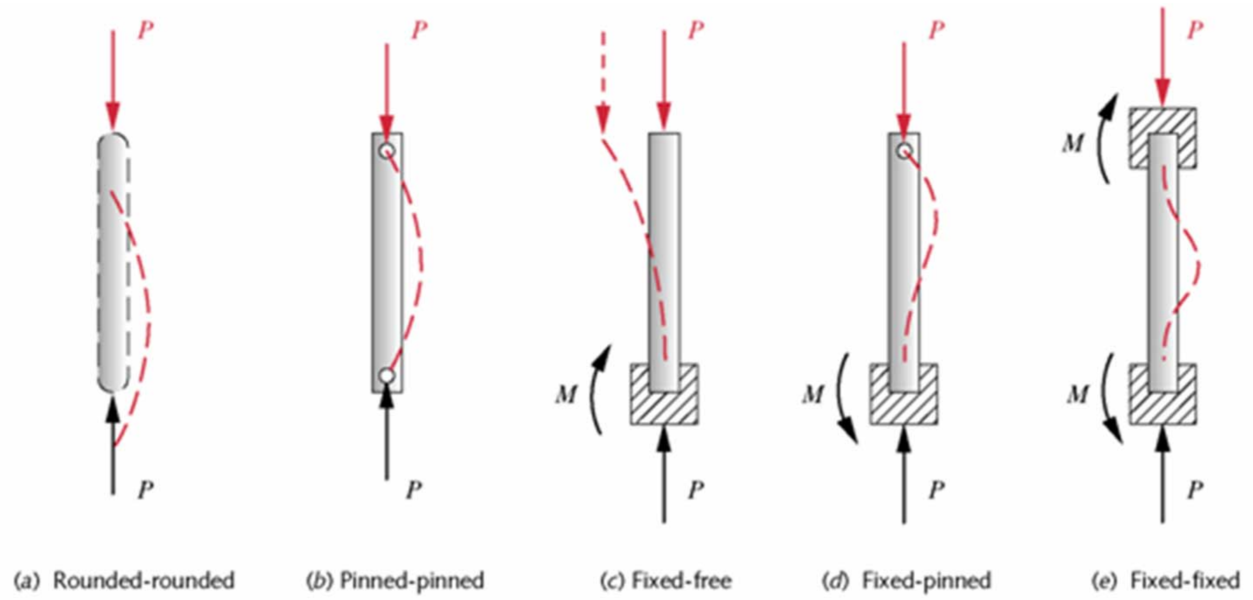
- Step 1: Problem Statement
  - Straight columns: structural elements (street light pole, traffic light post, water tower support)
  - Design a minimum mass tubular column of length *l* supporting a load P w/o buckling or overstressing
- Step 2: Data and Information Collection
  - Buckling load

$$P_{cr} = \frac{\pi^2 EI}{4l^2}$$



Buckling of an Euler Column





$$\frac{d^2y}{dx^2} = \frac{M}{EI} \rightarrow EI\frac{d^2y}{dx^2} = M = -Py$$

$$\frac{d^2y}{dx^2} + \left(\frac{P}{EI}\right)y = 0$$

$$y = c_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + c_2 \cos\left(\sqrt{\frac{P}{EI}}x\right)$$

+ boundary conditions

$$\rightarrow P_{cr} = \frac{\pi^2 EI}{l_{eff}^{\ 2}}$$

End Conditions	Theoretical Value	AISC* Recommends	Conservative Value
Rounded-Rounded	$l_{eff} = l$	$l_{eff} = l$	$l_{eff} = l$
Pinned-Pinned	$l_{eff} = l$	$l_{eff} = l$	$l_{eff} = l$
Fixed-Free	$l_{eff} = 2l$	$l_{eff} = 2.1l$	$l_{eff} = 2.4l$
Fixed-Pinned	$l_{eff} = 0.707l$	$l_{eff} = 0.80l$	$l_{eff} = l$
Fixed-Fixed	$l_{eff} = 0.5l$	$l_{eff} = 0.65l$	$l_{eff} = l$

Formulation 1

- Step 3: Design variables
  - *R* (mean radius of column) / *t* (wall thickness)
- Step 4: Cost function

$$\text{mass} = \rho(lA) = 2\rho l\pi Rt$$

$$\left[\text{assuming thin wall}\,(R >> t) \rightarrow A = 2\pi Rt;\; I = \pi R^3 t\right]$$

- Step 5: Constraints

$$\begin{cases} \sigma = \dfrac{P}{A} = \dfrac{P}{2\pi Rt} \le \sigma_a \\ P \le \dfrac{\pi^2 EI}{4l^2} = \dfrac{\pi^3 ER^3 t}{4l^2} \end{cases}$$

$$R_{\min} \le R \le R_{\max}; \quad t_{\min} \le t \le t_{\max}$$

Formulation 2

- Step 3: Design variables
  - R_o (outer radius of column) / R_i (inner radius of column)
- Step 4: Cost function

$$\text{mass} = \rho(lA) = \pi\rho l\left(R_0^2 - R_i^2\right) \quad \left[\ A = \pi\left(R_0^2 - R_i^2\right);\ I = \frac{\pi}{4}\left(R_0^4 - R_i^4\right)\right]$$

- Step 5: Constraints

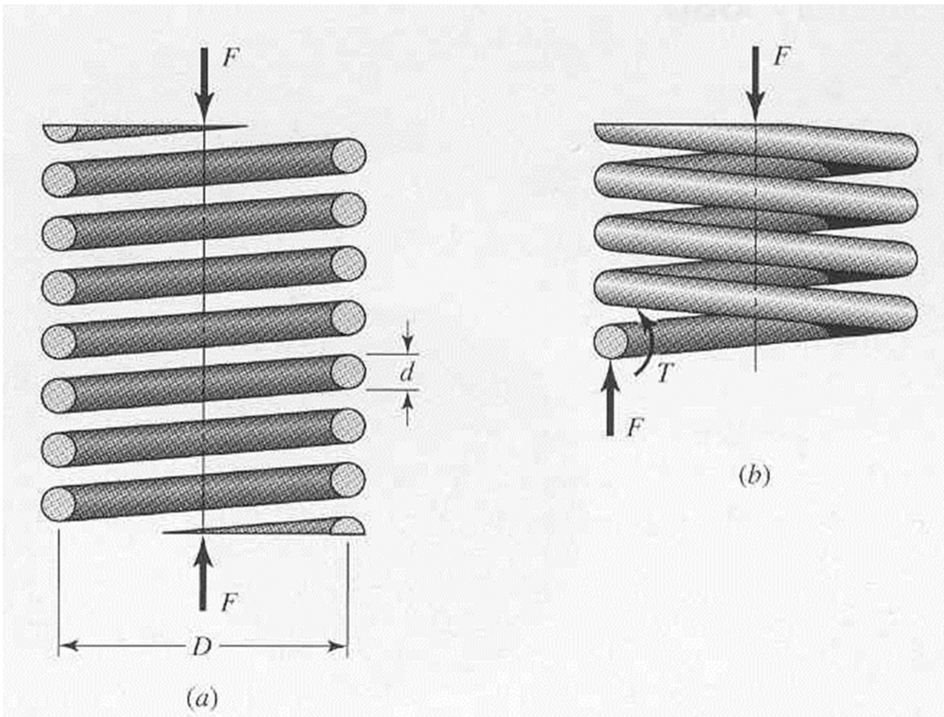
$$\begin{cases} \sigma = \dfrac{P}{A} = \dfrac{P}{\pi\left(R_0^2 - R_i^2\right)} \le \sigma_a \\ P \le \dfrac{\pi^2 EI}{4l^2} = \dfrac{\pi^3 E}{16 l^2}\left(R_0^4 - R_i^4\right) \end{cases}$$

$$\left(R_0\right)_{\min} \le R_0 \le \left(R_0\right)_{\max}; \quad \left(R_i\right)_{\min} \le R_i \le \left(R_i\right)_{\max}$$

$$R_0 > R_i; \quad \underbrace{\frac{R}{t} = \frac{R_0 + R_i}{2\left(R_0 - R_i\right)} \le k}_{\text{to avoid local buckling}} \quad (\text{thin-walled: } R >> t, k \ge 20)$$

Design of Coil Spring

- Step 1: Problem Statement
  - To design a minimum mass spring to carry a given axial load without material failure and while satisfying two performance requirement: the spring must deflect by at least Δ (in), and the frequency of surge waves must not be less than ω_0 (Hz)



Step 2: Data and Information Collection (1)

- Deflection along the axis of the spring: δ (in)
- Mean coil diameter: D (in)
- Wire diameter: d (in)
- Number of active coils: N
- Gravitational constant: g = 386 (in/s^2)
- Frequency of surge waves: ω (Hz)

Step 2: Data and Information Collection (2)

- Material property
  - Weight density: γ = 0.285 (lb/in^3)
  - Shear modulus: G = 1.15E7 (lb/in^2)
  - Mass density: ρ = 7.38342E-4 (lb-s^2/in^4)
  - Allowable shear stress: τ_a = 80000 (lb/in^2)
- Other data
  - Number of inactive coils: Q = 2
  - Applied load: P = 10 (lbs)
  - Minimum spring deflection: Δ = 0.5 (in)
  - Lower limit on surge wave frequency: ω_0 = 100 (Hz)
  - Limit on outer diameter of the coil: D_0 = 1.5 (in)

Design equations for the spring (1)

- Load-deflection

$$U = \underbrace{\frac{T^2 L}{2GJ}}_{torsion} + \underbrace{\frac{F^2 L}{2GA}}_{shear} = \frac{F^2 (D/2)^2 \pi D (N+Q)}{2G(\pi d^4/32)} + \frac{F^2 \pi D (N+Q)}{2G(\pi d^2/4)} = \frac{4F^2 D^3 (N+Q)}{d^4 G} + \frac{2F^2 D (N+Q)}{d^2 G}$$

$$\xrightarrow{\text{by Castigliago's theorm}} \delta = \frac{\partial U}{\partial F} = \frac{8FD^3 (N+Q)}{d^4 G} + \frac{4FD(N+Q)}{d^2 G}$$

$$\xrightarrow{C=\frac{D}{d}} \delta = \frac{8FD^3 (N+Q)}{d^4 G}\left(1 + \frac{1}{2C^2}\right) \approx \frac{8FD^3 (N+Q)}{d^4 G}$$

- Shear stress

$$\tau_{\max} = \frac{Tr}{J} + \frac{F}{A} = \frac{F(D/2)(d/2)}{(\pi d^4/32)} + \frac{F}{\pi d^2/4} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

$$= \frac{8FD + 4Fd}{\pi d^3} = \left(1 + \frac{d}{2D}\right)\frac{8FD}{\pi d^3} = K_s \frac{8FD}{\pi d^3}$$

$$\begin{cases} K_s = 1 + 0.5\dfrac{d}{D} \\ K_w = \dfrac{4D-1}{4(D-d)} + \dfrac{0.615d}{D} \end{cases}$$

Design equations for the spring (2)

- Frequency of surge waves

$$\frac{\partial^2 u}{\partial y^2} = \frac{W}{kgl^2}\frac{\partial^2 u}{\partial t^2}, \qquad B.C. \quad u(0,t)=0 \ \ and \ \ u(l,t)=0$$

$$W = AL\gamma = \left(\frac{\pi d^2}{4}\right)(\pi DN)\gamma = \frac{\pi^2 d^2 DN\gamma}{4}$$

$$\omega_m = m\pi\sqrt{\frac{kg}{W}}, \quad \text{fundamental frequency } (m=1)$$

$$f_1 = \frac{\omega_1}{2\pi} = \frac{1}{2}\sqrt{\frac{kg}{W}} = \frac{2}{\pi N}\frac{d}{D^2}\sqrt{\frac{Gg}{32\gamma}} = \frac{d}{2\pi ND^2}\sqrt{\frac{G}{2\rho}}$$

Problem Formulation

- Step 3: Identification of design variables
  - Wire diameter: d
  - Mean coil diameter: D
  - Number of active coils: N
- Step 4: Identification of an objective function
  - Mass

$$m = \rho AL = \rho\left(\frac{\pi d^2}{4}\right)\pi D(N+Q) = \frac{\pi^2 \rho d^2 D(N+Q)}{4}$$

- Step 5: Identification of constraints
  - Deflection: $\delta \geq \Delta$
  - Shear stress: $\tau \leq \tau_a$
  - Frequency of surge waves: $\omega \geq \omega_0$
  - Diameter: $D + d \leq D_0$
  - Side constraints: $d_{min} \leq d \leq d_{max}$, $D_{min} \leq D \leq D_{max}$, $N_{min} \leq N \leq N_{max}$

Mathematical Formulation

$$\underset{d,D,N}{\text{Minimize}}\; m = \frac{\pi^2 \rho d^2 D(N+Q)}{4}$$

$$\text{subject to } \frac{8FD^3(N+Q)}{d^4 G} \geq \Delta \rightarrow 1 - \frac{8FD^3(N+Q)}{d^4 G \Delta} \leq 0$$

$$\left[\frac{4D-d}{4(D-d)} + \frac{0.615d}{D}\right]\frac{8FD}{\pi d^3} \leq \tau_a \rightarrow \left[\frac{4D-d}{4(D-d)} + \frac{0.615d}{D}\right]\frac{8FD}{\pi d^3 \tau_a} - 1 \leq 0$$

$$\frac{d}{2\pi N D^2}\sqrt{\frac{G}{2\rho}} \geq \omega_0 \rightarrow 1 - \frac{d}{2\pi N D^2 \omega_0}\sqrt{\frac{G}{2\rho}} \leq 0$$

$$D + d \leq D_0 \rightarrow \frac{D+d}{D_0} - 1 \leq 0$$

$$d_{\min} \leq d \leq d_{\max}$$

$$D_{\min} \leq D \leq D_{\max}$$

$$N_{\min} \leq N \leq N_{\max}$$

Symmetric Three-Bar Truss (1)

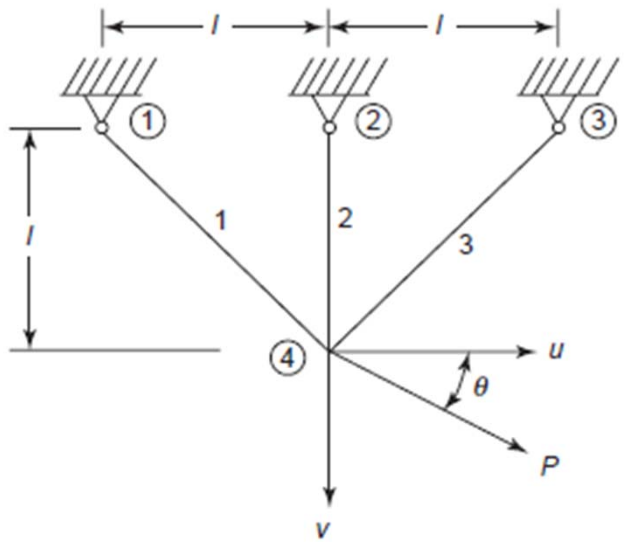
- Step 1: Problem Statement
  - Design for minimum volume to support a force P
  - Consideration of member crushing, member buckling, failure by excessive deflection of node 4, failure by resonance
- Step 2: Data and Information Collection
  - Equilibrium equations $\rightarrow$ displacements $\rightarrow$ forces carried by the members of the truss $\rightarrow$ stress

$$\sigma_1 = \frac{1}{\sqrt{2}}\left[\frac{P_u}{A_1} + \frac{P_v}{\left(A_1 + \sqrt{2}A_2\right)}\right]$$

$$\sigma_2 = \frac{\sqrt{2}P_v}{\left(A_1 + \sqrt{2}A_2\right)}$$

$$\sigma_3 = \frac{1}{\sqrt{2}}\left[-\frac{P_u}{A_1} + \frac{P_v}{\left(A_1 + \sqrt{2}A_2\right)}\right]$$

$$\omega = \frac{3EA_1}{\rho l\left(4A_1 + \sqrt{2}A_2\right)}$$



Symmetric Three-Bar Truss (2)

- Step 3: Design variables
  - A_1: cross-sectional area of material for members 1 and 3
  - A_2: cross-sectional area of material for members 2
- Step 4: Cost function
  - Material volume: $V = l\left(2\sqrt{2}A_1 + A_2\right)$
- Step 5: Constraints

$$\text{stress}: \sigma_1 \le \sigma_a, \sigma_2 \le \sigma_a \leftarrow \sigma_1 > \sigma_3$$

$$\text{displacement}: u \le \Delta_u, v \le \Delta_v$$

$$\text{natural frequency}: f_0 \ge \left(2\pi\omega_0\right)^2$$

$$\text{buckling}: -F_i \le \frac{\pi^2 EI}{l_i^2}, \quad I = \beta A^2$$

$$\text{side}: A_1, A_2 \ge A_{\min}$$

Standard Design Optimization Model

Find an n-vector $\mathbf{x} = (x_1, \ldots, x_n)$ of design variables

to minimize a cost function

$$f(\mathbf{x}) = f(x_1, \ldots, x_n)$$

subject to

$$\begin{cases} \text{the } p \text{ equality constraints} \\ h_j(\mathbf{x}) = h_j(x_1, \ldots, x_n) = 0; \quad j = 1, \ldots, p \\ \text{and the } m \text{ inequality constraints} \\ g_j(\mathbf{x}) = g_j(x_1, \ldots, x_n) \le 0; \quad i = 1, \ldots, m \end{cases}$$

bounds on design variables:

$$x_i \ge 0 \;\text{ or }\; x_{il} \le x_i \le x_{iu}; \quad i = 1, \ldots, n$$

Observations (1)

- Functions must depend on design variables.
- Number of independent equality constraints: $p \leq n$
  - $p > n$: overdetermined system of equations
    - redundant equality constraints
    - Inconsistent formulation
  - $p = n$: no optimization is necessary
- Inequality constraints written as "≤ 0"
  - No restriction on the number of inequality constraints
- Scaling effect
  - optimum design does not change. optimum cost function value, however, changes.
    - cost function by a positive constant
    - Inequality constraints by a positive constant
    - equality constraints by any constants

Observations (2)

- Maximization problem treatment

$$f(\mathbf{x}) = -F(\mathbf{x})$$

- "$\geq$ type" constraints

$$G_j(\mathbf{x}) \geq 0 \rightarrow g_j(\mathbf{x}) = -G_j(\mathbf{x}) \leq 0$$

- Discrete and Integer design variables
  - Approach 1
    - Solve the problem assuming continuous DVs
    - Assign nearest discrete/integer values
    - Check feasibility $\leftarrow$ numerous combinations
  - Approach 2 (adaptive numerical optimization)
    - Obtain optimum solution with continuous DVs
    - Assign only DVs close to their discrete/integer values
    - Optimize the problem until all DVs have proper values

Observations (3)

- Feasible set: collection of all feasible designs

$$S = \{\mathbf{x} \mid h_j(\mathbf{x}) = 0;\ j = 1, \ldots, p; \quad g_i(\mathbf{x}) \le 0; i = 1, \ldots, m\}$$

- Inequality constraint:

$$g_i(\mathbf{x}) \le 0 \rightarrow \begin{cases} \text{active/tight/binding} : g_i(\mathbf{x}^*) = 0 \\ \text{inactive} : g_i(\mathbf{x}^*) < 0 \\ \text{violated} : g_i(\mathbf{x}^*) > 0 \end{cases}$$

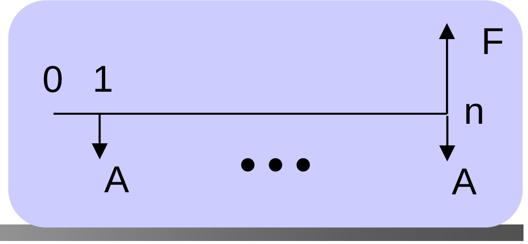
Supplements

Interest Formula (1)

- Three components of cash flow
  - Present(P), Future(F), Annuity(A)
  - 6 relationships
    - *n*: number of interest periods, e.g., months, years
    - *i*: return per dollar per period, e.g., annual interest rate
- F/P, P/F
  - Given P, Find F
    - Single payment compound amount factor (일회 지불 복리계수)
  - Given F, Find P
    - Single payment present worth factor (일회 지불 현가계수)

$$P \xrightarrow{iP} (1+i)P \xrightarrow{i(1+i)P} (1+i)^2 P \longrightarrow \cdots$$
$$F = (1+i)^n P = spcaf(i,n)P$$
$$P = (1+i)^{-n} F = sppwf(i,n)F$$

Interest Formula (2)



- F/A, A/F
  - Given A, Find F
    - Uniform series compound amount factor (연금 복리 계수)
  - Given F, Find A
    - Sinking fund deposit factor (감채 기금 계수)

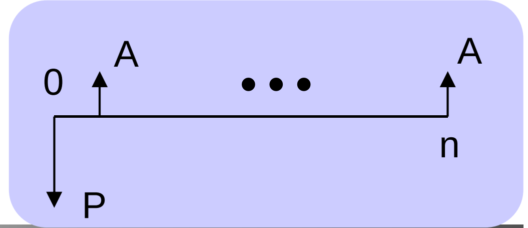
$$(1+i)^{n-1}A+(1+i)^{n-2}A+\cdots+A=F$$

$$\frac{(1+i)^{n-1}\left[1-(1+i)^{-n}\right]}{1-(1+i)^{-1}}=\frac{(1+i)^{n-1}\left[1-(1+i)^{-n}\right]}{\frac{i}{1+i}}=\frac{1}{i}\left[(1+i)^{n}-1\right]$$

$$F=\frac{1}{i}\left[(1+i)^{n}-1\right]A=uscaf\,(i,n)\,A$$

$$A=\frac{i}{\left[(1+i)^{n}-1\right]}F=sfdf\,(i,n)\,F$$

Interest Formula (3)



- P/A, A/P
  - Given A, Find P
    - Uniform series present worth factor (연금 현가 계수)
  - Given P, Find A
    - Capital recovery factor (자본 회수 계수)

$$F=\frac{1}{i}\left[(1+i)^{n}-1\right]A=(1+i)^{n}P \rightarrow P=\frac{1}{i}\left[1-(1+i)^{-n}\right]A=uspwf(i,n)A$$

$$A=\frac{i}{\left[1-(1+i)^{-n}\right]}P=crf(i,n)P$$

Summary: Interest Formula

To find	Given	Multiply by	Description
S_n	P	$spcaf(i,n) = (1+i)^n$	Single payment compound amount factor
P	S_n	$sppwf(i,n) = (1+i)^{-n}$	Single payment present worth factor
S_n	R	$uscaf(i,n) = \frac{1}{i}\left[(1+i)^n - 1\right]$	Uniform series compound amount factor
R	S_n	$sfdf(i,n) = \frac{i}{\left[(1+i)^n - 1\right]}$	Sinking fund deposit factor
P	R	$uspwf(i,n) = \frac{1}{i}\left[1-(1+i)^{-n}\right]$	Uniform series present worth factor
R	P	$crf(i,n) = \frac{i}{\left[1-(1+i)^{-n}\right]}$	Capital recovery factor