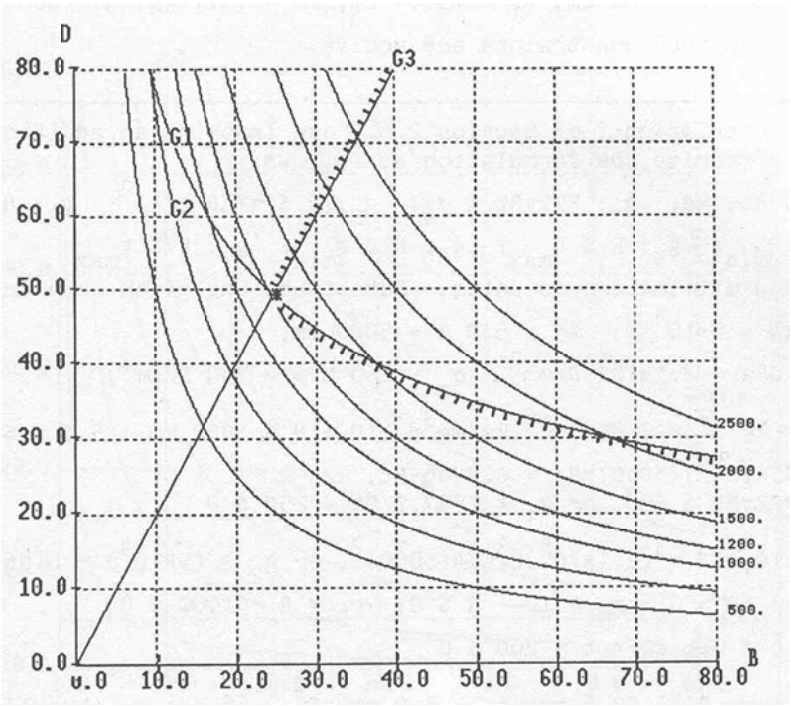
3.21 (←2.17)

(design variables) b = width (cm), d = depth (cm)

(objective function) minimize the cross-sectional area, f = bd

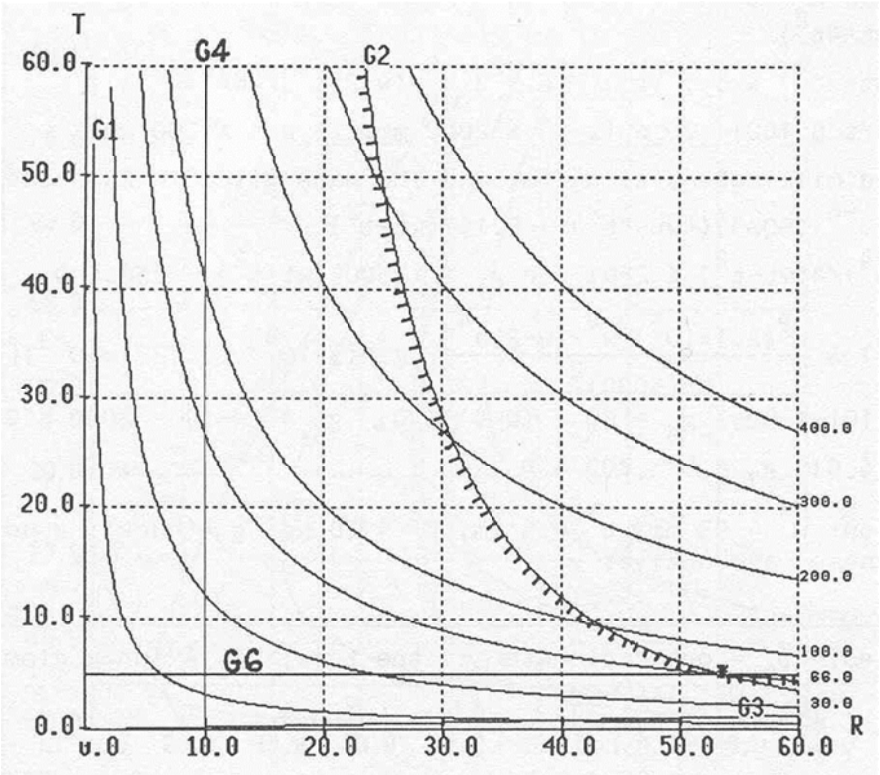
(constraints) $6M/bd^2 \le \sigma_a$, $3V/2bd \le \tau_a$, $d \le 2b$, $b \ge 0$, $d \ge 0$

$$\left.\begin{array}{l}\text{minimize } A = bd \\ \text{subject to} \\ \dfrac{6\left(1.4\times10^{7}\right)}{bd^{2}} \le 1.65\times10^{4} \\ \dfrac{3\left(2.4\times10^{4}\right)}{2bd} \le 5000 \\ d - 2b \le 0 \\ b \ge 0 \\ d \ge 0\end{array}\right\} \rightarrow \left\{\begin{array}{l}\text{minimize } A = bd \\ \text{subject to} \\ g_1 = \dfrac{6\left(8.0\times10^{6}\right)}{bd^{2}} - 800 \le 0 \\ g_2 = \dfrac{3\left(1.5\times10^{5}\right)}{2bd} - 300 \le 0 \\ g_3 = d - 2b \le 0 \\ g_4 = -b \le 0 \\ g_5 = -d \le 0\end{array}\right\} \rightarrow \left\{\begin{array}{c} b^{*} = 24.7cm \\ d^{*} = 49.3cm \\ f^{*} = \mathbf{1}216cm^{2} \\ (g_1 \text{ and } g_3 \text{ are active})\end{array}\right.$$



3.23 (units: N, mm)

$$\begin{aligned}
&\underset{R,t}{\text{minimize}} \quad f = 2\rho l\pi Rt \rightarrow 0.2466Rt \\
&\text{subject to} \\
&g_1 = \frac{P}{2\pi Rt} \le \sigma_a \rightarrow g_1 = \frac{7957.7}{Rt} - 250 \le 0 \\
&g_2 = P \le \frac{\pi^3 ER^3 t}{4l^2} \rightarrow g_2 = 50000 - 0.06511R^3 t \le 0 \\
&g_3 = \frac{R}{t} \le 50 \rightarrow g_3 = \frac{R}{t} - 50 \le 0 \\
&g_4 = R \ge R_{\min} \rightarrow g_4 = 10 - R \le 0 \\
&g_5 = R \le R_{\max} \rightarrow g_5 = R - 1000 \le 0 \\
&g_6 = t \ge t_{\min} \rightarrow g_6 = 5 - t \le 0 \\
&g_7 = t \le t_{\max} \rightarrow g_7 = t - 200 \le 0
\end{aligned} \quad \rightarrow \begin{cases} R^* = 53.6mm \\ t^* = 5.0mm \\ f^* = 66kg \\ (g_2 \text{ and } g_6 \text{ are active}) \end{cases}$$



3.24 (units: N, mm)

$$\underset{R_o,R_i}{\text{minimize}}\ \ f=\pi\rho l\left(R_o^{\,2}-R_i^{\,2}\right)\rightarrow 0.1233\left(R_o^{\,2}-R_i^{\,2}\right)$$

subject to

$$g_1=\frac{P}{\pi\left(R_o^{\,2}-R_i^{\,2}\right)}\le\sigma_a\rightarrow g_1=\frac{15915.5}{\left(R_o^{\,2}-R_i^{\,2}\right)}-250\le 0$$

$$g_2=P\le\frac{\pi^3E\left(R_o^{\,4}-R_i^{\,4}\right)}{16l^2}\rightarrow g_2=50000-0.016278\left(R_o^{\,4}-R_i^{\,4}\right)\le 0$$

$$g_3=\frac{\left(R_o+R_i\right)}{2\left(R_o-R_i\right)}\le 50\rightarrow g_3=\frac{\left(R_o+R_i\right)}{2\left(R_o-R_i\right)}-50\le 0$$

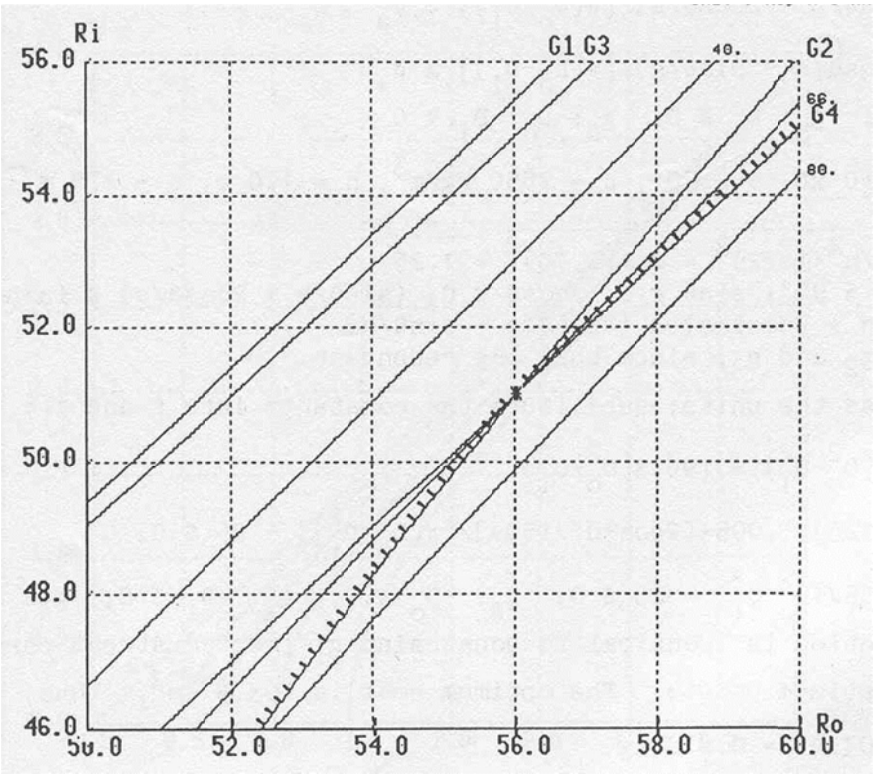
$$g_4=0.5\left(R_o+R_i\right)\ge R_{\min}\rightarrow g_4=10-0.5\left(R_o+R_i\right)\le 0$$

$$g_5=0.5\left(R_o+R_i\right)\le R_{\max}\rightarrow g_5=0.5\left(R_o+R_i\right)-1000\le 0$$

$$g_6=\left(R_o-R_i\right)\ge t_{\min}\rightarrow g_6=-R_o+R_i+5\le 0$$

$$g_7=\left(R_o-R_i\right)\le t_{\max}\rightarrow g_7=R_o-R_i-200\le 0$$

$$\rightarrow\begin{cases}R_o^{\,*}=56mm\\ R_i^{\,*}=51mm\\ f^{\,*}=66kg\\ \left(g_2\text{ and }g_6\text{ are active}\right)\end{cases}$$

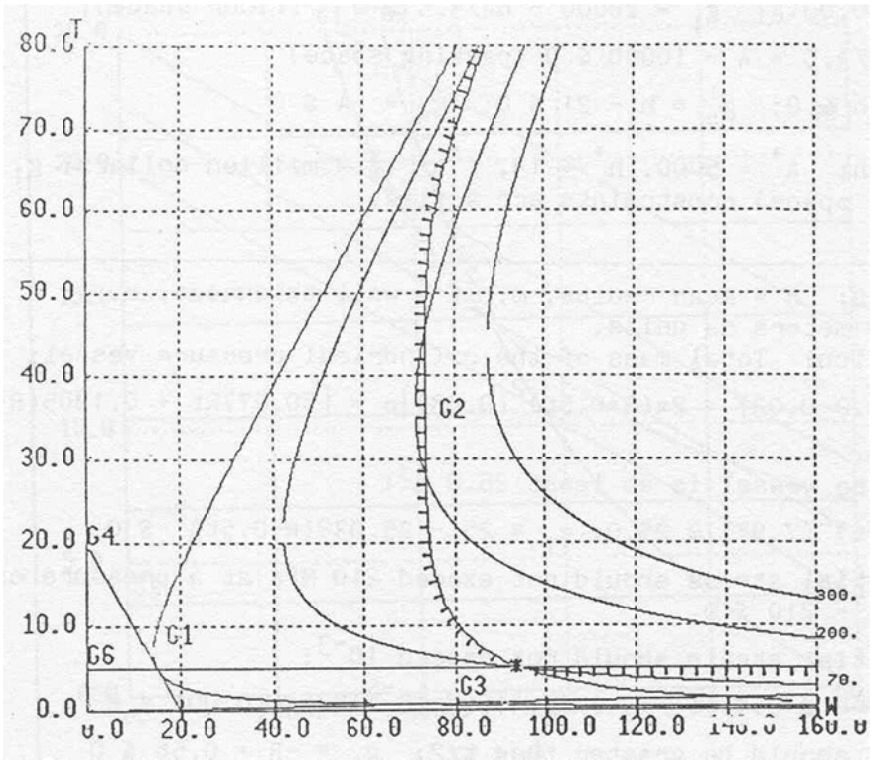


3.25 (units: N, mm)

$$A = w^2 - (w-2t)^2 = 4wt - 4t^2, I = \frac{w^4}{12} - \frac{(w-2t)^4}{12} = \frac{w^4 - (w-2t)^4}{12}$$

$$0.01 \le R \le 1.0m \rightarrow 0.01 \le \frac{w-t}{2} \le 1.0m \rightarrow 20 \le w - t \le 2000mm$$

$$\left.\begin{array}{l} \underset{w,t}{\text{minimize}} \;\; f = \rho l(4wt - 4t^2) \rightarrow 0.157(4wt - 4t^2) \\ \text{subject to} \\ g_1 = \dfrac{P}{4wt - 4t^2} \le \sigma_a \rightarrow g_1 = \dfrac{12500}{wt - t^2} - 250 \le 0 \\ g_2 = P \le \dfrac{\pi^3 E\left[w^4 - (w-2t)^4\right]}{48l^2} \\ \rightarrow g_2 = 50000 - (1.7271E-3)\left[w^4 - (w-2t)^4\right] \le 0 \\ g_3 = \dfrac{w-t}{2t} \le 50 \rightarrow g_3 = \dfrac{w}{t} - 101 \le 0 \\ g_4 = 20 - (w-t) \le 0 \\ g_5 = (w-t) - 2000 \le 0 \\ g_6 = 5 - t \le 0 \\ g_7 = t - 200 \le 0 \end{array}\right\} \rightarrow \left\{\begin{array}{c} w^* = 93mm \\ t^* = 5mm \\ f^* = 70kg \\ (g_2 \text{ and } g_6 \text{ are active}) \end{array}\right.$$



3.38 (←2.4)

(design variables) N =number of tubes, R =radius of each tube (cm)

(objective function) maximize surface area of tubes $N(2\pi R)l$ (cm^2)

(constraints) $R \geq 0.5$, $N(\pi R^2) \leq 2000$, $N \geq 0$ (integer)

$$\left.\begin{array}{l}\text{maximize } S = N(2\pi R)l \\ \text{subject to} \\ R \geq 0.5 \\ N(\pi R^2) \leq 2000 \\ N \geq 0\end{array}\right\} \rightarrow \left\{\begin{array}{l}\text{minimize } f = -2\pi lNR \\ \text{subject to} \\ g_1 = 0.5 - R \leq 0 \\ g_2 = \pi NR^2 - 2000 \leq 0 \\ g_3 = -N \leq 0\end{array}\right\} \rightarrow \left\{\begin{array}{c}R^* = 0.5 \\ N^* = 2546 \\ f^* = -8000 \\ (assume\ l = 1) \\ (g_1 \text{ and } g_2 \text{ are active})\end{array}\right.$$