8.87

$$\left.\begin{array}{l} \textit{Maximize } z = 10A + 8B \\ \text{subject to } g_1 = 0.4A + 0.5B \le 100 \\ \qquad g_2 = 0.6A + 0.5B \le 80 \\ \qquad g_3 = A \le 70 \\ \qquad g_4 = B \le 110 \\ \qquad A, B \ge 0 \end{array}\right\} \rightarrow \left\{\begin{array}{l} \textit{Minimize } f = -10A - 8B \\ \text{subject to } 0.4A + 0.5B + x_1 = 100 \\ \qquad 0.6A + 0.5B + x_2 = 80 \\ \qquad A + x_3 = 70 \\ \qquad B + x_4 = 110 \\ \qquad A, B, x_1, x_2, x_3, x_4 \ge 0 \end{array}\right.$$

basic	A	B	x_1	x_2	x_3	x_4	b
x_1	0.4	0.5	1	0	0	0	100
x_2	0.6	0.5	0	1	0	0	80
x_3	1	0	0	0	1	0	70
x_4	0	1	0	0	0	1	110
cost	-10	-8	0	0	0	0	f+0
x_1	0	0.5	1	0	-0.4	0	72
x_2	0	0.5	0	1	-0.6	0	38
A	1	0	0	0	1	0	70
x_4	0	1	0	0	0	1	110
cost	0	-8	0	0	10	0	f+700
x_1	0	0	1	-1	0.2	0	34
B	0	1	0	2	-1.2	0	76
A	1	0	0	0	1	0	70
x_4	0	0	0	-2	1.2	1	34
cost	0	0	0	16	0.4	0	f+1308

x* = (70, 76), f* = -1308, z* = 1308 (g2 and g3 are active)

(ranges)

$$g_1 : \left\{-\frac{34}{1} \quad -\frac{76}{0} \quad -\frac{70}{0} \quad -\frac{34}{0}\right\} \rightarrow -34 \le \Delta b_1 \le \infty \rightarrow 66 \le b_1 \le \infty$$

$$g_2 : \left\{-\frac{34}{-1} \quad -\frac{76}{2} \quad -\frac{70}{0} \quad -\frac{34}{-2}\right\} \rightarrow -38 \le \Delta b_2 \le 17 \rightarrow 42 \le b_2 \le 97$$

$$g_3 : \left\{-\frac{34}{0.2} \quad -\frac{76}{-1.2} \quad -\frac{70}{1} \quad -\frac{34}{1.2}\right\} \rightarrow -28.3 \le \Delta b_3 \le 63.3 \rightarrow 41.7 \le b_3 \le 133.3$$

$$g_4 : \left\{-\frac{34}{0} \quad -\frac{76}{0} \quad -\frac{70}{0} \quad -\frac{34}{1}\right\} \rightarrow -34 \le \Delta b_4 \le \infty \rightarrow 76 \le b_4 \le \infty$$

$$c_1 : \left\{\frac{0}{0} \quad \frac{16}{0} \quad \frac{0.4}{1} \quad \frac{0}{0}\right\} \rightarrow -\infty \le \Delta c_1 \le 0.4 \rightarrow -\infty \le c_1 \le -9.6$$

$$c_2 : \left\{\frac{0}{0} \quad \frac{16}{2} \quad \frac{0.4}{-1.2} \quad \frac{0}{0}\right\} \rightarrow -0.3 \le \Delta c_2 \le 8 \rightarrow -8.3 \le c_2 \le 0$$

$$\Delta f = -\lambda_1 \Delta b_1 - \lambda_2 \Delta b_2 - \lambda_3 \Delta b_3 - \lambda_4 \Delta b_4 = -0e_1 - 16e_2 - 0.4e_3 - 0e_4$$

(1) $\Delta b_1 = 120 - 100 = +20 \rightarrow \Delta f = 0 \rightarrow f^{new} = -1308\left(z^{new} = 1308\right)$

(2) $\Delta b_2 = 100 - 80 = +20 \rightarrow$ out of the range $\rightarrow$ resolve$\left(A^{new} = 70, B^{new} = 110, f^{new} = -1580\right)$

(3) $\Delta b_3 = 60 - 70 = -10 \rightarrow \Delta f = +4 \rightarrow f^{new} = -1304\left(z^{new} = 1304\right)$

$A^{new} = 70 + (-10)(1) = 60, B^{new} = 76 + (-10)(-1.2) = 88$

(4) $\Delta c_1 = -8 - (-10) = +2 \rightarrow$ out of the range $\rightarrow$ resolve$\left(A^{new} = 41.7, B^{new} = 110, f^{new} = -1213.3\right)$

8.60

Minimize $f = 5x_1 + 4x_2 - x_3$
Subject to $x_1 + 2x_2 - x_3 \geq 1$
$2x_1 + x_2 + x_3 \geq 4$
$x_1,\ x_2 \geq 0$; x_3 is unrestricted in sign.

Solution:
Standard LP form:
Minimize $f = 5y_1 + 4y_2 - y_3 + y_4$
Subject to $y_1 + 2y_2 - y_3 + y_4 - y_5 + y_7 = 1$
$2y_1 + y_2 + y_3 - y_4 - y_6 + y_8 = 4$
$y_i \geq 0; i = 1$ to 8
The optimum solution is $x_1^* = 0.0,\ x_2^* = 1.667,\ x_3^* = 2.333$ and $f^* = 4.33$, where both constrair are active.

Table E8.60

Basic	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	b	ratio
y_7	1	2	-1	1	-1	0	1	0	1	0.5
y_8	2	1	1	-1	0	-1	0	1	4	4
Cost	5	4	-1	1	0	0	0	0	f-0	
Arti	-3	-3	0	0	1	1	0	0	w-5	
y_2	0.5	1	-0.5	0.5	-0.5	0	0.5	0	0.5	negative
y_8	1.5	0	1.5	-1.5	0.5	-1	-0.5	1	3.5	2.33333
Cost	3	0	1	-1	2	0	-2	0	f-2	
Arti	-1.5	0	-1.5	1.5	-0.5	1	1.5	0	w-3.5	
y_2	1	1	0	0	- 1/3	- 1/3	1/3	1/3	1.6667	
y_3	1	0	1	-1	1/3	- 2/3	- 1/3	2/3	2.3333	
Cost	2 (c_1')	0 (c_2')	0 (c_3')	0 (c_4')	5/3 (c_5')	2/3 (c_6')	-5/3 (c_7')	- 2/3 (c_8')	f-4.3333	
Arti	0	0	0	0	0	0	1	1	w-0	End phs1

8.115

Referring to Exercise 8.60 and the final tableau in Table E8.60, we can find the ranges for RHS by *Theorem 8.6* as follows:

For $b_1 = 1$: $-\frac{5/3}{1/3} \leq \Delta_1 \leq \frac{7/3}{1/3}$ or $-5.0 \leq \Delta_1 \leq 7.0$;

For $b_2 = 4$: $\max\{-\frac{5/3}{1/3}, -\frac{8/3}{2/3}\} \leq \Delta_2 \leq \infty$ or $-4 \leq \Delta_2 \leq \infty$