확률과 통계 4주차 과제 모범답안

1. 확률변수 X,Y 의 결합밀도함수는 다음과 같다.

$$f(x,y)=\begin{cases}\frac{1}{8}\cdot(6-x-y) & 0<x<2, 2<y<4\\ 0 & otherwise\end{cases}$$

(1) X의 주변밀도함수 $g(x)$와 Y의 주변밀도함수 $h(y)$를 구하라. (각 2.5점)

$$g(x)=\int_{-\infty}^{\infty}f(x,y)dy=\int_{2}^{4}f(x,y)dy$$

$$=\int_{2}^{4}\frac{1}{8}\cdot(6-x-y)dy=\frac{1}{8}[6y-xy-\frac{y^2}{2}]_2^4=\frac{3-x}{4}$$

$$\therefore g(x)=\begin{cases}\frac{3-x}{4} & ,0<x<2\\ 0 & ,otherwise\end{cases}$$

$$h(y)=\int_{-\infty}^{\infty}f(x,y)dx=\int_{0}^{2}f(x,y)dx$$

$$=\int_{0}^{2}\frac{1}{8}\cdot(6-x-y)dx=\frac{1}{8}[6x-\frac{x^2}{2}-yx]_0^2=\frac{5-y}{4}$$

$$\therefore h(y)=\begin{cases}\frac{5-y}{4} & ,2<y<4\\ 0 & otherwise\end{cases}$$

(2) $E(X)$와 $E(Y)$를 구하라. (각 2.5점)

$$E(X) = \int_{-\infty}^{\infty} x \cdot g(x)dx$$

$$= \frac{1}{4}\int_{0}^{2}(3x - x^2)dx = \frac{1}{4}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^2 = \frac{5}{6}$$

$$\therefore E(X) = \frac{5}{6}$$

$$E(Y) = \int_{-\infty}^{\infty} y \cdot h(y)dy$$

$$= \frac{1}{4}\int_{2}^{4}(5y - y^2)dy = \frac{1}{4}\left[\frac{5y^2}{2} - \frac{y^3}{3}\right]_2^4 = \frac{17}{6}$$

$$\therefore E(Y) = \frac{17}{6}$$

(3) $Var(X)$와 $Var(Y)$를 구하라. (각 2.5점)

$$Var(X) = EX^2 - E(X)^2$$

$$= \frac{1}{4}\int_0^2 (3x^2 - x^3)dx - \left(\frac{5}{6}\right)^2 = 1 - \left(\frac{25}{36}\right) = \frac{11}{36}$$

$$\therefore\ Var(X) = \frac{11}{36}$$

$$Var(Y) = EY^2 - E(Y)^2$$

$$= \frac{1}{4}\int_2^4 (5y^2 - y^3) - \left(\frac{17}{6}\right)^2 = \frac{25}{3} - \left(\frac{25}{36}\right) = \frac{11}{36}$$

$$\therefore\ Var(Y) = \frac{11}{36}$$

(4) $Cov(X,Y)$를 구하라. (2.5 점)

$$Cov(X,Y) = E(X,Y) - E(X)E(Y)$$

$$= \frac{1}{8}\int_2^4\int_0^2 (6xy - yx^2 - y^2x)dxdy - \left(\frac{5}{6}\right)\left(\frac{17}{6}\right)$$

$$= \frac{7}{3} - \left(\frac{85}{36}\right) = -\frac{1}{36}$$

$$\therefore Cov(X,Y) = -\frac{1}{36}$$

(5) 상관계수를 구하라. (2.5 점)

$$\rho_{X,Y} = \frac{Cov(X,Y)}{\sqrt{VarX}\sqrt{VarY}} = \frac{-\frac{1}{36}}{\sqrt{\frac{11}{36}}\sqrt{\frac{11}{36}}} = -\frac{1}{11}$$

$$\therefore \rho_{X,Y} = -\frac{1}{11}$$