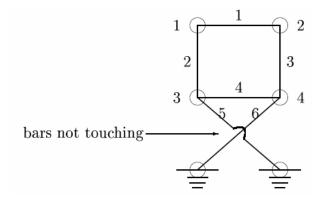
- 1. Solve the differential equation $-u''(x) = \delta(x)$ with u(-2) = 0 and u(3) = 0. The pieces u = A(x+2) and u = B(x-3) meet at x = 0. Show that the vector U = (u(-1), u(0), u(1), u(2)) solves the corresponding matrix problem KU = F = (0, 1, 0, 0). (20 pts)
- 2. The 2 by 2 matrix K_2 has eigenvalues 1 and 3 in Λ . Its *unit* eigenvectors q_1 and q_2 are the columns of Q. Multiply $Q\Lambda Q^T$ to recover K_2 . (10 pts)
- 3. The function f(x, y) = 2xy certainly has a saddle point and not a minimum at (0, 0). What symmetric matrix *S* produces this *f*? What are its eigenvalues ? (20 pts)
- 4. Suppose the measurements at t = -1, 1, 2 are b = (5, 13, 17). Compute \hat{u} and the closest line and e. The error is e = 0 because this b is _____. (20 pts)
- 5. This problem is about the symmetric matrix $H = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$
 - (1) By elimination find the triangular L and diagonal D in $H = LDL^{T}$. (5 pts)
 - (2) What is the smallest number q that could replace the corner entry $H_{33} = 1$ and still leave H positive semidefinite? (5 pts)
 - (3) *H* comes from the 3-step framework for a hanging line of springs:

displacement \xrightarrow{A} elongation \xrightarrow{C} spring forces $\xrightarrow{A^{T}}$ external force f

What are the specific matrices A and C in $H = A^T CA$? (5 pts)

- (4) What are the requirements on any *m* by *n* matrix A and symmetric matrix C for $A^{T}CA$ to be positive definite? (5 pts)
- 6. This truss doesn't look safe to me. Those angles are 45. The matrix A will be 6 by 8 when the displacements are fixed to zero at the bottom.
 - (1) How many independent solutions to e = Au = 0? (2 pts)
 - (2) Write numerical vectors $u = (u_1^H, u_1^V, \dots, u_4^H, u_4^V)$ that solve Au = 0 to give those mechanisms in (1). (3 pts)
 - (3) What is the first row of $A^{T}A$ if unknowns are taken in that usual order used in (2)? (5 pts)



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