Midterm Exam 05/06/2008

1. Solve the differential equation $-u''(x) = \delta(x)$ with $u(-2) = 0$ and $u(3) = 0$. The pieces $u = A(x+2)$ and $u = B(x-3)$ meet at $x = 0$. Show that the vector $U = (u(-1), u(0), u(1), u(2))$ solves the corresponding matrix problem $KU = F = (0, 1, 0, 0)$. (20 pts)

2. The 2 by 2 matrix K_2 has eigenvalues 1 and 3 in Λ. Its *unit* eigenvectors q_1 and q_2 are the columns of Q. Multiply $Q\Lambda Q^T$ to recover K_2. (10 pts)

3. The function $f(x, y) = 2xy$ certainly has a saddle point and not a minimum at (0, 0). What symmetric matrix S produces this f? What are its eigenvalues ? (20 pts)

4. Suppose the measurements at $t = -1, 1, 2$ are $b = (5, 13, 17)$. Compute $\hat{u}$ and the closest line and e. The error is $e = 0$ because this b is _______. (20 pts)

5. This problem is about the symmetric matrix $H = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

   (1) By elimination find the triangular L and diagonal D in $H = LDL^T$. (5 pts)

   (2) What is the smallest number q that could replace the corner entry $H_{33} = 1$ and still leave H positive semidefinite? (5 pts)

   (3) H comes from the 3-step framework for a hanging line of springs:

   displacement $\xrightarrow{A}$ elongation $\xrightarrow{C}$ spring forces $\xrightarrow{A^T}$ external force f

   What are the specific matrices A and C in $H = A^T CA$? (5 pts)

   (4) What are the requirements on any m by n matrix A and symmetric matrix C for $A^T CA$ to be positive definite? (5 pts)

6. This truss doesn't look safe to me. Those angles are 45. The matrix A will be 6 by 8 when the displacements are fixed to zero at the bottom.

   (1) How many independent solutions to $e = Au = 0$? (2 pts)

   (2) Write numerical vectors $u = (u_1^H, u_1^V, \ldots, u_4^H, u_4^V)$ that solve $Au = 0$ to give those mechanisms in (1). (3 pts)

   (3) What is the first row of $A^T A$ if unknowns are taken in that usual order used in (2)? (5 pts)