Final Exam

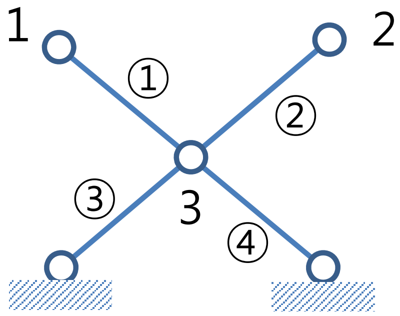
06/11/2012

1.

(1) A truss is in the shape of an X with 45 angles and 2 fixed nodes at the bottom. Based on the number *m* of bars and n of unknown displacements, how many independent solutions do you expect to $Au = 0$? (5 pts)

(2) Give the components $u_1^H, u_1^V, \ldots, u_3^V$ for a full set of independent solutions of $Au = 0$. Draw three mechanisms. (10 pts)

(3) What is row 1 of the matrix *A* corresponding to upper left bar 1? (10 pts)



2.

(1) Check that $f = (x+iy)^n$ solves Laplace's equation. Substitute it into the equation to confirm. (5 pts)

(2) Find the real and imaginary parts u and s of the function $1/z = 1/(x+iy)$. Give the answers u and s in x, y coordinates and also in polar r, θ coordinates. (10 pts)

(3) Solve Laplace's equation outside the unit circle $r^2 = x^2 + y^2 = 1$ if the boundary condition is $u = u_0 = y$ on the circle. (5 pts)

3.

(1) Explain what condition the components $(v_1, v_2) = (\partial u/\partial x, \partial u/\partial y)$ of a gradient field must satisfy, and give the reason why. (5 pts)

(2) Find a potential $u(x, y)$ if $v(x, y) = (1, 2) = \text{constant velocity}$. Using this $u(x, y)$, find a stream function $s(x, y)$ so that the Cauchy-Riemann equations are satisfied: $\frac{\partial u}{\partial x} = \frac{\partial s}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial s}{\partial x}$ (10 pts)

(3) Draw a few equipotential curves $u(x, y) = \text{constant}$ and a few stream lines $s(x, y) = \text{constant}$. (10 pts)

4.

(1) Find the weak form of the 1-D equation $-\frac{d}{dx}\left(\frac{du}{dx}\right)=\delta\left(x-\frac{1}{2}\right)$ with $du/dx(0)=0$ and $u(1)=0$. Describe what boundary conditions $u(x)$ and $v(x)$ are required to satisfy in the weak form. (10 pts)

(2) Suppose $h=\frac{1}{2}$ and we use two continuous piecewise linear functions (hat-type functions) ϕ_1 and ϕ_2:

trial function ϕ_i = test function V_i = hat-type functions = 1 at one node

Draw these functions. Find the 2 by 2 stiffness matrix K and the 2 component vector F. (10 pts)

(3) Solve $KU=F$ to find the finite element solution $U=(U_1,U_2)$ at the nodes. Draw the graph of this solution $U(x)=U_1\phi_1+U_2\phi_2$. This is the exactly correct solution $u(x)$ to the differential equation. (10 pts)