Final Exam

06/15/2017

1.

(1) Is the vector field $w(x,y)=\left(x^2-y^2,2xy\right)$ equal to the gradient of any function $u(x)$? What is the divergence of w? If $u(x,y)$ and $s(x,y)$ are a Cauchy-Riemann pair, show that $w(x,y)=\left(s(x,y),u(x,y)\right)$ will be a gradient field and also have divergence zero. (15 pts)

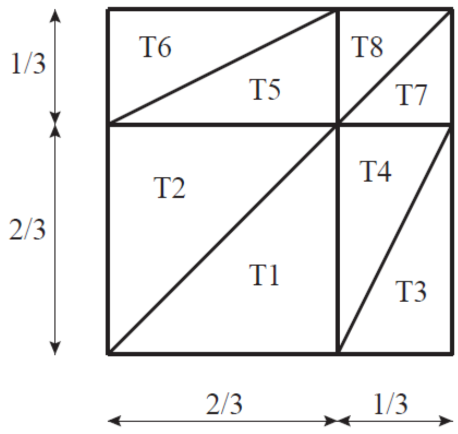
(2) Take real and imaginary parts of $f(x+iy)=\left(x+iy+\frac{1}{x+iy}\right)$ to find two solutions of Laplace's equation. Write those two solutions also in polar coordinates. (10 pts)

2. Solve Poisson's equation with $f(x,y)=1$ on a square of side length 1 by *finite differences* considering two *interior* nodes in the *x* direction and three *interior* nodes in the *y* direction. The boundary conditions are: (25 pts)

- $u=1$ along the horizontal sides (*x* direction) including the corners of the square
- $u=0$ along the vertical sides (*y* direction) excluding the corners of the square

** Do not compute the actual solution. Just write down the matrices involved in the calculation.

3. Solve Poisson's equation with $f(x, y) = 1$ on a square of side length 1 by *finite element method* considering the non-uniform triangular grid given in the sketch below. The boundary conditions are u = 1 everywhere. (25 pts)



4. Find the curve of fixed length 5/4 that joins the points (0, 0) and (1, 0), lies above the x-axis, and encloses the maximum area between itself and the maximum area between itself and the x-axis. Draw the curve. (25 pts)