Midterm Exam

04/20/2017

1. Suppose $\lambda_1 = 1$ and $\lambda_2 = 2$ are the eigenvalues of a matrix *A*, $y_1^T = [1 \quad 0]$ and $y_2^T = [1 \quad 1]$ are the corresponding eigenvectors.

(1) Calculate the matrix *A*. (4 pts)

(2) Calculate the matrix A^8, its eigenvalues and its eigenvectors. (4 pts)

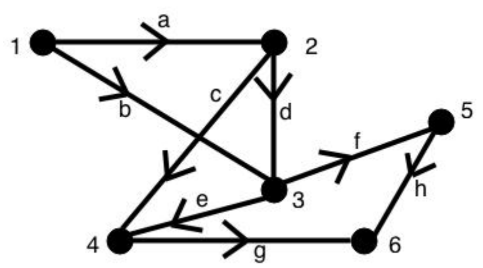
(3) Suppose a new matrix *B* with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 2$ and eigenvectors $y_1^T = 2^{-1/2}[-1 \quad 1]$ and $y_2^T = 2^{-1/2}[-1 \quad -1]$. Can you compute *B* without using the inverse? Explain your answer and, if possible, compute *B* (4 pts)

(4) In the question above, what if y_1 and y_2 are not unit vectors? Can you still do the same? (2 pts)

(5) Suppose $C = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$ Can you diagonalize *C* in terms of its eigenvalues and eigenvectors? Explain your answer and, if possible, perform the diagonalization. (4 pts)

(6) For *A*, A^8, *B* and *C*, determine if each matrix is positive definite, semidefinite or indefinite. (2 pts)

2. Consider the following electrical circuit with six nodes (numbered) and eight edges (lettered):



(1) Write down the 8 × 6 incidence matrix *A*. (3 pts)

(2) Describe the vectors *x* such that *Ax* = 0 and the vectors *w* such that $A^T w = 0$. (5 pts)

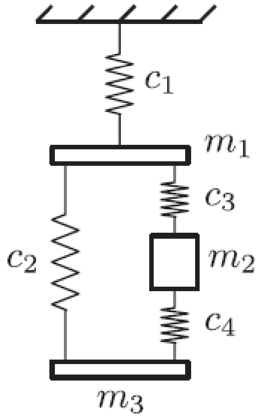
(3) Give the physical interpretation of $A^T w = 0$, where $w = (0,0,1,-1,0,-1,1,-1)^T$. (3 pts)

(4) Let *A* be the incidence matrix, *e* the vector of potential differences, and *w* the vector of edge currents. State Kirchhoff's current and voltage law and as relations involving *A*, *e*, and *w*. Suppose no external forces or batteries. (You may like to write down the laws in words first.) (3 pts)

(5) Show the following for vectors *x*, *y*, and *z*: If *y* = *Ax* and $A^T z = 0$, then $y^T z = 0$. (3 pts)

(6) Explain, using the result from the previous question, why Kirchhoff's current implies Kirchhoff's voltage law. (3 pts)

3. For the following system of springs and masses,



(1) What matrix A gives the spring stretching $e = Au$ from the displacements u of the masses? (3 pts)

(2) Find the stiffness matrix K of the system. (8 pts)

(3) Is there any zero entry in K? Why? (2 pts)

(4) Suppose $c_1 = c,\ c_2 = 2c,\ c_3 = 3c,\ c_4 = 4c$ (with $c > 0$), demonstrate whether or not K is singular. (5 pts)

(5) Under the same conditions as in the previous question (4), demonstrate whether or not K is positive definite. (2 pts)

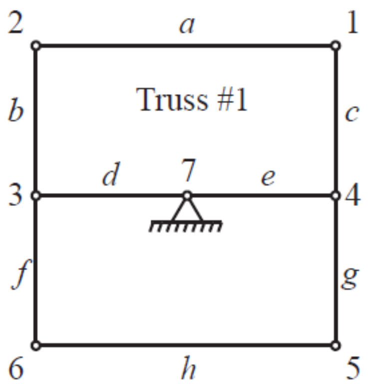
4. Set up the problem to fit a parabola $\hat{u} = C + Dt + Et^2$ through these (t, b) points, minimizing the sum of the squares of the error: (0,1), (1,1), (2,4).

(1) Give the matrix A and the vector b for the least squares formulation of the problem. (8 pts)

(2) Calculate $A^T A$ and $A^T b$, and give the Matlab command that would give you the coefficients of the fitted parabola. (8 pts)

(3) What is the minimum value $E = e_1^2 + e_2^2 + e_3^2$? (5 pts)

5. Consider the following trusses with numbered nodes lettered edges:



(1) Is Truss #1 stable? If not, identify the mechanisms and rigid motions and provide the corresponding u's that conform the null space of the incidence matrix A. (10 pts)

(2) In Truss #1, add (draw) the minimum number of bars necessary to block all the mechanisms. (5 pts)

(3) Following from the previous question, what is the rank of the new matrix A? Guess and explain your answer. Do not write the new incidence matrix. (5 pts)