Midterm Exam - Lab

04/18/2013

Submit the compressed file as `(ID)-(name).zip` to [ftp://cdl.hanyang.ac.kr → CAE/중간고사-Lab] folder. It should contain the final results of each problem (equations and graphs) using PowerPoint (`ID.ppt`), Matlab files (`problem#-#.m`), and COMSOL files (`problem#-#.mph`)

1. (20 pts) Solve the following initial value problem over the interval from $t = 0$ to 2 where $y(0)=1$. Display all your results on the same graph.

$$\frac{dy}{dt} = yt^3 - 1.5y$$

(1) Analytically.

(2) Using Euler's method with $h = 0.5$ and 0.25.

(3) Using the fourth-order RK method with $h = 0.5$.

(4) Using the Matlab built-in functions.

[Fourth-order RK]

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

$$\begin{cases} k_1 = f(x_i, y_i) \\ k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h) \\ k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h) \\ k_4 = f(x_i + h, y_i + k_3 h) \end{cases}$$

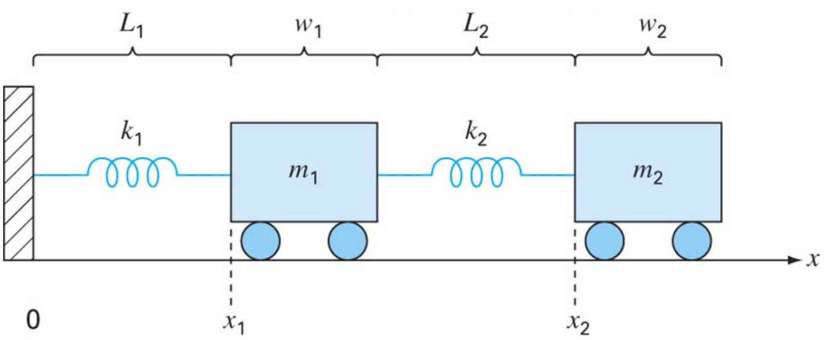
2. (30 pts) Two masses are attached to a wall by linear springs as shown in the figure.

(1) Derive the following force balances based on Newton's second law.

$$\begin{cases} \dfrac{d^2x_1}{dt^2} = -\dfrac{k_1}{m_1}(x_1 - L_1) + \dfrac{k_2}{m_1}(x_2 - x_1 - w_1 - L_2) \\ \dfrac{d^2x_2}{dt^2} = -\dfrac{k_2}{m_2}(x_2 - x_1 - w_1 - L_2) \end{cases}$$

(2) Compute the positions of the masses as a function of time using the following parameter values: $k_1 = k_2 = 5,\ m_1 = m_2 = 2,\ w_1 = w_2 = 5,\ L_1 = L_2 = 2$. Set the initial conditions as $x_1 = L_1$ and $x_2 = L_1 + w_1 + L_2 + 6$. Perform the simulation from $t = 0$ to 20. Construct time-series plots of both the displacements and the velocities. In addition, produce a phase-plane plot of x_1 versus x_2. (Use Matlab built-in functions.)

(3) Repeat (2) using COMSOL Multiphysics.

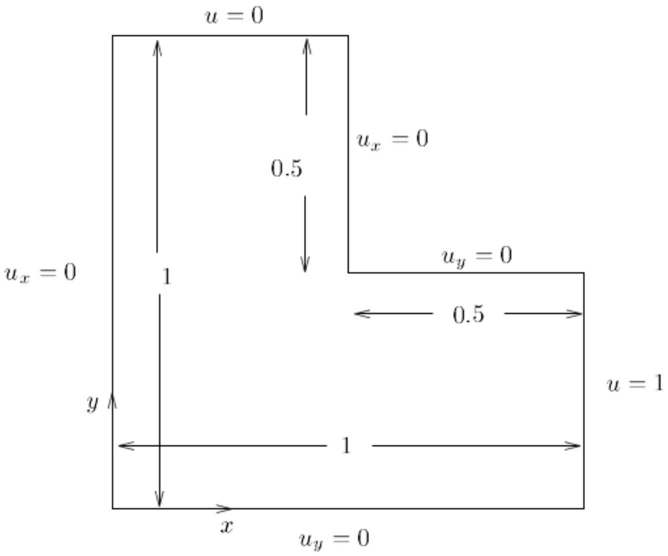


3. (20 pts) [Matlab] Solve the partial differential equation $\dfrac{\partial u}{\partial t} = 0.5\dfrac{\partial u}{\partial x} - 0.4u$ defined on the semi-infinite domain, $-\infty < x < \infty$ and $0 \le t < \infty$, with the boundary condition given at $t = 0$ as

$$u(x,0) = \begin{cases} 1 & \text{if } 3.5 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Use the 2-point forward difference scheme to discretize both $\partial u/\partial t$ and $\partial u/\partial x$. Choose $\Delta x = 0.1$ and $\Delta t = 0.1$. Integrate your system forward in t to find the solution, $u(x,t)$ at $t = 0.5$, 1, and 2. Plot these solutions (as a function of x) along with the "initial" state $u(x,0)$, over the interval of $0 \le x \le 5$.

4. (15 pts) [COMSOL] Solve $u_{xx} + u_{yy} = 0$ with the geometry, dimensions and boundary conditions indicated in the figure. Plot (1) the surface $u(x, y)$, and (2) several $u = \text{constant}$ contours.



5. (15 pts) [COMSOL] Solve $u_{xx} + u_{yy} = u_t$ with $u(x, y, t)$ in a square $0 \le x \le 1,\ 0 \le y \le 1$. The initial condition is $u(x, y, 0) = \sin[\pi x(1-x) y(1-y)]$, and the boundaries are all at $u = 0$. Plot the u contours for $t = 0,\ 0.5,\ 1.0$.