Midterm Exam

04/16/2013

1. (2 pts each)

* The errors associated with both calculations and measurements can be characterized with regard to their ((1)) and ((2)). ((1)) refers to how closely a computed or measured value agrees with the true value. ((2)) refers to how closely individual computed or measured values agree with each other.

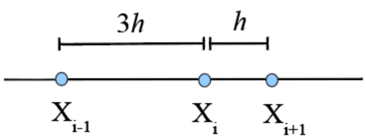
* The numerical solution of ODEs involves two types of error: ((3)) errors caused by the limited numbers of significant digits that can be retained by a computer. ((4)) errors caused by the nature of the techniques employed to approximate values of y.

* Fractional quantities are typically represented in computers using floating-point form. In this approach, the number is expressed as a fractional part, called a(n) ((5)), and an integer part, called a(n) ((6)), as in $m \cdot b^e$.

* The condition of a mathematical problem relates to its sensitivity to changes in its input values. We say that a computation is numerically unstable if the uncertainty of the input values is grossly magnified by the numerical method. These ideas can be studied using a first-order Taylor series $f(x) = f(\tilde{x}) + (\ (7)\ )$ This relationship can be employed to estimate the relative error of $f(x)$ as in $\left[f(x) - f(\tilde{x})\right]/f(x) \cong (\ (8)\ )$. The relative error of x is given by $(x-\tilde{x})/\tilde{x}$. A condition number can be defined as the ratio of these relative errors: Condition number = ((9)).

* ((10)) ODEs are both individual and systems of ODEs that have both fast and slow components to their solutions. We introduce the idea of an implicit solution technique as one commonly used remedy for this problem.

2. (10 pts) Consider the non-uniform grid (shown in the diagram below) with $x_i - x_{i-1} = 3h$ and $x_{i+1} - x_i = h$. Derive a 3-point finite difference formula for the first derivative of $f(x_i)$ at $x = x_i$ that has a truncation error of $O(h^2)$. Your formula should have the form: $f'(x_i) = Af(x_{i-1}) + Bf(x_i) + Cf(x_{i+1}) + O(h^2)$



3. (20 pts) Given the differential equation $y' = 2y$ with $y(0) = 2$

(1) Find an estimate of $y(1)$ using Euler's method with a single step.

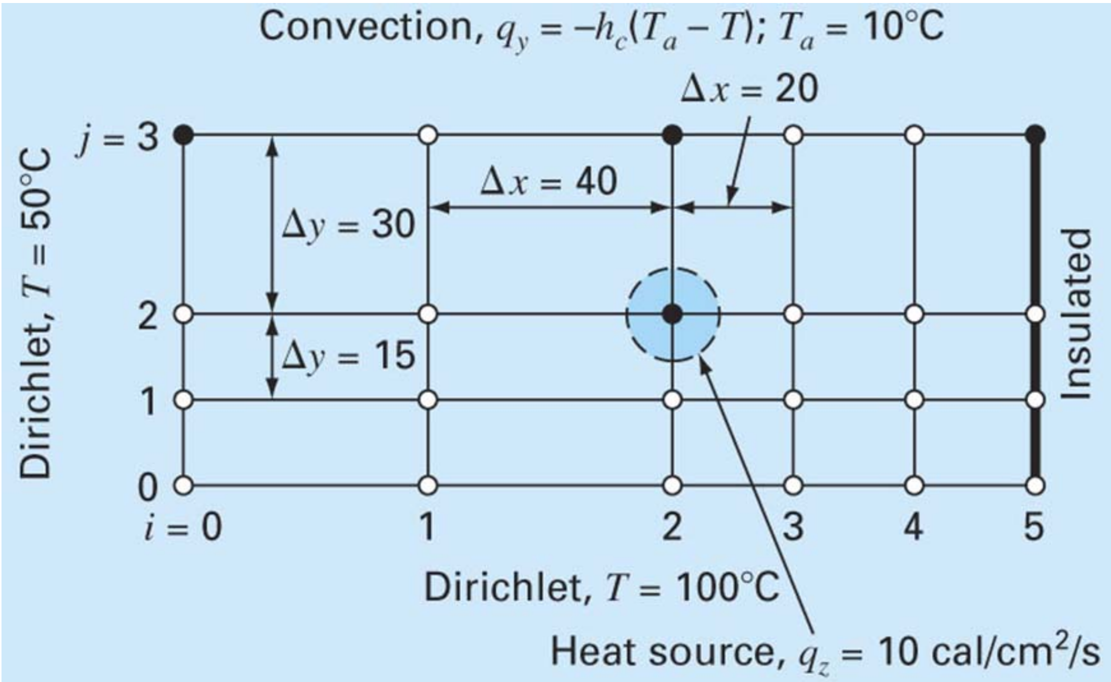
(2) Find an estimate of $y(1)$ using Heun's method.

(3) Write down the Matlab code to calculate $y(1)$ using ode function.

4. (10 pts) Given the differential equation $y' + y = t$ with $y(0) = 1$, find an estimate of $y(1)$ with the implicit Euler method.

5. (15 pts) Consider the boundary value problem $\frac{d^2u}{dx^2} + (15 + 8x)u = 0$ with $u'(0) = 0.5$ and $u(1) = 0.3$ for $u(x)$ over the interval of $0 \le x \le 1$. Use the 3-point central difference scheme to represent u'' in the differential equation and 2-point forward difference scheme to represent u' in the first boundary condition. Using $h = 0.25$, construct the set of simultaneous equations in matrix form.

6. (10 pts) Write equations for the node (2,2) in the figure. Note all units are cgs. The convection coefficients is $h_c = 0.01$ cal/(cm$^2 \cdot C \cdot s$) and the thickness of the plate is 0.5 cm.



7. (15 pts) Find the general solution of the following PDE by the method of separation of variables.

$$\frac{\partial^2 u}{\partial x \partial y} - xyu = 0$$