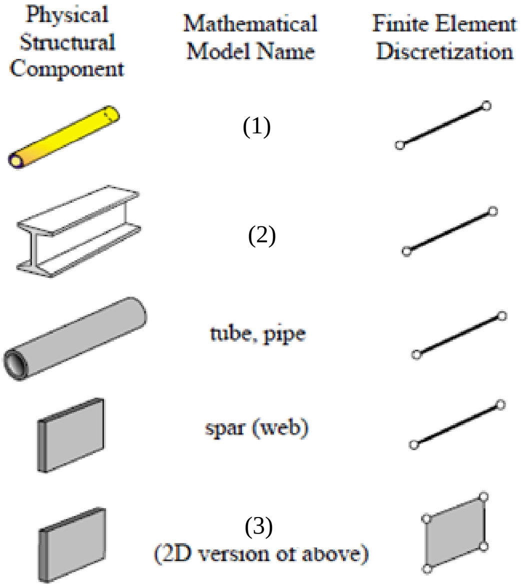
Final Exam

06/17/2014

1. Fill in the blanks. (2 pts each)



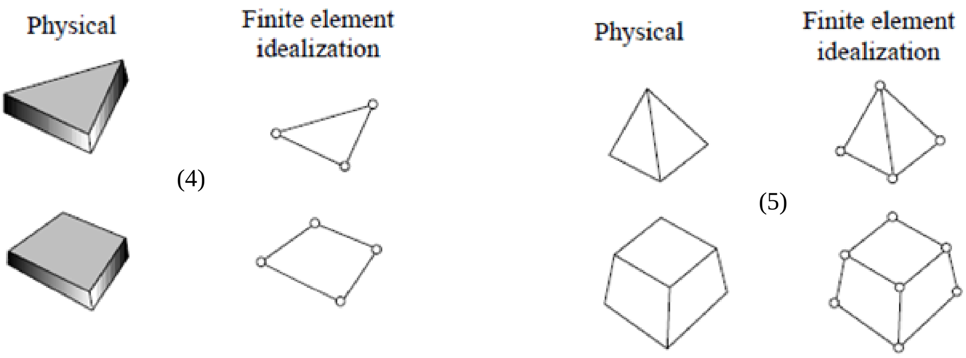


Table 6.1. Significance of **u** and **f** in Miscellaneous FEM Applications

Application Problem	*State (DOF) vector* **u** *represents*	*Conjugate vector* **f** *represents*
Structures and solid mechanics	(6)	Mechanical force
Heat conduction	Temperature	(7)
Acoustic fluid	Displacement potential	Particle velocity
Potential flows	(8)	Particle velocity
General flows	Velocity	Fluxes
Electrostatics	Electric potential	(9)
Magnetostatics	(10)	Magnetic intensity

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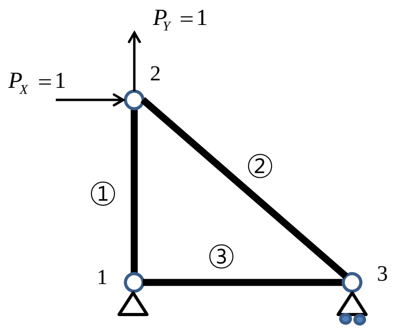
2. (20 pts) Consider the three-member example truss. (Young's modulus E = 100)



(1) Assemble the master stiffness equations.

(2) Apply the displacement BCs and derive the reduced system. (Do not solve the equations.)

(3) Recover the axial forces in the three members.

(4) Recover all reaction forces.

Node #	coordinate	
	x	y
1	0.0	0.0
2	0.0	1.0
3	1.0	0.0

Element #	Node #		Cross sectional area	length
①	1	2	1	1
②	2	3	1	$\sqrt{2}$
③	1	3	1	1

3. (20 pts) Suppose that the assembled stiffness equations for a one-dimensional finite element model before imposing constraints are $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$. This system is to be solved subject to the multipoint constraint $u_1 = u_3$.

(1) Impose the constraint by the master-slave method taking u_1 as master, and solve the resulting 2x2 system of equations.

(2) Impose the constraint by the penalty function method, leaving the weight w as a free parameter. Write down the equation only and do not solve.

(3) Impose the constraint by the Lagrange multiplier method. Show the 4x4 multiplier-augmented system of equations.

(4) Explain the characteristics of above three methods.

4. (20 pts) The simplest triangular element for plane stress is the three-node triangle with *linear shape functions*, with degrees of freedom located at the corners.

(1) Write down the linear interpolation for the displacement components u_x and u_y at an arbitrary point *P*.

(2) Write down the strain-displacement equations and the stress-strain equations. [Note that the strains are *constant* over the element. This is the origin of the name *constant strain triangle* constant strain triangle (CST) given it in many finite element publications.]

(3) Derive the stiffness matrix of Turner triangle with uniform thickness h.

(4) Derive the consistent nodal force vector when only internal body forces are considered.

[Hint: $\begin{bmatrix} 1 \\ x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix} \rightarrow \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix} = \frac{1}{2A} \begin{bmatrix} x_2y_3 - x_3y_2 & y_2 - y_3 & x_3 - x_2 \\ x_3y_1 - x_1y_3 & y_3 - y_1 & x_1 - x_3 \\ x_1y_2 - x_2y_1 & y_1 - y_2 & x_2 - x_1 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ y \end{bmatrix}$]

5. (20 pts) Construct the six shape functions for the 6-node transition quadrilateral element in the figure. [Hint: for the corner nodes, use two corrections to the shape functions of the 4-node bilinear quadrilateral. Check compatibility and completeness.]