Midterm Exam - Lab

04/24/2014

Submit the compressed file as `(ID)_(name).zip` to [ftp://cdl.hanyang.ac.kr → CAE/Midterm_Lab] folder. It should contain the final results of each problem (equations and graphs) using PowerPoint (`ID.ppt`), Matlab files (`problem#-#.m`), and COMSOL files (`problem#-#.mph`)

1. [MATLAB] Solve the following initial value problem over the interval from $x = 0$ to 4 where $y(0)=0.5$.

$$\frac{dy}{dx} + 0.6y = 10e^{-(x-2)^2 / \left[2(0.075)^2\right]}$$

(1) Predict the curve shape of the above equation by using the Matlab built-in functions. (10 pts)

(2) Obtain solutions from Euler's method, Heun's method and fourth-order RK method with $h = 0.2$. Compare the three results by using graphical method. (30 pts)

(3) Suggest the reasonable value of step size $h$ for three different methods if you can. (5 pts)

$$[\text{Euler's method}]$$
$$y_{i+1} = y_i + f\left(x_i, y_i\right)h$$

$$[\text{Heun's method}]$$
$$\text{Predictor}: y_{i+1}^0 = y_i + f\left(x_i, y_i\right)h$$
$$\text{Corrector}: y_{i+1} = y_i + \frac{f\left(x_i, y_i\right) + f\left(x_{i+1}, y_{i+1}^0\right)}{2}h$$

$$[\text{Fourth-order RK}]$$
$$y_{i+1} = y_i + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)h$$
$$\begin{cases} k_1 = f(x_i, y_i) \\ k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h) \\ k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h) \\ k_4 = f(x_i + h, y_i + k_3 h) \end{cases}$$

Midterm Exam - Lab

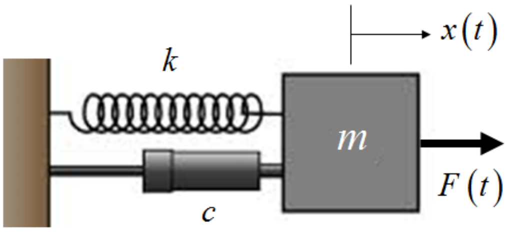
04/24/2014

2. [MATLAB] Compute the response of the system in the following figure for the case that the damping is linear and the spring is a nonlinear softening spring of the form

$$f_k(x) = kx - k_1 x^3$$

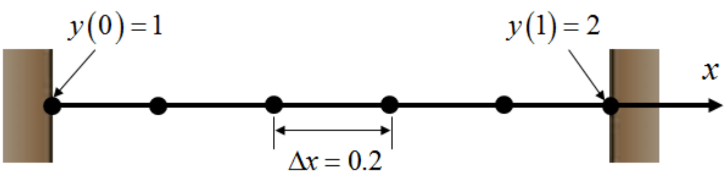
The system is subject to a harmonic excitation of 1,500N at a frequency of approximately one-third the natural frequency ($\omega = \omega_n/2.964$) and initial conditions of $x_0 = 0.01\text{m}$ and $v_0 = 0.1\text{m/s}$. The system has a mass of 100 kg, a damping coefficient of 170 kg/s, and a linear stiffness coefficient of 2,000 N/m. the value of $k_1$ is taken to be 520 N/m$^3$.

Derive the equation of motion based on Newton's second law and compute the solution until $t = 20\text{s}$ by using the Matlab built-in functions. Compare it to the linear solution ($k_1 = 0$). (20 pts)




3. Solve the following boundary value problem. Let the spatial interval ($\Delta x$) to 0.2.

$$y'' + 5y' + 4y = 2$$




(1) [MATLAB] Find solution by using finite-difference method. Approximate the differential terms to the centered difference formulas as follows. (25 pts)

$$y'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta x^2} \quad , \quad y' = \frac{y_{i+1} - y_{i-1}}{2\Delta x}$$

(2) [COMSOL] Find solution by using 'Coefficient Form PDE' module in COMSOL. Compare two results from MATLAB and COMSOL. (10 pts)