

1. Fill in the blanks. (2 pts each)

approximation of the first derivative

2. Describe the difference between round-off errors and truncation errors. (10 pts)

3. Evaluate y = ((x-7)x+8)x-0.35 at x = 1.37. Use 3-digit arithmetic with chopping. Evaluate the percent relative error. (10 pts)

Midterm Exam

4. You are interested in finding the behavior of the mass-damper-spring system shown in the picture below. The system consists of two masses, two dampers, and two linear springs. The weights of the masses are m_1 and m_2 , the damping coefficients are c_1 and c_2 , the spring constants are k_1 and k_2 , and the neutral length of the spring are l_1 and l_2 respectively. An external force, $f \sin(\omega t)$, is applied to the second mass.

- (1) Find a set of ODEs, or governing equations of motion, of the system. (10 pts)
- (2) The set of governing equations of motion that you found in the previous problem are second order ODEs. These equations need to be reduced to a set of coupled first-order equations for applying a Runge-Kutta method. What are the set of coupled first-order equations? (10 pts)



5. Consider the following first order ODE. Integrate the ODE using the mid-point method and the fourth order Runge-Kutta method with an initial condition, y = 1 at x = 1, and the interval of numerical integration, h = 2. $y' + 2xy - x^2 = 0$

(1) Integrate the ODE using the mid-point method once. (10 pts) What are k_1 and k_2 ? What is the representative slope, ϕ ? What is the value of y after one iteration, i.e., y(3)?

(2) Integrate the same ODE using the fourth-order Runge-Kutta method once. (10 pts) What are k_1 , k_2 , k_3 and k_4 ? What is the representative slope, ϕ ? [Hint: $\phi = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$] What is the value of y after one iteration, i.e., y(3)?

Midterm Exam

04/22/2014

6. The steady-state temperature distribution of a plate can be represented by an elliptic partial differential equation called the Laplace equation: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$

Suppose you define four by five grid points and specify the temperatures on the boundary and one internal point as shown in the picture below. Find the eight by eight matrix equation to be solved in order to find the steady-state temperature distribution of a plate, T_1 , T_2 , T_3 , T_4 , T_5 , T_6 , T_7 and T_8 . (20 pts)



7. Describe the difference between explicit and implicit approximations. (10 pts)