Final Exam

12/15/2016

1. (20 pts) Figure 1 shows a pin-jointed plane stress discretized with 2 elements and 3 nodes. Node 3 is fixed whereas 1 and 2 move over rollers as shown. The only nonzero applied load acts upward on node 1. Solve this problem by the Direct Stiffness Method. Start from the element stiffness equations, already incorporated the $E^e A^e / L^e$ factor below. The element stiffness equations in global coordinates are

$$\begin{bmatrix} 50 & -50 & -50 & 50 \\ -50 & 50 & 50 & -50 \\ -50 & 50 & 50 & -50 \\ 50 & -50 & -50 & 50 \end{bmatrix} \begin{bmatrix} u_{x1}^{(1)} \\ u_{y1}^{(1)} \\ u_{x2}^{(1)} \\ u_{y2}^{(1)} \end{bmatrix} = \begin{bmatrix} f_{x1}^{(1)} \\ f_{y1}^{(1)} \\ f_{x2}^{(1)} \\ f_{y2}^{(1)} \end{bmatrix}, \quad \begin{bmatrix} 50 & 50 & -50 & -50 \\ 50 & 50 & -50 & -50 \\ -50 & -50 & 50 & 50 \\ -50 & -50 & 50 & 50 \end{bmatrix} \begin{bmatrix} u_{x2}^{(2)} \\ u_{y2}^{(2)} \\ u_{x3}^{(2)} \\ u_{y3}^{(2)} \end{bmatrix} = \begin{bmatrix} f_{x2}^{(2)} \\ f_{y2}^{(2)} \\ f_{x3}^{(2)} \\ f_{y3}^{(2)} \end{bmatrix}$$

(1) Assemble the master stiffness equations. (This result is reused in Problem 2 below.)

(2) Apply the given force and displacement BCs to get a reduced system of 2 equations and show it.

(3) Solve the reduced stiffness system for the unknown displacements and show the complete node displacement vector.

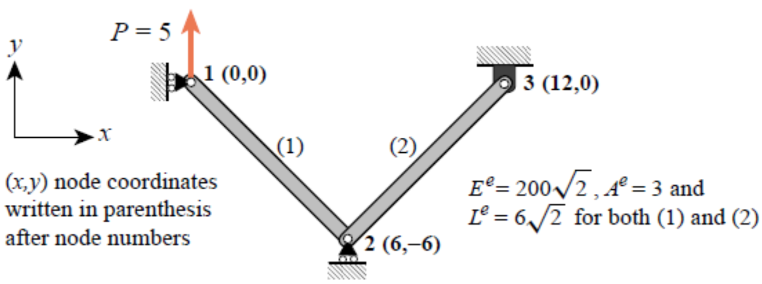


Figure 1

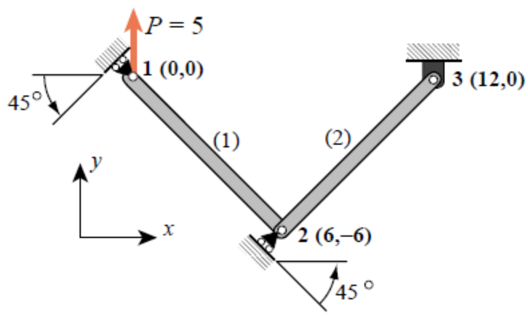


Figure 2

2. (30 pts) This problem reuses the truss structure solved in Problem 1. Everything remains the same except that the support at 1 and 2 are changed to skew rollers at $\pm 45°$ with respect to x, as shown in Figure 2. These two support conditions become multifreedom constraints (MFCs). The departing point is the master stiffness system obtained upon removing node 3, which is fixed, but before applying the MFCs.

(1) Write down the two MFCs.

(2) Apply the MFCs by the master-slave (M/S) method by picking two slaves. Write down the M/S transformation in matrix form, but do *not* proceed further.

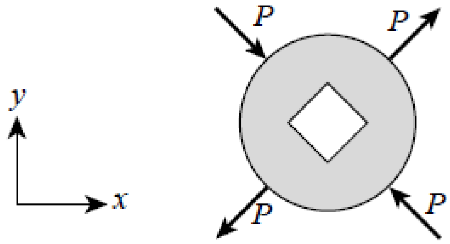
(3) Apply the MFCs by the penalty function method. Write down the stiffness equations of the two penalty elements needed for this method (use same weight w for each), but do *not* proceed further. (Do not forget to include the RHS of the penalty element equations.)

(4) Apply the MFCs by Lagrange multipliers. Show the resulting equation system, but do *not* solve.

3. (15 pts) Figure 3 shows a flat plate of constant thickness and uniform material, loaded as indicated. All edges are free: there are no external supports. The external boundary is a circle and the square hole is centered.

(1) Try to reduce the portion to be discretized as much as possible by identifying symmetry and/or antisymmetry lines. Sketch a diagram of the whole plate that identifies such lines.

(2) Sketch a coarse finite element mesh over the portion you picked in (1). Draw this separately from the sketch in (1). Mark (using roller and/or fixed-point symbols) how you would apply displacement boundary conditions on the nodes of the mesh sketched.



(Loads act at $\pm 45°$ from x.)

Figure 3

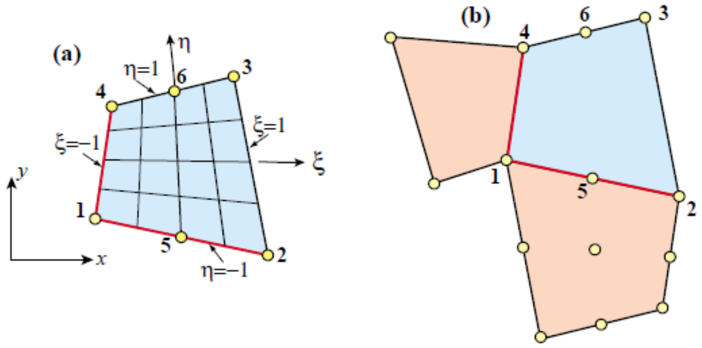


Figure 4

4. (35 pts) The plane stress isoparametric quadrilateral shown in Figure 4 is used as transition element in many FEM codes. It has 6 nodes: four at the corners, and two (5 and 6) at the midpoint of the sides 1-2 and 3-4, respectively. The quadrilateral coordinates of node 5 are $\xi = 0,\ \eta = -1$, and that of node 6 are $\xi = 0,\ \eta = 1$.

(1) Construct the shape function $N_1^e(\xi,\eta)$ for corner node 1 using the product of three lines: two opposite sides and one of the 2 medians.

(2) Check whether the N_1^e constructed in item (1) satisfies C^0 continuity along the sides that contain node 1, namely 1–5–2 and 1–4 (those sides are colored red in the figure), in the sense that the polynomial variation along those sides has sufficient number of nodes to define it uniquely.

(3) Suppose you have derived all shape functions of the 6-node element, and that C^0 compatibility is OK. Which operation would be required to check completeness? (state it, but don't do it).

(4) Restricting attention to the shape function derived in item (1) for simplicity, will *interelement compatibility* be satisfied along those two sides? Explain.

(5) Assume that it will be numerically integrated. What is the minimum number of Gauss points n_G needed so that the element is rank sufficient? (This is not necessarily an actual rule.)

(6) Which $p \times p$ (p is a positive integer) two-dimensional Gauss quadrature rule can be chosen to achieve that?