Midterm Exam

11/03/2016

1. Fill in the blanks. (2 pts each)

A mathematical model can be broadly defined as a formulation or equation that expresses the essential features of a physical system or process in mathematical terms. In a very general sense, it can be represented as a functional relationship of the form: ((1)) variable = f (((2)) variables, ((3)), forcing functions)

where the ((1)) variable is a characteristic that usually reflects the behavior or state of the system; the ((2)) variables are usually dimensions, such as time and space, along which the system's behavior is being determined; the ((3)) are reflective of the system's properties or composition; and the forcing functions are external influences acting upon the system.

Most of the chemical engineering applications will focus on ((4)) balances for reactors. The ((4)) balance is derived from the conservation of ((4)). Both the civil and mechanical engineering applications will focus on models developed from the conservation of ((5)). For civil engineering, ((6)) balances are utilized to analyze structures such as the simple truss. The same principles are employed for the mechanical engineering applications to analyze the transient up-and-down motion or vibrations of an automobile. Finally, the electrical engineering applications employ both ((7)) and ((8)) balances to model electric circuits. The ((7)) balance, which results from the conservation of charge, is similar in spirit to the flow balance. Just as flow must balance at the junction of pipes, electric ((7)) must balance at the junction of electric wires. The ((8)) balance specifies that the changes of voltage around any loop of the circuit must add up to zero.

Numerical errors arise from the use of approximations to represent exact mathematical operations and quantities. These include ((9)) errors, which result when approximations are used to represent exact mathematical procedures, and ((10)) errors, which result when numbers having limited significant figures are used to represent exact numbers.

2. Consider the function $f(x) = x^2$ at $x = 2$. (20 pts)

(1) Compare the forward and centered difference approximations to derivative of the function with a step size $h = 1$ with the exact derivative.

(2) Write the Taylor series of the function at $x = 2$, and check whether the error in the forward differences result agrees with the remainder term.

3. Newton's law of cooling says that the temperature of a body(T) changes at a rate that is proportional to the difference between its temperature and the surrounding medium (usually the ambient) (T_a): (20 pts)

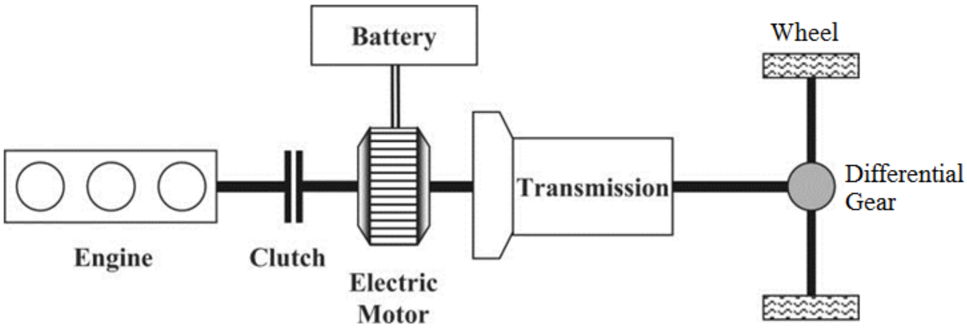
(1) Write the governing equation where k is the heat transfer coefficient.

(2) Obtain the formula for temperature vs. time.

(3) Integrate numerically and compare, for the temperature of a cup of coffee that is initially at 68℃. (use two steps with $k = 0.017/\text{min},\ T_a = 20°C,\ \Delta t = 10$)

4. Because of their widespread application in engineering, our treatment of PDEs will focus on linear, second-order equations. For two independent variables, such equations can be expressed in the following general form: $A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D = 0$ where A, B, and C are functions of x and y and D is a function of x, y, u, $\partial u/\partial x$, and $\partial u/\partial y$. Classify PDEs into three categories depending on the values of the coefficients of the second-derivative terms, and describe characteristics of each category in terms of engineering problem contexts. (15 pts)

5. Consider below the parallel hybrid electric vehicle system.



$$J_{engine} = 0.2\ \text{kg}\cdot\text{m}^2,\ J_{motor} = 0.05\,\text{kg}\cdot\text{m}^2,\ J_{wheel} = 1\ \text{kg}\cdot\text{m}^2 (\text{sum of wheels})$$

$$T_{engine} = 30\ \text{Nm},\ T_{motor} = 50\,\text{Nm},\ Z_{ring} = 80,\ Z_{sun} = 40,\ GR_{differential} = 4$$

$$M_{vehicle} = 1500\ \text{kg}, R_{tire} = 0.3\,\text{m},\ C_d = 0.25, A_{front} = 1.8\,\text{m}^2, \rho_{air} = 1.2\,\text{kg/m}^3, \mu_{roll} = 0.01,\ g = 9.81\ \text{m/s}^2$$

$$V_{battery} = 250\ \text{V},\ C_{nom} = 50{,}000\,\text{As}$$

(1) Describe the advantage/disadvantage things of parallel HEV in comparison with series HEV. (4 pts)

(2) Calculate the total equivalent inertia at wheel when clutch is closed. (drive : sun gear, driven : carrier, fix : ring gear) (8 pts)

(3) Calculate the wheel torque and vehicle acceleration speed. ($V_{vehicle} = 20\ \text{m/s}$) (8 pts)

(4) Calculate the SOC rate $\frac{dSOC}{dt}$ [%/s] when the vehicle speed is 10 m/s. (without loss) (5 pts)