

1. Describe two numerical errors with examples: round-off errors and truncation errors. (10 pts)
2. Solve the following problem over the interval from $x = 0$ to 1 using a step size of 0.5 where $y(0) = 1$.

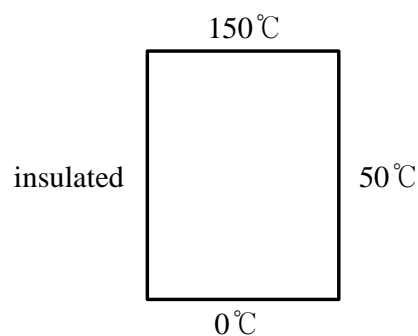
$$\frac{dy}{dx} = (1 + 2x)\sqrt{y} \quad (20 \text{ pts})$$

- (1) Analytically
- (2) Euler's method
- (3) Heun's method without the corrector
- (4) Midpoint method
- (5) Fourth-order RK method [Hint: $\phi = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$]

3. Transform the following differential equation and initial conditions to a set of first order differential equations. (5 pts)

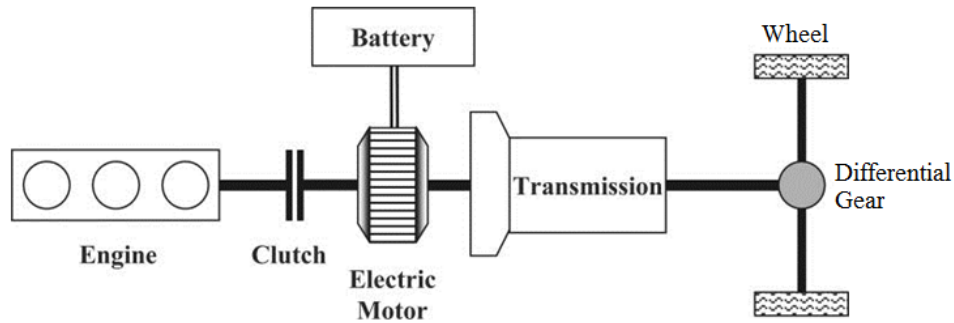
$$\frac{d^3 z}{dx^3} + z^2 \frac{d^2 z}{dx^2} - \left(\frac{dz}{dx} \right)^3 - 2z = 0 \quad \text{where } z(0) = 1, \frac{dz}{dx}(0) = 0, \frac{d^2 z}{dx^2}(0) = -1$$

4. Suppose you define three by four grid points (square grid) and specify the temperatures on the boundary as shown in the figure below. Write the matrix equation to be solved in order to find the steady-state temperature distribution of a plate. (20 pts)



5. Describe the explicit method and the implicit method to solve the heat-conduction equation ($k \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$) in terms of approximations for the second derivative in space and the first derivative in time, and the numerical stability. (20 pts)

6. Consider the following parallel hybrid electric vehicle system. A vehicle speed is 10 m/s.



Inertia/mass : $J_{engine} = 0.2 \text{ kg}\cdot\text{m}^2$, $J_{motor} = 0.05 \text{ kg}\cdot\text{m}^2$, $J_{wheel} = 1 \text{ kg}\cdot\text{m}^2$ (sum of wheels), $m_{body} = 1,500 \text{ kg}$

Powertrain : $T_{engine} = 30 \text{ Nm}$, $T_{motor} = 50 \text{ Nm}$, $GR_{TM} = 2$, $GR_{differential} = 4$, $R_{tire} = 0.3 \text{ m}$

Clutch : $\mu_{disk} = 0.5$, $r_{in} = 0.1 \text{ m}$, $r_{out} = 0.2 \text{ m}$

Motor : $K_t = 2 \text{ Nm/A}$, $K_e = 0.4 \text{ Vs}$, $R = 10 \text{ }\Omega$

Resistance: $C_d = 0.25$, $A_{front} = 1.8 \text{ m}^2$, $\rho_{air} = 1.2 \text{ kg/m}^3$, $\mu_{roll} = 0.01$, $g = 9.81 \text{ m/s}^2$

Battery: $C_{nom} = 50,000 \text{ As}$, $V_{battery} = 250 \text{ V}$

- (1) When a driving mode is converted EV to HEV, a clutch is engaging and generates the friction torque on the clutch disk. Calculate the clutch output torque at this condition. (normal force on disk = 2,000 N) (4 pts)
- (2) Calculate the source voltage of motor. (At first, calculate a motor speed.) (6 pts)
- (3) On HEV mode, calculate a vehicle acceleration speed. (9 pts)
- (4) If the vehicle will be driving only EV mode at this vehicle speed, how much is the driving distance[m] until the battery is completely discharged? (without loss, initial SOC = 50 %) (6 pts)