

Submit the compressed file as (ID)_(name).zip to [<http://cdl.hanyang.ac.kr> → CAE/Midterm_Lab] folder. It should contain the final results (graphs) of each problem using PowerPoint (ID.ppt), MATLAB file (problem#_#.m), Simulink file (problem#_#.slx), and AMESim file (problem#_#.ame).

1. [MATLAB] Solve a following initial value problem over the interval from $y=0$ to 4 , where $y(0)=1$ and a step size of $\Delta x=0.2$.

$$\frac{dy}{dx} = -2x^3 + 12x^2 - 20x + 8.5$$

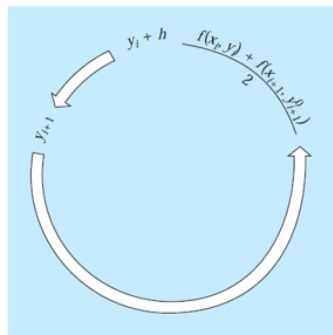
- (1) Obtain solutions using Euler's method. (5 pts)
- (2) Obtain solutions using Heun's method with updating corrector. Use a convergence criterion for the corrector is an absolute percent relative error between the result from the prior and the present iteration of the corrector less than 0.01 %. (10 pts)
- (3) Compare two results with an analytic solution using a graphical method as shown in the following figure. (5 pts)

Analytic solution: $y(x) = -0.5x^4 + 4x^3 - 10x^2 + 8.5x + 1$

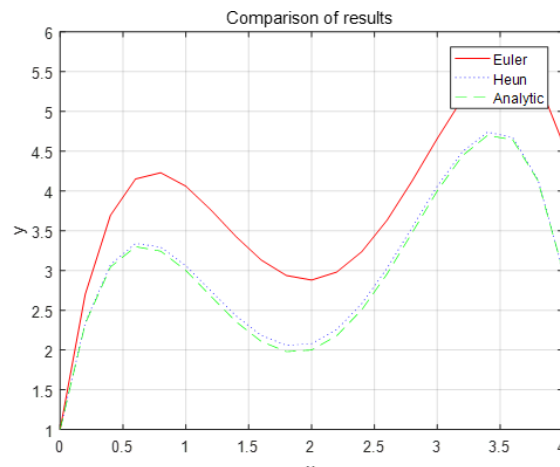
- (4) Comparing the results between Euler's method and analytic solution, calculate the relative error at each point. Suggest the reasonable value of step size Δx which the maximum error is less than 5 %. Develop a script to find this step size automatically not by using trial and error. (10 pts)

[Euler's method] $y_{i+1} = y_i + f(x_i, y_i)\Delta x$ [Relative error] $error = \frac{|Sol_{numerical} - Sol_{analytic}|}{Sol_{analytic}}$

[Heun's method] Predictor $y_{i+1}^0 = y_i + f(x_i, y_i)\Delta x$, Corrector $y_{i+1} = y_i + \frac{f(x_i, y_i) + f(x_{i+1}, y_{i+1}^0)}{2}\Delta x$



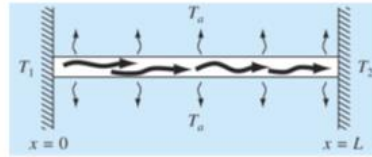
$$|\varepsilon_a| = \left| \frac{y_{i+1}^j - y_{i+1}^{j-1}}{y_{i+1}^j} \right| 100\%$$



2. Consider a boundary value problem with respect to the heat balance for a long, thin rod. The governing equation at steady state, boundary conditions, and parameter values are as follows:

- Heat balance for a long, thin rod

- Not insulated along its length
- Steady state



$$\frac{d^2T}{dx^2} + h'(T_a - T) = 0$$

$$\left. \begin{aligned} T(0) &= T_1 = 40^\circ\text{C} \\ T(L) &= T_2 = 200^\circ\text{C} \end{aligned} \right\} \text{boundary conditions}$$

$$T_a = 20^\circ\text{C} \text{ (temperature of the surrounding air)}$$

$$L = 10 \text{ m}$$

$$h' = 0.01 \text{ m}^{-2} \text{ (heat transfer coefficient)}$$

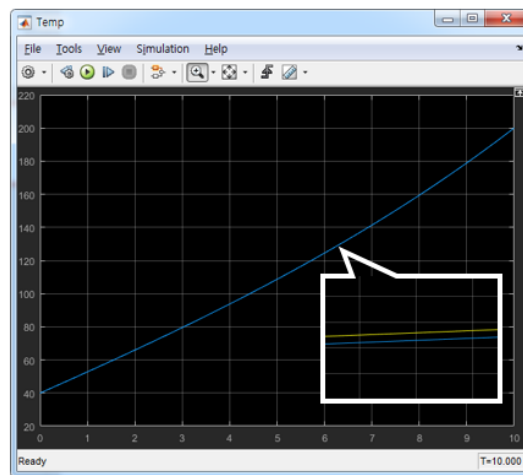
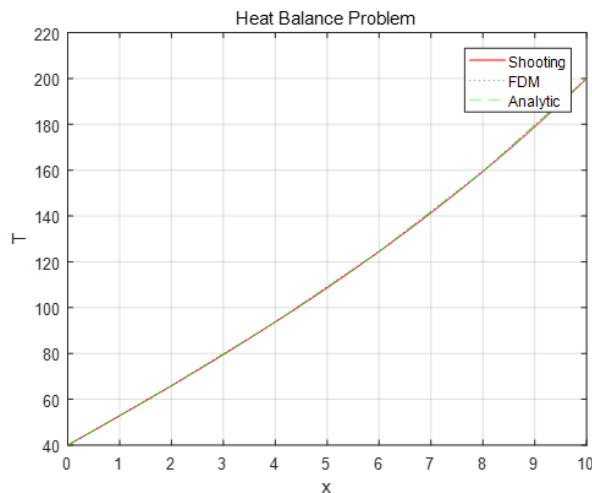
- (1) [MATLAB] Obtain solutions using the shooting method which includes MATLAB built-in function (ode45) and linear interpolation. (10 pts)
- (2) [MATLAB] Obtain solutions using forward finite difference method with step size of $\Delta x = 2$. (10 pts)

$$\text{[Finite difference method]} \quad \frac{d^2T}{dx^2} = -h'(T_a - T) \rightarrow \frac{T_{i-1} - 2T_i + T_{i+1}}{\Delta x^2} = -h'(T_a - T_i)$$

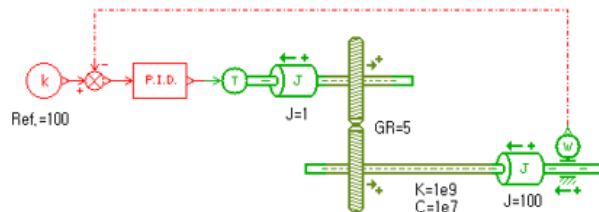
- (3) [MATLAB] Compare the results of (1) and (2) with the analytic solution using a graphical method as shown in the following figure. Use a step size of $\Delta x = 0.1$ for the analytic solution. (5 pts)

$$\text{Analytic solution: } T(x) = 73.4523e^{0.1x} - 53.4523e^{-0.1x} + 20$$

- (4) [Simulink] Construct a Simulink model to obtain the solution. Compare the result with the exact solution by using scope block in Simulink library. Use an initial condition as the result of (1). (10 pts).



3. [AMESim] Consider a simple powertrain model by AMESim. The parameter values in the model are as follows.



error	Kp=1	Kp=5	Kp=10
Ki=0.1			
Ki=1		4.99	
Ki=5			

- (1) Construct this AMESim model. Parameterize the proportional (Kp) and integral (Ki) gain values in a PID block by using the Global Parameters option. (5 pts)
- (2) From the AMESim model, when you change the Kp and Ki values as this table, check the velocity error at 5 second and fill in the table. Which case of the parameters shows the minimum error? (5 pts)
4. [Simulink] Consider the following powertrain system test from 0 to 100 km/h by AMESim. Construct an equivalent Simulink model for this system. Find the time when the vehicle reaches about 100 km/h. (Use “ode23tb” solver in Model Configuration Parameters) (25 pts)

