Midterm Exam

10/30/2018

1. Consider the following initial value problem problem $\frac{dy}{dx} = yx^2 - 1.1y,\ y(0) = 1$. (15 pts)

(1) Perform one step of the explicit Euler method with $\Delta x = 1$.

(2) Perform one step of the implicit Euler method with the same step.

(3) Perform one step of Heun's method without the corrector with the same step.

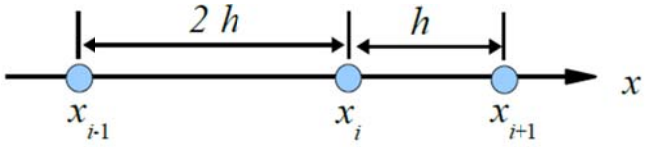
2. We will solve the initial value problem, $\frac{du}{dx} = -2u + x + 4,\ u(0) = 1$ to obtain $u(0.2)$ using $\Delta x = 0.2$ (i.e., we will march forward by just one Δx) (20 pts)

(1) Use 3rd order Runge-Kutta method. [Hint: $\phi = \frac{1}{6}(k_1 + 4k_2 + k_3)$ where $k_3 = f(x_i + h, y_i - k_1 h + 2k_2 h)$]

(2) Use 4th order Runge-Kutta method. [Hint: $\phi = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$]

(3) Compare both with the exact solution.

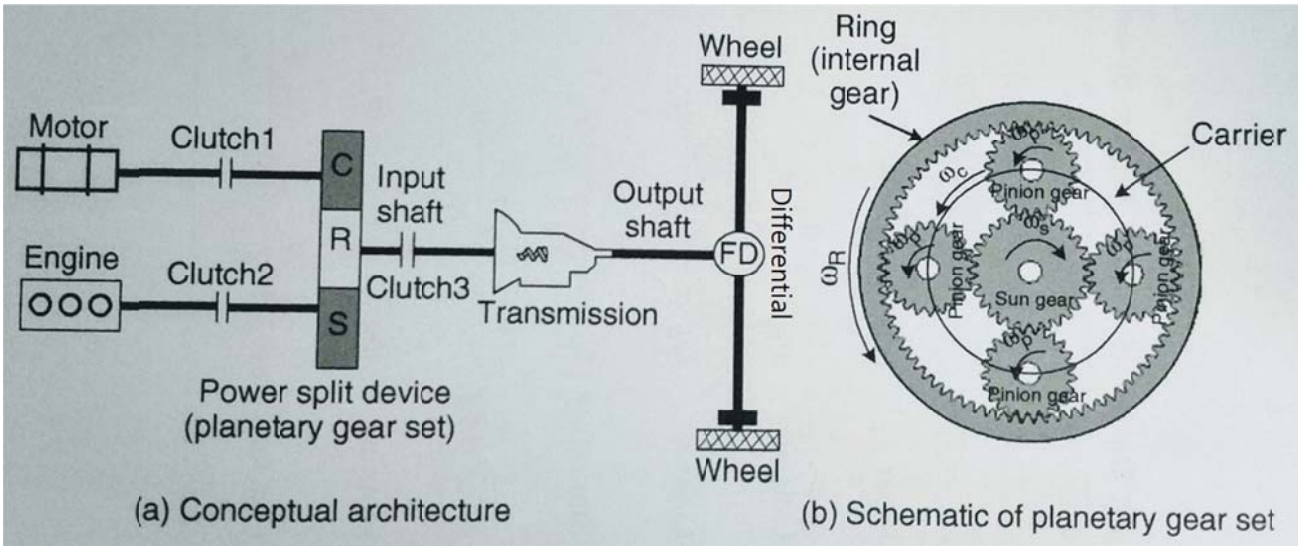
3. Derive a three-point finite difference formula for the second derivative, in which $f''(x_i)$ is expressed as a combination of $f(x_{i-1})$, $f(x_i)$ and $f(x_{i+1})$ with $x_i - x_{i-1} = 2h$ and $x_{i+1} - x_i = h$. (10 pts)



4. Write the matrix equation to obtain a numerical solution of the following boundary value problem with $\Delta x = 0.2$: $\frac{d^2u}{dx^2} = 1,\ u'(0) = 0.5,\ u(1) = 2$. [Do NOT solve the equation.] (15 pts)

5. Find the general solution of the following PDE by the method of separation of variables: $xy\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} + yu = 0$. (15 pts)

6. Refer to the following power-split hybrid electric vehicle system.



Inertia/mass : $J_{engine} = 0.2$ kg•m^2, $J_{motor} = 0.05$ kg•m^2, $J_{wheel} = 1$ kg•m^2 (sum of wheels), $m_{body} = 1{,}500$ kg

Powertrain : $Z_{sun} = 30, Z_{ring} = 60$ (power-split device), $GR_{transmission} = 3$, $GR_{differential} = 4$, $R_{tire} = 0.3$ m

Resistance: $C_d = 0.25, A_{front} = 1.8\,\text{m}^2, \rho_{air} = 1.2\,\text{kg/m}^3, \mu_{roll} = 0.01$, $g = 9.81$ m/s^2

Battery: $C_{nom} = 50{,}000$ As, $V_{battery} = 250$ V

(1) On EV mode, calculate the total equivalent inertia at wheel. (At first, define the state of power-split device. Clutch 1&3 are closed and a sun gear is fixed.) (6 pts)

(2) When the traction motor torque is 50 Nm, calculate a vehicle acceleration speed. ($V_{vehicle} = 20$ m/s) (8 pts)

(3) On HEV mode (All clutches are closed.), when the engine and motor RPM are 3000 and 2000, respectively, calculate a vehicle speed. (At first, calculate an input shaft speed.) (5 pts)

(4) This vehicle is currently driving a downhill road (EV mode). A driver is working the braking pedal to maintain the vehicle speed of 10 m/s, and the motor is charging a battery. When the sum of regenerative braking torque at wheels is 1000 Nm (negative value), calculate the motor torque and speed. How much is a SOC after driving of 20 s. (without loss, current SOC = 50 %) (6 pts)