Final Exam

12/17/2019

1. (10 pts) Fill in the blanks.

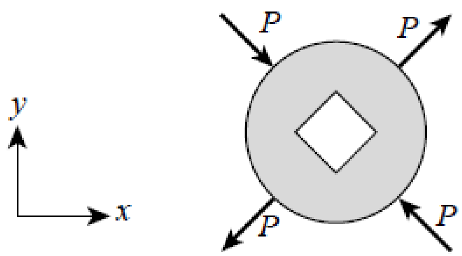
Table 6.1. Significance of **u** and **f** in Miscellaneous FEM Applications

Application Problem	*State (DOF) vector* **u** *represents*	*Conjugate vector* **f** *represents*
Structures and solid mechanics	(1)	Mechanical force
Heat conduction	Temperature	(2)
Acoustic fluid	Displacement potential	Particle velocity
Potential flows	(3)	Particle velocity
General flows	Velocity	Fluxes
Electrostatics	Electric potential	(4)
Magnetostatics	(5)	Magnetic intensity

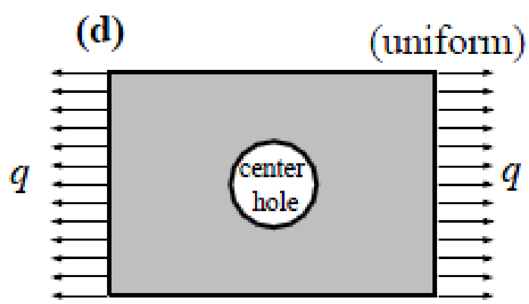
2. (20 pts) Consider the two-dimensional problems illustrated in the figures.

(1) Identify all the symmetry and anti-symmetry lines.

(2) Draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or anti-symmetry lines.



(Loads act at $\pm 45°$ from x.)



3. (15 pts) Derive the stiffness matrix for a tapered bar element in which the cross section area varies linearly along the element length: $A = A_i(1-\zeta) + A_j\zeta$ where A_i and A_j are the areas at the end nodes, and $\zeta = \bar{x}/l$ is the dimensionless coordinate. Show that this yields the same answer as that of a stiffness of a constant-area bar with cross section $\frac{1}{2}(A_i + A_j)$. [Hint: $\mathbf{K}^e = \int_0^1 EA\mathbf{B}^T\mathbf{B}l\,d\zeta$]

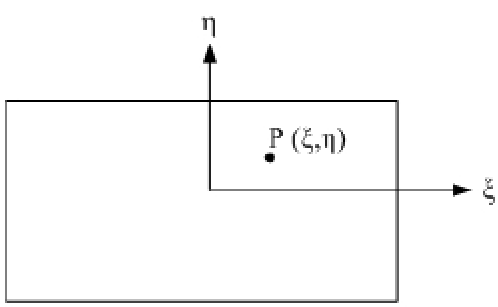
4. (15 pts) Suppose that the assembled stiffness equations for a one-dimensional finite element model before imposing constraints are $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$. This system is to be solved subject to the multipoint constraint $u_1 = u_3$.

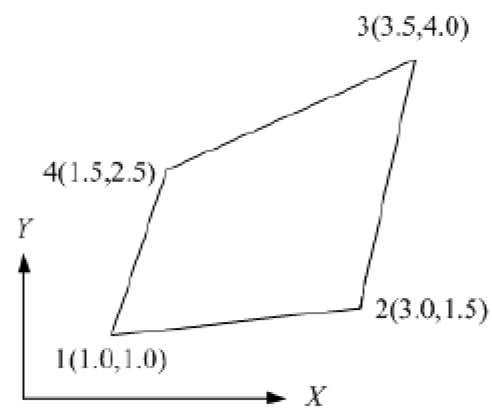
(1) Impose the constraint by the master-slave method taking u_1 as master, and solve the resulting 2x2 system of equations.

(2) Impose the constraint by the penalty function method, leaving the weight w as a free parameter. Write down the equation only and do not solve.

(3) Impose the constraint by the Lagrange multiplier method. Show the 4x4 multiplier-augmented system of equations.

5. (10 pts) Determine the Cartisian coordinate of the point P($\xi = 0.8$, $\eta = 0.9$) as shown in the figure.





6. (30 pts) The plane stress isoparametric quadrilateral shown in Figure is used as transition element in many FEM codes. It has 6 nodes: four at the corners, and two (5 and 6) at the midpoint of the sides 1-2 and 3-4, respectively. The quadrilateral coordinates of node 5 are $\xi = 0,\ \eta = -1$, and that of node 6 are $\xi = 0,\ \eta = 1$.

(1) Construct the shape function $N_1^e(\xi,\eta)$ for corner node 1 using the product of three lines: two opposite sides and one of the 2 medians.

(2) Check whether the N_1^e constructed in item (1) satisfies C^0 continuity along the sides that contain node 1, namely 1–5–2 and 1–4 (those sides are colored red in the figure), in the sense that the polynomial variation along those sides has sufficient number of nodes to define it uniquely.

(3) Suppose you have derived all shape functions of the 6-node element, and that C^0 compatibility is OK. Which operation would be required to check completeness? (state it, but don't do it).

(4) Restricting attention to the shape function derived in item (1) for simplicity, will *interelement compatibility* be satisfied along those two sides? Explain.

(5) Assume that it will be numerically integrated. What is the minimum number of Gauss points n_G needed so that the element is rank sufficient? (This is not necessarily an actual rule.)

(6) Which $p \times p$ (p is a positive integer) two-dimensional Gauss quadrature rule can be chosen to achieve that?