Midterm Exam

10/20/2020

1. Solve the following initial value problem with $h = 0.2$ to obtain $u(1.2)$: (15 pts)

$$\frac{du}{dx} = -0.3u^2 + x^2 u - 0.5, u(1) = 3$$

   (1) Use Euler's explicit method
   (2) Use the midpoint method
   (3) Use the 4th order Runge-Kutta method

   [Hint: $\phi = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\; where\; k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h)$ and $k_4 = f(x_i + h, y_i + k_3 h)$]

2. Write the matrix equation to obtain a numerical solution of the following boundary value problem: (15 pts)

$$\frac{d^2u}{dx^2} - 3\frac{du}{dx} + 2u = 0, u(0) = 0\; and\; u(1) = 1 - e$$

   (1) Discretize the equation with $\Delta x = 0.2$ using 3-point central difference scheme to represent for $\frac{d^2u}{dx^2}$ and 2-point central difference scheme for $\frac{du}{dx}$. [Do NOT solve the equation.]
   (2) Solve the original equation analytically and plot the solution.

3. The following PDE is defined on the semi-infinite domain with $x \in (-\infty, \infty), t \in [0, \infty]$: (15 pts)

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$$

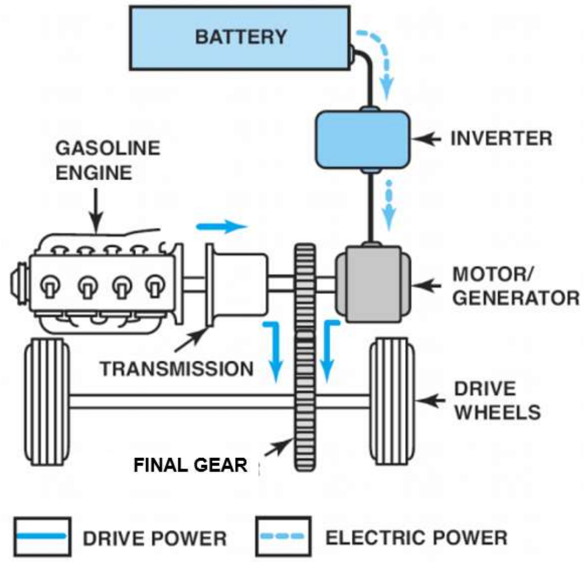
   (1) Discretize the equation by using the two-point forward difference scheme for $\partial u/\partial t$ and the three-point central difference scheme for $\partial^2 u/\partial x^2$. Write the resulted finite difference formula for the PDE into the form $u_{i,j+1} = Pu_{i-1,j} + Qu_{i,j} + Ru_{i+1,j}\; where\; u_{i,j} \equiv u(i\Delta x, j\Delta t)$.
   What are the $P, Q\; and\; R$ as a function of $(\Delta x, \Delta t)$?
   (2) Choosing $\Delta x = 0.5, \Delta t = 0.1$ and the boundary condition (given at $t = 0$), $u_{5,0} = 1\; and\; u_{i,0} = 0\; for\; all\; i \neq 5$, determine the values of all non-zero $u_{i,j}\; at\; j = 1\; and\; 2, i.e., at\; t = 0.1\; and\; 0.2$.

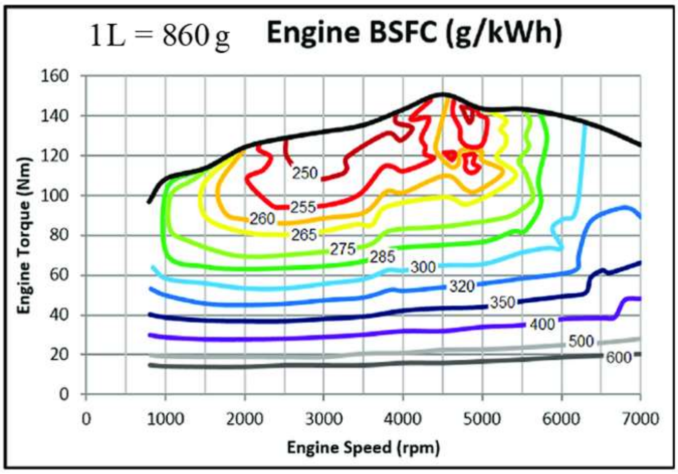
4. Find the general solution of the following PDE using the method of separation of variables: (10 pts)

$$\frac{\partial^2 u}{\partial x \partial y} - 2u = 0$$

5. Answer the following questions. (15 pts)
   (1) List at least three common arithmetic operations that affect round-off errors.
   (2) Compare explicit and implicit schemes.

6. Consider the following parallel hybrid electric vehicle system. Be sure to check the units of parameter values.





inertia/mass: $J_{eng} = 0.2\,\text{kgm}^2$, $J_{motor} = 0.1\,\text{kgm}^2$, $M_{veh} = 1{,}500\,\text{kg}$

engine/motor: $T_{eng} = 50\,\text{Nm}$, $T_{mot} = 80\,\text{Nm}$, $\eta_{mot} = 0.9$

driveline: $Z_{ring(TM)} = 80$, $Z_{sun(TM)} = 40$, $GR_{final} = 4$, $R_{tire} = 0.3\,\text{m}$

battery: $V_{OCV} = 350\ \text{V}$, $R_{in} = 0.1\ \Omega$, $C_{nom} = 50{,}000\ \text{As}$

resistance: $A = 2\,m^2$, $C_d = 0.3$, $\rho = 1.2\,\text{kg/m}^3$, $\mu_{roll} = 0.01$, $g = 9.81\,\text{m/s}^2$

(1) A transmission consists of the planetary gear set. Calculate the total equivalent inertia at wheel. (drive: sun gear, driven: carrier, fixed: ring gear) (5 pts)

(2) When a vehicle speed is 36 km/h, calculate a vehicle acceleration speed. (8 pts)

(3) Assume the vehicle speed and motor torque are constant. After 5 minutes under this condition, calculate final SOC. (initial SOC is 50%.) (7 pts)

(4) After driving as (3), calculate the fuel efficiency (km/L). Here, ignore the motor, and consider only the engine operation. (8 pts)

(5) Convert the fuel efficiency of (4) to MPG unit. (1 mile = 1.609 km, 1 gallon = 3.785 L) (2 pts)