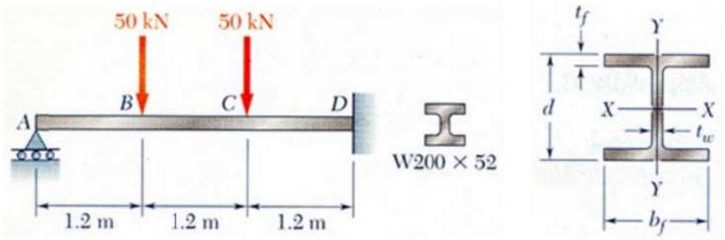
Final Exam - Lab

12/21/2021

Submit the compressed file as `(ID)_(name).zip` to [ftp://cdl.hanyang.ac.kr → CAE/Final_Lab] folder. It should contain the final results of each problem (equations and graphs) using PowerPoint `(ID.ppt)` and COMSOL files (`problem#-#.mph`).

1. [Beam deflection] For the beam and boundary conditions shown, compute the deflection at point B.

   Analytic solution : $d_B = 3.19mm$ (24 pts total)



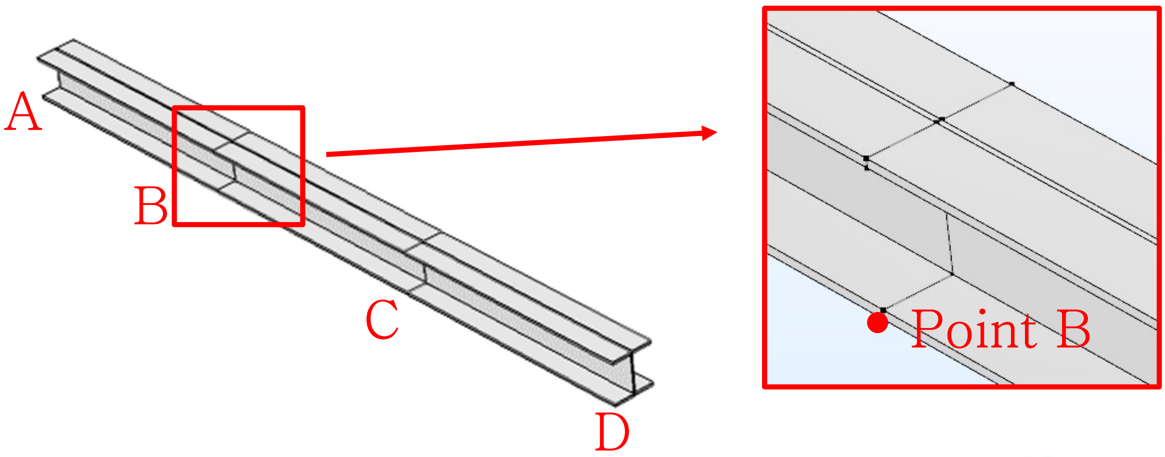
$E = 200\,GPa,\ \nu = 0.33$

$d = 206\,mm,\ b_f = 204\,mm$

$t_f = 12.6\,mm,\ t_w = 7.9\,mm$

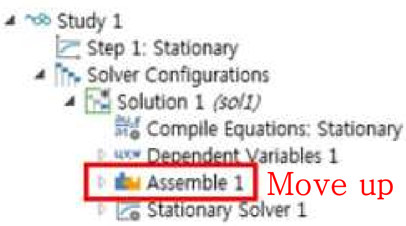
1) Use 2D Beam module (number of elements : 30) (8 pts)

2) Use 3D Solid Mechanics module (mesh option : normal) (8 pts)



3) Use “MATLAB” with the eliminated stiffness matrix (K) and load vector (f) of 2D Beam (beam) results from “assemble” option. (The number of elements is 9) (8 pts)

Hint :



$$u = K^{-1} f$$

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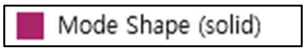
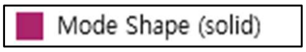
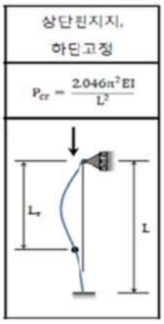
2. [Linear buckling] For the column and boundary condition shown, compute critical load for the column. (16 pts)

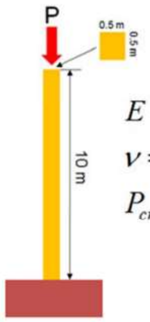
   1) Use 2D Solid Mechanics module and show mode shape using 

   (8 pts)

   2) Use 3D Solid Mechanics module module and show mode shape using 

   (8 pts)



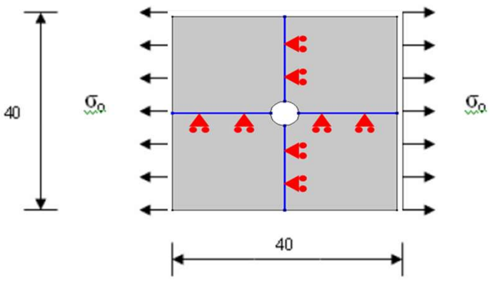


$E = 200\,GPa$

$\nu = 0.33$

$P_{cr} = 2.1e8\,N$

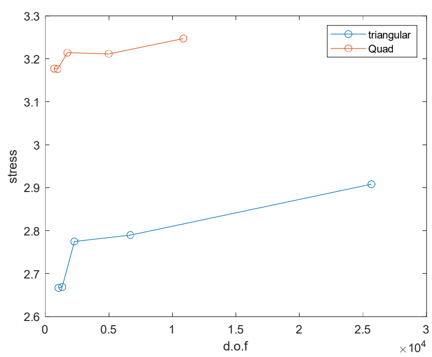
3. [Plane stress] Consider portion of plate within concentric circle so that stress field is not perturbed by hole. (20 pts total)



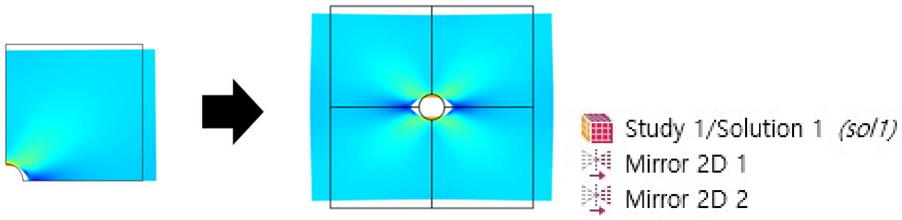
- 2D approximation
  - Plane stress
- Material Properties
  - E = 200 x 10^9
  - ν = 0.3
- Element Properties
  - Thickness = 1
- Loads: σ_0 = 1 Nm

   1) Compute the maximum normal stress (solid.sx) with hole radius = 2.5. Check the stress by mesh dependency applying free triangular and quad elements (linear order). Plot the graph using MATLAB as d.o.f vs stress changing mesh size with two cases.(mesh option : normal ~ extremely fine) (10 pts)

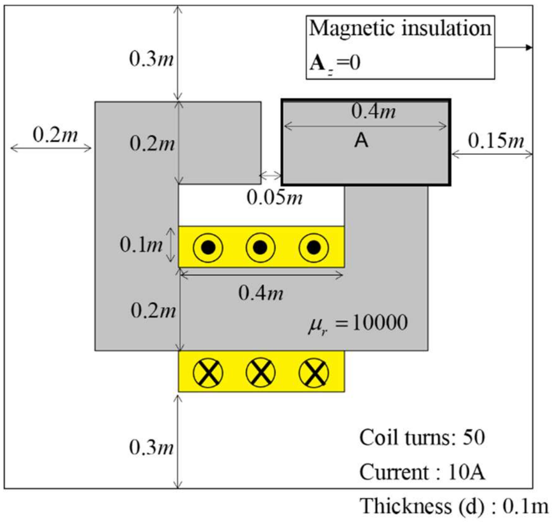
[MATLAB plot]

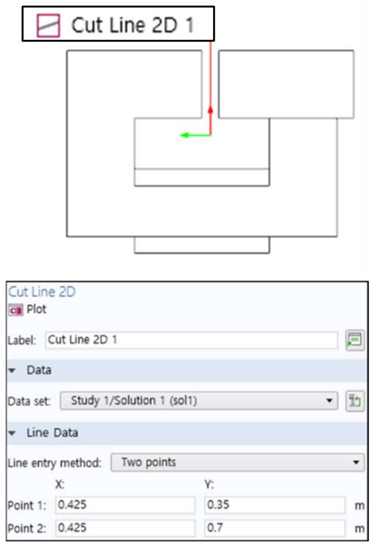


2) Perform the analysis using only the quarter model with the symmetric boundary condition, and show the full model result through mirror 2d post-processing. (10 pts)



4. [Magnetic Actuator] For the actuator and boundary conditions shown, solve the Poisson equation by [Magnetic Fields (mf)]. (Use **Fine** discretization level.) (40 pts total)





1) Draw a Magnetic flux density norm distribution along the [Cut Line 2D 1] by [1D Plot Group / Line Graph] (10 pts total)

2) Evaluate the magnetic force using Maxwell stress tensor method by [Boundary Probe 1 (bnd1) / Boundary Probe 2 (bnd2) / Global Variable Probe 2 (var2)] (15 pts total) Analytic solution : 1.4289 N

Hint:

Magnetic force (Maxwell Stress Tensor Method)

$$f_x = \frac{d}{\mu_0}\left[\int \frac{B_x B_x}{2} dy + \int B_x B_y dx\right]$$

3) Solve the Dirichlet Boundary Condition 1 problem for Poisson equation by Weak Form PDE (w)

Plot a difference error with Magnetic Fields (mf) and Weak Form PDE (w) by using 2D Plot Group / Surface 1

Show that both solutions are equal. (15 pts total)

Hint:

Poission equation

$$\int_{\Omega} \mathbf{A}^* \left\{ \frac{-1}{\mu_0 \mu_r} \left(\nabla^2 \mathbf{A} \right) - \mathbf{J} \right\} dV = 0$$

Integration by parts

$$\int_{\Omega} \nabla \cdot \left\{ \frac{-1}{\mu_0 \mu_r} \mathbf{A}^* \left(\nabla \mathbf{A} \right) \right\} dV + \int_{\Omega} \left\{ \frac{1}{\mu_0 \mu_r} \left(\nabla \mathbf{A}^* \cdot \nabla \mathbf{A} \right) \right\} dV - \int_{\Omega} \mathbf{A}^* \mathbf{J} dV = 0$$

Divergence Theorem

$$\int_{\Gamma} \left\{ \frac{-1}{\mu_0 \mu_r} \mathbf{A}^* \left(\nabla \mathbf{A} \right) \right\} \cdot \mathbf{n} dA + \int_{\Omega} \left\{ \frac{1}{\mu_0 \mu_r} \left(\nabla \mathbf{A}^* \cdot \nabla \mathbf{A} \right) \right\} dV = \int_{\Omega} \mathbf{A}^* \mathbf{J} dV$$

Boundary Condition ($\mathbf{A} = 0$ on Γ)

$$\int_{\Omega} \left\{ \frac{1}{\mu_0 \mu_r} \left(\nabla \mathbf{A}^* \cdot \nabla \mathbf{A} \right) - \mathbf{A}^* \mathbf{J} \right\} dV = 0 \quad (\text{Weak form})$$

Difference error plot expression

$$\text{difference error} = \mathbf{B}_{norm}^{mf} - \mathbf{B}_{norm}^{Weak}$$

$$\mathbf{B}_{norm} = \sqrt{\mathbf{B}_x{}^2 + \mathbf{B}_y{}^2}$$

Expression

Expression:

mf.normB-sqrt(dtang(u,x)^2+dtang(u,y)^2)