Final Exam

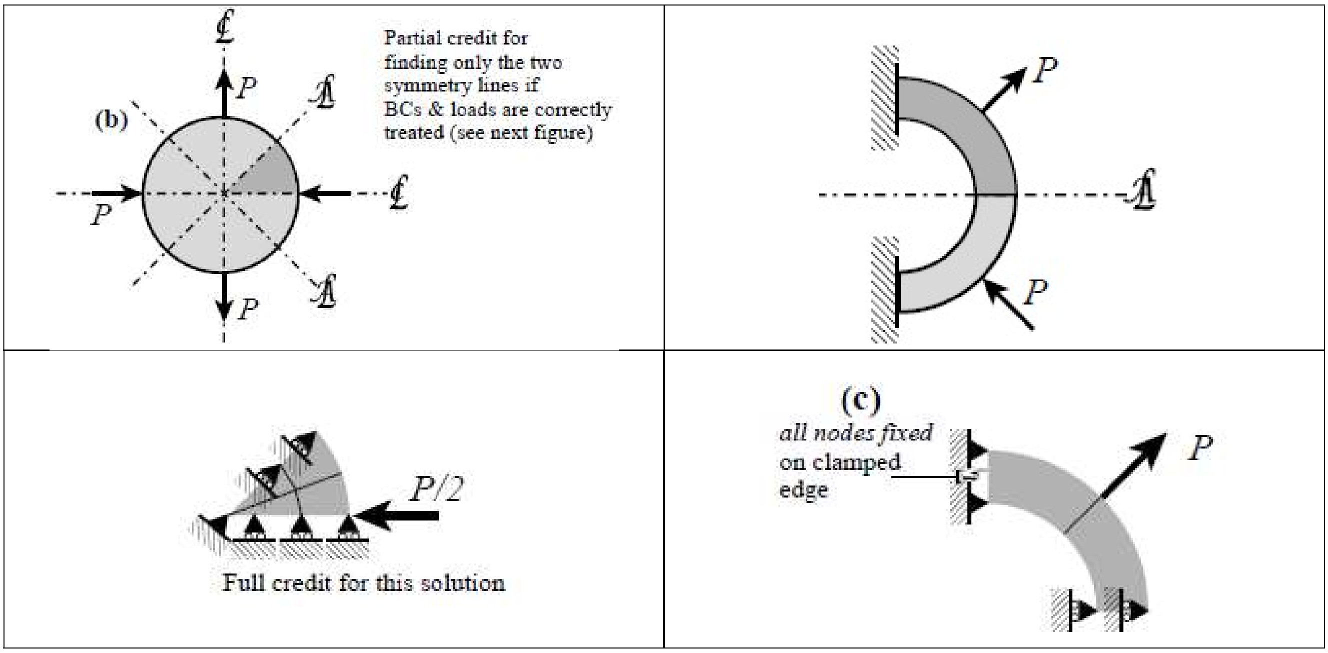
12/21/2021

1. (5 pts each)

(1) ① the determinant of the Jacobian relates the parametric differential volume to the physical differential volume ($d\Omega = |J|\, d\square$), ②The inverse of Jacobian converts the parametric gradient to the physical gradient ($\frac{\partial}{\partial x} = J^{-1}\frac{\partial}{\partial \square}$)

(2) [strong form] governing equation must hold at each point over the span ($r(x)=0$), [weak form] relax that condition so that it is satisfied only in an average sense ($J=\int r(x)v(x)\,dx = 0$)

2. (5 pts each) 요소절점에 경계조건과 하중조건이 작용되어야 함



3.

$$(1)\; K = \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1+k_2+k_3 & -k_3 & -k_2 \\ 0 & -k_3 & k_3 & 0 \\ 0 & k_2 & 0 & k_2 \end{bmatrix} \rightarrow K = 10^3 \begin{bmatrix} 10 & -10 & 0 & 0 \\ -10 & 25 & -10 & -5 \\ 0 & -10 & 10 & 0 \\ 0 & -5 & 0 & 5 \end{bmatrix} \text{ (10 pts)}$$

$$(2)\; K = 10^3 \begin{bmatrix} 10 & -10 & 0 & 0 \\ -10 & 25 & -10 & -5 \\ 0 & -10 & 10 & 0 \\ 0 & -5 & 0 & 5 \end{bmatrix} \rightarrow K_{reduced} = 10^3 \begin{bmatrix} 25 & -5 \\ -5 & 5 \end{bmatrix} \text{ (5 pts)}$$

4. (답이 틀리더라도) 문제를 끝까지 풀어내는 과정 훈련: 최종 변위, 응력값 필요, 고정벽에서부터 아래 방향을 x축으로 설정하여 1차원 문제로 해결 (2D 변환행렬 등 불필요)

$$(1)\ \begin{cases} k_1 = \dfrac{E_1A_1}{L_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \dfrac{(0.001)(70\times10^9)}{1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 10^6\begin{bmatrix} 70 & -70 \\ -70 & 70 \end{bmatrix} \\ k_2 = \dfrac{E_2A_2}{L_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \dfrac{(0.004)(70\times10^9)}{1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 10^6\begin{bmatrix} 280 & -280 \\ -280 & 280 \end{bmatrix} \end{cases} \quad (4\text{ pts})$$

$$(2)\ K = 10^6\begin{bmatrix} 70 & -70 & 0 \\ -70 & 70+280 & -280 \\ 0 & -280 & 280 \end{bmatrix} = 10^6\begin{bmatrix} 70 & -70 & 0 \\ -70 & 350 & -280 \\ 0 & -280 & 280 \end{bmatrix} \quad (4\text{ pts})$$

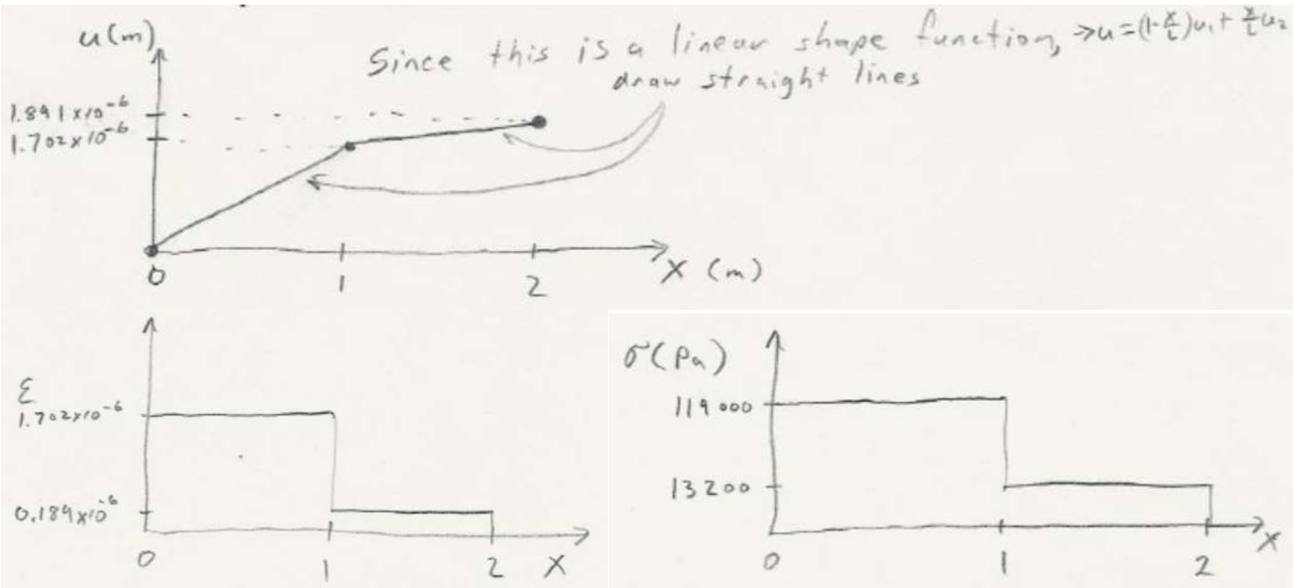
$$(3)\ \begin{cases} q_1 = \rho_1A_1g = (2700)(0.001)(9.81) = 26.49\ N/m \\ q_2 = \rho_2A_2g = (2700)(0.004)(9.81) = 105.9\ N/m \end{cases} \rightarrow F = \begin{Bmatrix} \dfrac{q_1L_1}{2} \\ \dfrac{q_1L_1}{2}+\dfrac{q_2L_2}{2} \\ \dfrac{q_2L_2}{2} \end{Bmatrix} = \begin{Bmatrix} 13.25 \\ 66.20 \\ 52.95 \end{Bmatrix} N \quad (5\text{ pts})$$

$$(4)\ 10^6\begin{bmatrix} 70 & -70 & 0 \\ -70 & 350 & -280 \\ 0 & -280 & 280 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 13.25+R_1 \\ 66.20 \\ 52.95 \end{Bmatrix} \rightarrow 10^6\begin{bmatrix} 350 & -280 \\ -280 & 280 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 66.20 \\ 52.95 \end{Bmatrix}$$

$$\rightarrow \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = 10^{-6}\begin{Bmatrix} 1.702 \\ 1.891 \end{Bmatrix} m \quad (4\text{ pts})$$

$$(5)\ \text{first row: } 10^6\left[70(0) - 70(1.702\times10^{-6}) + 0(1.891\times10^{-6})\right] = 13.25 + R_1 \rightarrow R_1 = -132.4N \quad (5\text{ pts})$$

$$(6)\ \begin{cases} \varepsilon_1 = \dfrac{u_2-u_1}{L_1} = 1.702\times10^{-6} \\ \varepsilon_2 = \dfrac{u_3-u_2}{L_2} = 0.189\times10^{-6} \end{cases} \rightarrow \begin{cases} \sigma_1 = E_1\varepsilon_1 = (70\times10^9)(1.702\times10^{-6}) = 119{,}000Pa \\ \sigma_2 = E_2\varepsilon_2 = (70\times10^9)(0.189\times10^{-6}) = 132{,}000Pa \end{cases} \quad (4\text{ pts}) \qquad (7)$$

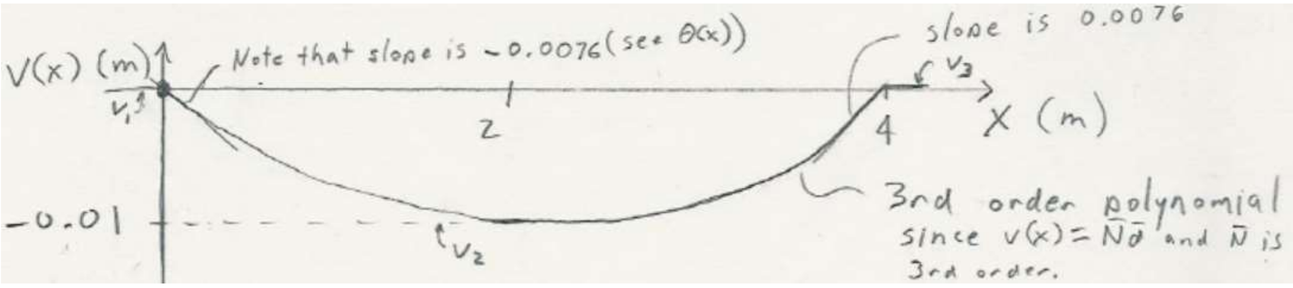


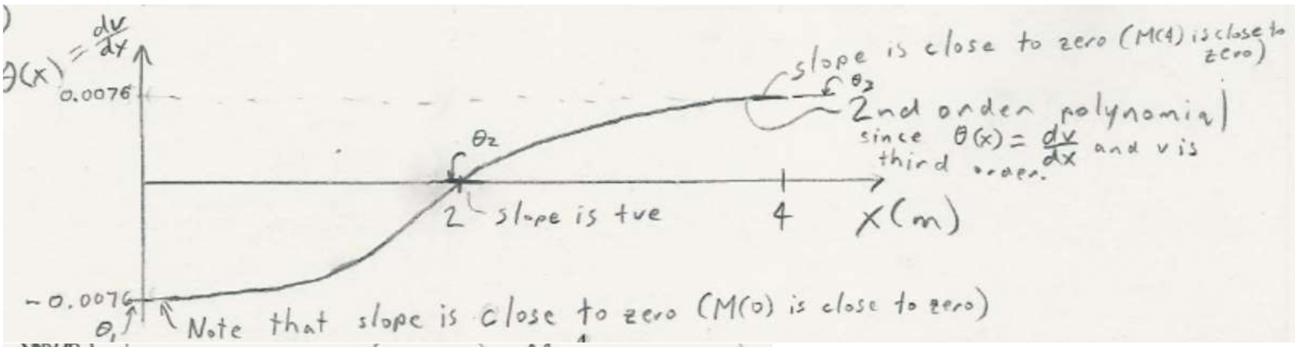
5. 고체역학에서 학습한 beam(보)에 대한 이해 확인

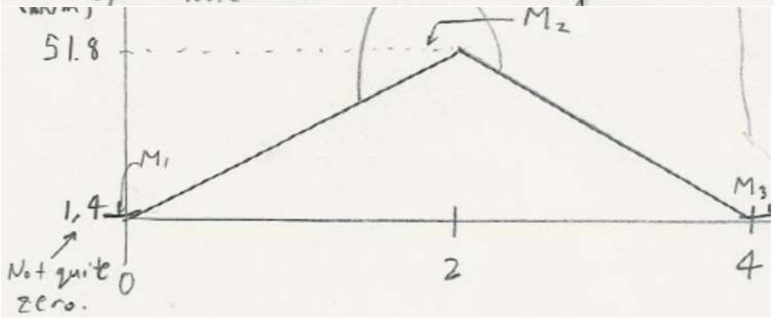
(1) $q=-100\,N/m,\ l=2m$

$$\mathbf{f}^e=\frac{1}{2}ql\begin{Bmatrix}1\\ \frac{1}{6}l\\ 1\\ -\frac{1}{6}l\end{Bmatrix}=\begin{Bmatrix}-100N\\ -33.3Nm\\ -100N\\ 33.3Nm\end{Bmatrix}(4\text{ pts})\rightarrow F=\begin{Bmatrix}-100\\ -33.3\\ -200\\ 0\\ -100\\ 33.3\end{Bmatrix}(4\text{ pts})$$

(2) (4 pts each)
$$\begin{cases} v(x)=Nd\rightarrow \text{3rd order}\\ \theta(x)=\dfrac{dv}{dx}=\dfrac{dN}{dx}d\rightarrow \text{2nd order}\\ M(x)=EI\dfrac{d^2v}{dx^2}=EIBd\rightarrow \text{1st order}\end{cases}$$







6. (5 pts each)

(1) $N_1 = c_1(\eta-1)(\xi-1)\xi \rightarrow N_1(-1,-1) = -4c_1 = 1 \rightarrow c_1 = -\frac{1}{4} \rightarrow N_1 = -\frac{1}{4}(\eta-1)(\xi-1)\xi$ (4 pts)

(2) compatibility $\begin{cases} \text{edge1-4: } N_1(\xi=-1) = -\frac{1}{2}(\eta-1) \rightarrow \text{linear} \xleftrightarrow{OK} \text{two points} \\ \text{edge1-5-2: } N_1(\eta=-1) = -\frac{1}{2}(\xi-1)\xi \rightarrow \text{quadratic} \xleftrightarrow{OK} \text{three points} \end{cases}$ (4 pts)

(3) completeness: $\sum_{i=1}^{6} N_i = 1$ (3 pts)

(4) $r = \min(n_F - n_R, 3n_G) = \min(12-3, 3n_G) \rightarrow n_G \geq 3$ (4 pts)