Final Exam

12/21/2021

1. (10 pts) Answer the following:

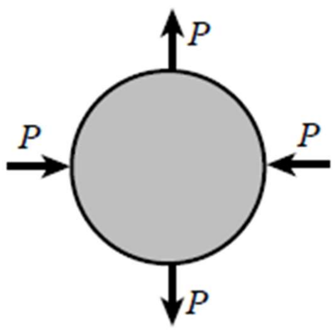
(1) Primary uses of the geometric Jacobian in a typical element

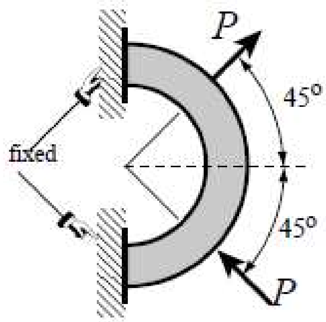
(2) Difference between the strong form and the weak form

2. (10 pts) Consider the two-dimensional problems illustrated in the figures.

(1) Identify all the symmetry and anti-symmetry lines.

(2) Draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or anti-symmetry lines.

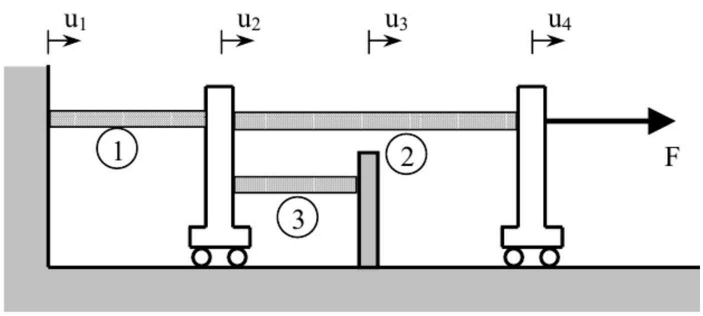




3. (15 pts) For the one-dimensional problem shown, calculate:

(1) The global stiffness matrix before the application of boundary conditions.

(2) The reduced stiffness matrix after the application of boundary conditions.



$$\begin{cases} k_1 = 10,000\, N/mm \\ k_2 = 5,000\, N/mm \\ k_3 = 10,000\, N/mm \\ F = 500\, N \end{cases}$$

4. (30 pts) For the figure on the right:

(1) Solve for the two elemental stiffness matrices.

(2) Assemble the global stiffness matrix.

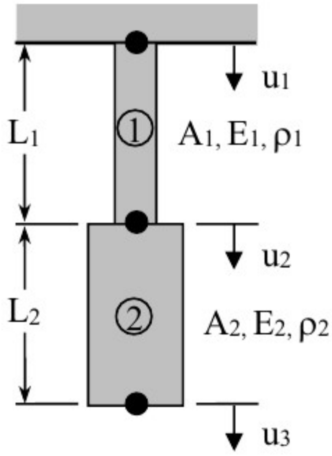
(3) Compute the global applied force vector considering only the gravitational force acting on the rod elements.

(4) After applying the appropriate restraint condition(s), solve for the nodal displacements.

(5) Solve for the reaction forces(s) at the restraint(s).

(6) Solve for the element strains and stresses.

(7) Plot the displacement, strain and stress of both elements as a function of the distance from the top.

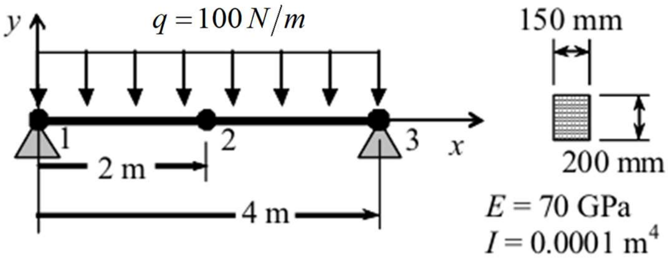


$$\begin{bmatrix} L_1 = L_2 = 1m,\ E_1 = E_2 = 70\times10^9\,Pa,\ \rho_1 = \rho_2 = 270\,kg/m^3 \\ A_1 = 0.001m^2,\ A_2 = 0.004m^2 \end{bmatrix}$$

5. (20 pts) For the beam element shown (with shape functions given below), the nodal displacements have been calculated in meters and radians as:

(1) Give equivalent nodal forces and moments that represent the distributed force on the beam.

(2) Plot $v(x)$, $\theta(x)$ and $M(x)$ for the entire beam (both elements). In the plots, display actual values at node positions and indicate the order of the polynomial for each plot.



$$d = \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -0.0076 \\ -0.010 \\ 0 \\ 0 \\ 0.0076 \end{Bmatrix},\quad N^T = \begin{bmatrix} 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} \\ x - \frac{2x^2}{L} + \frac{x^3}{L^2} \\ \frac{3x^2}{L^2} - \frac{2x^3}{L^3} \\ -\frac{x^2}{L} + \frac{x^3}{L^2} \end{bmatrix}$$

6. (15 pts) The plane stress isoparametric quadrilateral shown in Figure is used as transition element in many FEM codes. It has 6 nodes: four at the corners, and two (5 and 6) at the midpoint of the sides 1-2 and 3-4, respectively. The quadrilateral coordinates of node 5 are $\xi = 0,\ \eta = -1$, and that of node 6 are $\xi = 0,\ \eta = 1$.

(1) Construct the shape function $N_1^e(\xi,\eta)$ for corner node 1 using the product of three lines: two opposite sides and one of the 2 medians.

(2) Check whether the N_1^e constructed in item (1) satisfies C^0 continuity along the sides that contain node 1, namely 1–5–2 and 1–4 (those sides are colored red in the figure), in the sense that the polynomial variation along those sides has sufficient number of nodes to define it uniquely.

(3) Suppose you have derived all shape functions of the 6-node element, and that C^0 compatibility is OK. Which operation would be required to check completeness? (state it, but don't do it).

(4) Assume that it will be numerically integrated. What is the minimum number of Gauss points n_G needed so that the element is rank sufficient? (This is not necessarily an actual rule.)