Midterm Exam

11/02/2021

1.

$$\frac{du}{dx}=f(x,u)=-u^2+\sin x,\ u_0=0\rightarrow u(0.2)=?$$

(계산값(특히 radian, 설명없이 sin(0.1)≈0.1) 틀린 경우 -1)

(1) Euler's explicit method: $\Delta x=0.1\rightarrow u(0.2)=u_2$ (5 pts)

$$u_1=u_0+f(x_0,u_0)\Delta x=0+(-0^2+\sin 0)(0.1)=0$$

$$u_2=u_1+f(x_1,u_1)\Delta x=0+(-0^2+\sin(0.1))(0.1)=0.00998334$$

(2) 4-th order Runge-Kutta: $\Delta x=0.2\rightarrow u(0.2)=u_1$ (10 pts)

$$\begin{cases} k_1=f(x_0,u_0)=0 \\ k_2=f\left(x_0+\frac{1}{2}\Delta x,u_0+\frac{1}{2}k_1\Delta x\right)=f(0.1,0)=\sin(0.1)=0.0998334 \\ k_3=f\left(x_0+\frac{1}{2}\Delta x,u_0+\frac{1}{2}k_2\Delta x\right)=f(0.1,(0.1)\sin(0.1))=-[(0.1)\sin(0.1)]^2+\sin(0.1)=0.0997337 \\ k_4=f(x_0+\Delta x,u_0+k_3\Delta x)=f(0.2,0.0997337(0.2))=0.1982714 \end{cases}$$

$$u_1=u_0+\frac{1}{6}(k_1+2k_2+2k_3+k_4)\Delta x=0.0199135$$

2.

$$(1)\ \left.\begin{matrix} \frac{d^2u}{dx^2}\approx\frac{u_{i+1}-2u_i+u_{i-1}}{(\Delta x)^2}\text{(2 pts)} \\ \frac{du}{dx}\approx\frac{u_{i+1}-u_{i-1}}{2(\Delta x)}\text{(2 pts)} \end{matrix}\right\}\rightarrow\begin{cases} \frac{u_{i+1}-2u_i+u_{i-1}}{(\Delta x)^2}+5\frac{u_{i+1}-u_{i-1}}{2(\Delta x)}+6u_i=0 \\ \left[\frac{1}{(\Delta x)^2}-\frac{5}{2(\Delta x)}\right]u_{i-1}+\left[6-\frac{2}{(\Delta x)^2}\right]u_i+\left[\frac{1}{(\Delta x)^2}+\frac{5}{2(\Delta x)}\right]u_{i+1}=0 \end{cases}$$

$$\Delta x=0.25,\ u_0=2,\ \frac{u_1-u_{-1}}{2(\Delta x)}=-5\rightarrow u_{-1}=u_1+2.5\text{(2 pts)}$$

$$\left.\begin{matrix} i=0:\ 6u_{-1}-26u_0+26u_1=0 \\ i=1:\ 6u_0-26u_1+26u_2=0 \\ i=2:\ 6u_1-26u_2+26u_3=0 \\ i=3:\ 6u_2-26u_3+26u_4=0 \end{matrix}\right\}\rightarrow\begin{bmatrix} 32 & 0 & 0 & 0 \\ -26 & 26 & 0 & 0 \\ 6 & -26 & 26 & 0 \\ 0 & 6 & 26 & 26 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}=\begin{Bmatrix} 37 \\ -12 \\ 0 \\ 0 \end{Bmatrix}\text{(4 pts)}$$

$$(2)\ \left.\begin{matrix} u=c_1e^{-2x}+c_2e^{-3x} \\ u'=-2c_1e^{-2x}-3c_2e^{-3x} \end{matrix}\right\}\rightarrow\left\{\begin{matrix} u(0)=c_1+c_2=2 \\ u'(0)=-2c_1-3c_2=-5 \end{matrix}\right\}\rightarrow\left\{\begin{matrix} c_1=1 \\ c_2=1 \end{matrix}\right\}\rightarrow u=e^{-2x}+e^{-3x}\text{(3 pts)}$$

graph: $u(0)=2\xrightarrow{\text{monotonically decreasing}}u(1)=e^{-2}+e^{-3}$ (2 pts) (그래프에 값표시 없으면 -1)

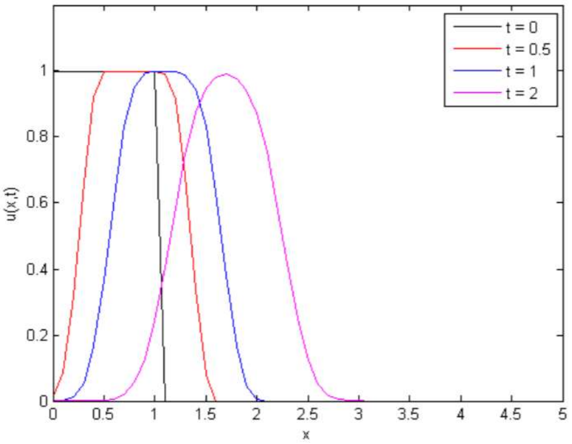
3.

$$(1)\ \left.\begin{aligned}\frac{\partial u}{\partial t}\approx\frac{u_i^{l+1}-u_i^l}{\Delta t}\\ \frac{\partial u}{\partial x}\approx\frac{u_i^l-u_{i-1}^l}{\Delta x}\end{aligned}\right\}\rightarrow\begin{cases}\dfrac{\partial u}{\partial t}=A\dfrac{\partial u}{\partial x}\rightarrow\dfrac{u_i^{l+1}-u_i^l}{\Delta t}=A\dfrac{u_i^l-u_{i-1}^l}{\Delta x}\\ u_i^{l+1}=\left(1+A\dfrac{\Delta t}{\Delta x}\right)u_i^l-A\dfrac{\Delta t}{\Delta x}u_{i-1}^l\end{cases}$$

(8 pts)

$$(2)\ A=-0.6,\ \Delta x=\Delta t=0.1,\ u_i^0=\begin{cases}1\left(i=0,\ldots,10\right)\\ 0\text{ otherwise}\end{cases}\rightarrow u_i^{l+1}=0.4u_i^l+0.6u_{i-1}^l$$

$$l=0:\ u_i^1=0.4u_i^0+0.6u_{i-1}^0$$



(7 pts, 공학적 센스 확인)

4. (10 pts)

$$u\left(x,y\right)=G\left(x\right)H\left(y\right)\rightarrow y\frac{\partial u}{\partial x}+x\frac{\partial u}{\partial y}=0\rightarrow yH\frac{dG}{dx}+xG\frac{dH}{dy}\rightarrow\frac{1}{xG}\frac{dG}{dx}=-\frac{1}{yH}\frac{dH}{dy}=c$$

$$\begin{cases}\dfrac{1}{xG}\dfrac{dG}{dx}=c\rightarrow\dfrac{1}{G}\dfrac{dG}{dx}=cx\rightarrow\ln G=\dfrac{1}{2}cx^2+k_1^*\rightarrow G=k_1e^{\frac{1}{2}cx^2}\\ \dfrac{1}{yH}\dfrac{dH}{dy}=-c\rightarrow\dfrac{1}{H}\dfrac{dH}{dy}=-cy\rightarrow\ln H=\dfrac{1}{2}cy^2+k_2^*\rightarrow H=k_2e^{-\frac{1}{2}cy^2}\end{cases}$$

$$u\left(x,y\right)=G\left(x\right)H\left(y\right)=ke^{\frac{1}{2}c\left(x^2-y^2\right)}$$

5.

(1) (5 pts each)

Round-ff error: ①Common arithmetic operations (Addition, subtraction, multiplication, division), ②Large computations, ③Adding a large and a small number, ④Subtractive cancellation, ⑤Smearing, ⑥Inner products

Truncation error: Taylor series approximation

(2) (5 pts each)

$$\frac{dy}{dt} = -ay \rightarrow \begin{cases} \text{explicit (forward): } \dfrac{y_{i+1} - y_i}{\Delta t} = -ay_i \rightarrow y_{i+1} = (1 - a\Delta t) y_i \rightarrow \text{stability: } |1 - a\Delta t| \le 1 \\ \text{implicit (backward): } \dfrac{y_{i+1} - y_i}{\Delta t} = -ay_{i+1} \rightarrow y_{i+1} = \dfrac{1}{1 + a\Delta t} y_i \rightarrow \text{unconditionally stable} \end{cases}$$

6.

(1) $\omega_s = 0 \rightarrow \dfrac{\omega_c}{\omega_r} = \dfrac{Z_r}{Z_s + Z_r} = GR_{PSD} = \dfrac{2}{3}$ (2pts), $J_{vehicle} = mR_{tire}^2 = 1500 \times 0.3^2 = 135\ \text{kg}\cdot\text{m}^2$ (2pts)

$$J_{eq} = J_{motor} \times (GR_{PSD} \times GR_{TM} \times GR_{diff})^2 + J_{wheel} + J_{vehicle} = 0.05 \times \left(\frac{2}{3} \times 3 \times 4\right)^2 + 1 + 135 = 139.2\,\text{kg}\cdot\text{m}^2$$ (2pts)

(2) $T_{wheel} = T_{motor} \times (GR_{PSD} + GR_{TM} + GR_{diff}) = 50 \times \left(\dfrac{2}{3} \times 3 \times 4\right) = 400\ \text{Nm}$ (2pts)

$$F_{drag} = \frac{1}{2} C_d A_{front} \rho_{air} V_{vehicle}^2 + \mu_{roll} M_{vehicle} g = \frac{1}{2} \times 0.25 \times 1.8 \times 1.2 \times 20^2 + 0.01 \times 1500 \times 9.81 = 255.15\ \text{N}$$ (2pts)

$$T_{drag} = F_{drag} R_{tire} = 255.15 \times 0.3 = 76.55\ \text{Nm}$$,

$$a_{vehicle} = \frac{T_{wheel} - T_{drag}}{J_{eq}} R_{tire} = \frac{400 - 76.55}{139.2} \times 0.3 = 0.697\ \text{m/s}^2$$ (4pts)

(3) $\omega_s : (\text{engine rpm})$, $\omega_c : (\text{motor})$, $\omega_s + 2\omega_r - 3\omega_c = 0 \rightarrow \omega_r = \dfrac{3(2000) - 3000}{2} = 1500\ \text{RPM}$ (2pts)

$$\omega_{wheel} = \frac{\omega_{ring}}{GR_{TM} \times GR_{diff}} = \frac{1500}{3 \times 4} = 125\ \text{RPM} = 13.09\,\text{rad/s}$$ (2pts)

$$V_{vehicle} = R_{tire} \omega_{wheel} = 0.3 \times 13.09 = 3.93\ \text{m/s}$$ (1pt)

(4) $$\omega_{motor} = \frac{V_{vehicle}}{R_{tire}} \times GR_{diff} \times GR_{TM} \times GR_{PSD} = 266.67\,\text{rad/s} \quad \text{(1pt)}$$

$$T_{motor} = T_{regen} \times \frac{1}{GR_{diff} \times GR_{TM} \times GR_{PSD}} = -125\,\text{Nm} \quad \text{(1pt)}$$

$$I_{battery} = \frac{T_{regen}\omega_{wheel}}{V_{battery}} = \frac{-1000 \times 10 / 0.3}{250} = -133.33\ \text{A} \quad \text{(1pt)}$$

$$\frac{dSOC}{dt} = -I_{battery}\frac{100}{C_{nom}} = 133.33\frac{100}{50,000} = 0.267\ \text{\%/s} \quad \text{(2pts)}$$

$$SOC_{final} = SOC_{initial} + \frac{dSOC}{dt}\Delta t = 50 + 0.267 \times 20 = 55.34\ \% \quad \text{(1pt)}$$