Midterm Exam

11/02/2021

1. Solve the following initial value problem to obtain $u(0.2)$ by using (15 pts)

$$\frac{du}{dx} = -u^2 + \sin(x), \; u(0) = 0$$

   (1) Euler's explicit method with $\Delta x = 0.1$

   (2) Classical 4th order Runge-Kutta method with $\Delta x = 0.2$

   [Hint: $\phi = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\; where\; k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h)$ and $k_4 = f(x_i + h, y_i + k_3 h)$]

2. Write the matrix equation to obtain a numerical solution of the following boundary value problem: (15 pts)

$$\frac{d^2u}{dx^2} + 5\frac{du}{dx} + 6u = 0, \;\; u(0) = 2 \text{ and } u'(0) = -5 \;\; (u' = du/dx)$$

   (1) Discretize the equation with $\Delta x = 0.25$ $(0 \le x \le 1)$ using 3-point central difference scheme to represent for $\frac{d^2u}{dx^2}$ and 2-point central difference scheme for $\frac{du}{dx}$. [Do NOT solve the equation.]

   (2) Solve the original equation analytically and plot the solution roughly.

3. The following PDE is defined on the semi-infinite domain with $x \in (-\infty, \infty), t \in [0, \infty]$: (15 pts)

$$\frac{\partial u}{\partial t} = A\frac{\partial u}{\partial x}, \;\; u(x,0) = \begin{cases} 1 & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   (1) Discretize the equation by using the two-point forward difference scheme for $\partial u/\partial t$ and the two-point backward difference scheme for $\partial u/\partial x$ and write the resulted finite difference formula.

   (2) Plot the solution at $t = 0.5, 1.0$ and 2.0 along with the "initial" state ($u\,(x, 0)$) for the domain of $0 \le x \le 5$ when $A = -0.6$, $\Delta x = 0.1$, $\Delta t = 0.1$. [Do NOT integrate. Use your intuition.]

4. Find the general solution of the following PDE using the method of separation of variables: (10 pts)

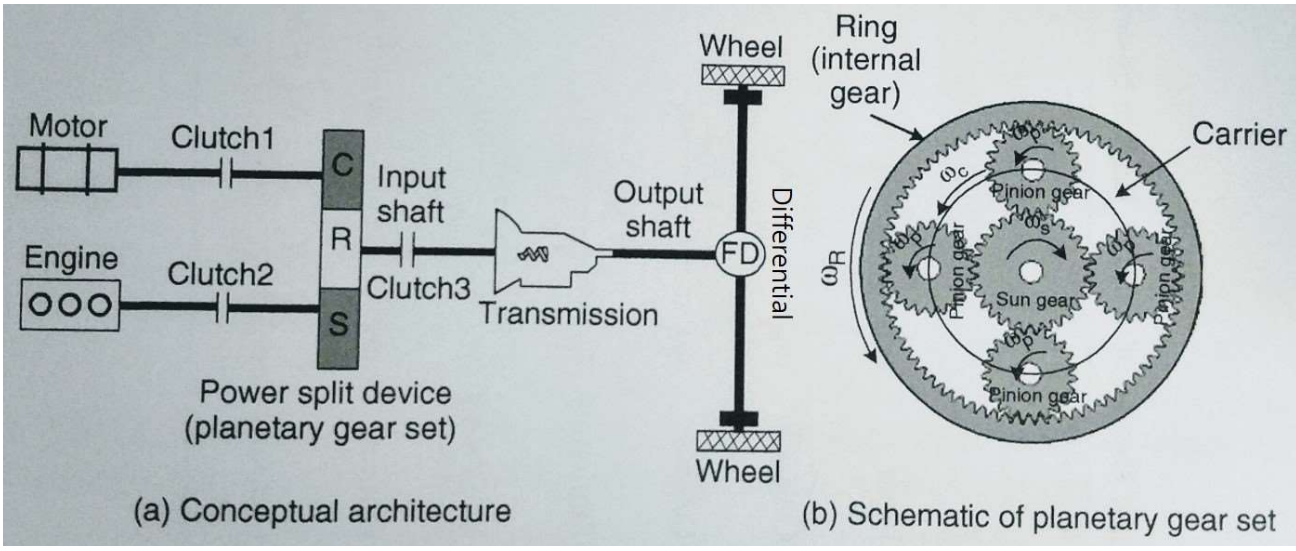
$$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = 0$$

5. Describe the following questions. (20 pts)

(1) difference between round-off errors and truncation errors including examples.

(2) stability between explicit and implicit schemes of $\frac{\partial y}{\partial t} = -ay$ and $y(0) = y_0$.

6. Consider the following parallel hybrid electric vehicle system. Be sure to check the units of parameter values.



Inertia/mass : $J_{engine} = 0.2\ \text{kg}\cdot\text{m}^2$, $J_{motor} = 0.05\,\text{kg}\cdot\text{m}^2$, $J_{wheel} = 1\ \text{kg}\cdot\text{m}^2$(sum of wheels), $M_{vehicle} = 1{,}500\ \text{kg}$

Powertrain : $Z_{sun} = 30$, $Z_{ring} = 60$ (power-split device), $GR_{transmission} = 3$, $GR_{differential} = 4$, $R_{tire} = 0.3\ \text{m}$

Resistance: $C_d = 0.25$, $A_{front} = 1.8\,\text{m}^2$, $\rho_{air} = 1.2\,\text{kg/m}^3$, $\mu_{roll} = 0.01$, $g = 9.81\ \text{m/s}^2$

Battery: $C_{nom} = 50{,}000\ \text{As}$, $V_{battery} = 250\ \text{V}$

motion equation of planetary gear: $\omega_s + \frac{Z_r}{Z_s}\omega_r - \frac{Z_s + Z_r}{Z_s}\omega_c = 0$

(1) On EV mode, calculate the total equivalent inertia at wheel. (Hint: define the state of power-split device. Clutch 1 and 3 are engaged and a sun gear is fixed.) (6 pts)

(2) When the traction motor torque is 50 Nm and a vehicle speed is 72 km/h, calculate a vehicle acceleration speed. (8 pts)

(3) On HEV mode (All clutches are engaged.), when the engine and motor RPM are 3000 and 2000, respectively, calculate a vehicle speed. (Hint: calculate an input shaft speed.) (5 pts)

(4) This vehicle is currently driving a downhill road (EV mode). A driver is working the braking pedal to maintain the vehicle speed of 36 km/h, and the motor is charging a battery. When the sum of regenerative braking torque at wheels is 1000 Nm (negative value), calculate the motor torque and speed. How much is a SOC after driving of 20 s. (without loss, current SOC = 50 %) (6 pts)