Final Exam

12/19/2023

1. (30 pts) Answer the following:

(1) What is the difference between rod(bar, truss) and beam elements?

(2) If an element has a quadratic shape function in x, will the strain also be a quadratic function of x? (If not, why?)

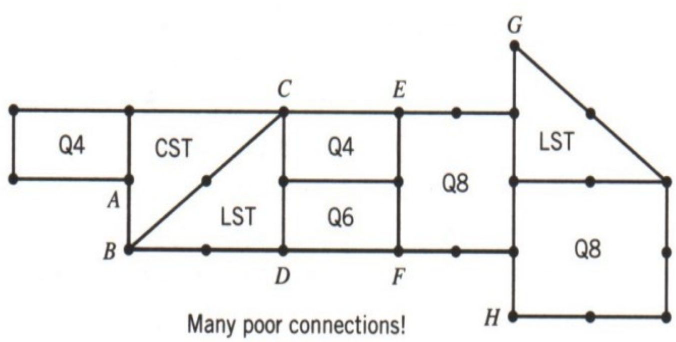
(3) What is the advantage of using isoparametric elements?

(4) List two causes of singular stiffness matrices.

(5) Sketch the shape function for the edge node of a quadratic quadrilateral.

(6) Explain rank-sufficient Gauss rules for 3-node triangular and 4-node quadrilateral elements.

2. (15 pts) Identify four problems with the mesh shown.



Name	Element	Number of nodes
CST	Constant Strain Triangle	3
LST	Linear Strain Triangle	6
Q4	Quadrilateral	4
Q6	bilinear quadratic Quadrilateral	4
Q8	quadratic Quadrilateral	8

3. (15 pts) Suppose that the assembled stiffness equations for a one-dimensional finite element model before imposing constraints are $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$. This system is to be solved subject to the multipoint constraint $u_1 = u_3$.

(1) Impose the constraint by the master-slave method taking u_1 as master, and solve the resulting 2x2 system of equations.

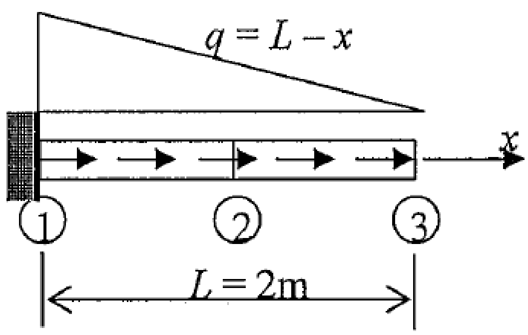
(2) Impose the constraint by the penalty function method, leaving the weight w as a free parameter. Write down the equation only and do not solve.

(3) Impose the constraint by the Lagrange multiplier method. Show the 4x4 multiplier-augmented system of equations.

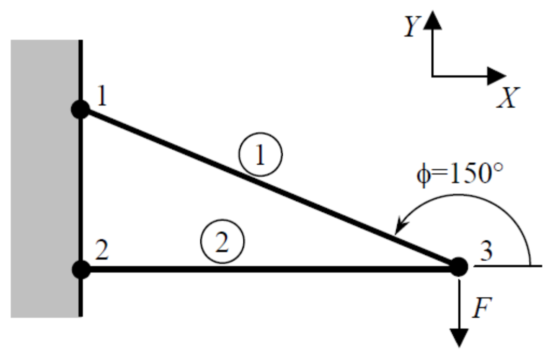
4. (10 pts) A bar of $L = 2m$ carries linearly varying axial loads, as shown in the figure. Consider two equal-length bar elements, using axial displacements as nodal DOFs. Calculate work-equivalent nodal loads $\{\mathbf{F}\} = \{F_1, F_2, F_3\}$.

5. (10 pts) Given the truss structure shown, calculate the stress and strain in truss element ① if:

$$A_1 = 0.0004m^2,\ E_1 = 200\times10^9\,Pa,\ L_1 = 2m,\ \{u_3, v_3\} = \{-1, -2\}\times10^{-3}$$

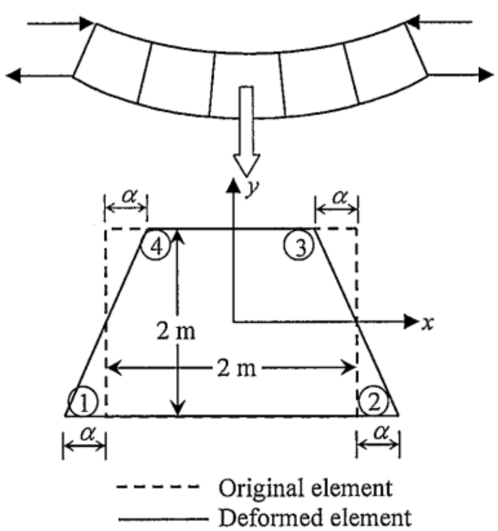


(Problem 4)



(Problem 5)

6. (20 pts) A beam under pure bending is modeled using Q6 elements. For a Q6 element shown, nodal displacements are given as $v_1 = v_2 = v_3 = v_4 = 0,\ u_1 = u_3 = -\alpha,\ u_2 = u_4 = \alpha$. Determine internal DOFs a_1, a_2, a_3 and a_4 such that the element satisfies the pure bending conditions: i.e., ε_{xx} is a function of y only, $\varepsilon_{yy} = \gamma_{xy} = 0$.



$$\begin{cases} N_1 = \frac{1}{4}(1-x)(1-y),\ N_2 = \frac{1}{4}(1+x)(1-y) \\ N_3 = \frac{1}{4}(1+x)(1+y),\ N_4 = \frac{1}{4}(1-x)(1+y) \\ N_5 = 1-x^2,\ N_6 = 1-y^2 \end{cases}$$

$$u = \sum_{i=1}^{4} N_i u_i + N_5 a_1 + N_6 a_2$$

$$v = \sum_{i=1}^{4} N_i v_i + N_5 a_3 + N_6 a_4$$