Midterm Exam

10/24/2023

1. Solve the following set of differential equations assuming that at $x=0,\ y_1=4$ and $y_2=6$. Integrate to $x=0.5$ with a step size of 0.5 (20 pts)

$$\frac{dy_1}{dx}=-0.5y_1 \qquad \frac{dy_2}{dx}=4-0.3y_2-0.1y_1$$

   (1) Euler's explicit method

   (2) Classical 4th order Runge-Kutta method

   [Hint: $\phi=\frac{1}{6}(k_1+2k_2+2k_3+k_4)\ where\ k_3=f(x_i+\frac{1}{2}h, y_i+\frac{1}{2}k_2h)$ and $k_4=f(x_i+h, y_i+k_3h)$]

2. Solve the following boundary value problem: (20 pts)

$$\frac{d^2u}{dx^2}=1,\ \ u(0)=0 \text{ and } u(1)=0$$

   (1) Obtain the analytic solution.

   (2) Write the matrix equation to obtain a numerical solution. Discretize the equation with $\Delta x=0.2$ $(0\le x\le 1)$ using 3-point central difference scheme to represent for $\frac{d^2u}{dx^2}$. [Do NOT solve the equation.]

   (3) Solve the matrix equation by hand and compare with the analytic solution using graph.

   (4) Write the matrix equation to obtain a numerical solution for $u'(0)=0.5$ and $u(1)=2$ using the forward difference scheme for u'. [Do NOT solve the equation.]

3. Consider Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ defined on $x\in[0,1]$ and $y\in[0,1]$ with the boundary conditions $u(x,0)=1,\ u(x,1)=2,\ u(0,y)=1$ and $u(1,y)=2$. Derive the matrix equation to solve the unknown u at interior points for $\Delta x=\Delta y=1/3$. [Do NOT solve the equation.] (20 pts)

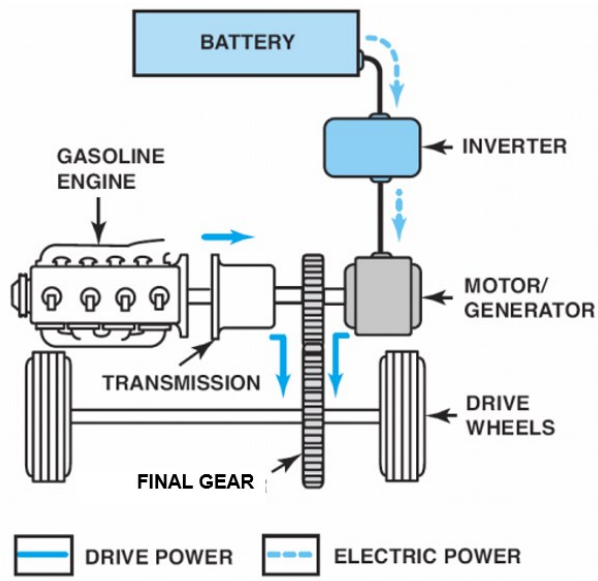
4. For $u(x,t)$ defined on $x\in[0,1]$ and $t\in[0,\infty)$, solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(0,t)=0,\ u(1,t)=0$ and $u(x,0)=\sin(4\pi x)$. (20 pts)

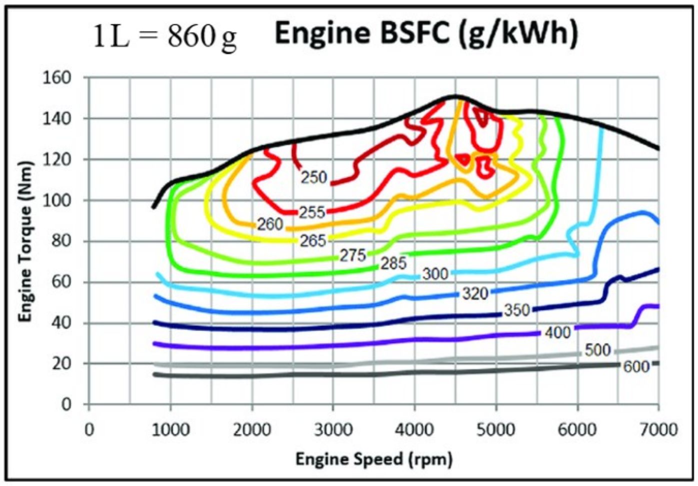
5. Describe the following questions. (10 pts)

(1) difference between round-off errors and truncation errors including examples.

(2) stability between explicit and implicit schemes of $\frac{\partial y}{\partial t} = -ay$ and $y(0) = y_0$.

6. Consider the following parallel hybrid electric vehicle system. Be sure to check the units of parameter values.





inertia/mass: $J_{eng} = 0.2\,\text{kgm}^2$, $J_{motor} = 0.1\,\text{kgm}^2$, $M_{veh} = 1{,}500\,\text{kg}$

engine/motor: $T_{eng} = 50\,\text{Nm}$, $T_{mot} = 80\,\text{Nm}$, $\eta_{mot} = 0.9$

driveline: $Z_{ring(TM)} = 80$, $Z_{sun(TM)} = 40$, $GR_{final} = 4$, $R_{tire} = 0.3\,\text{m}$

battery: $V_{OCV} = 350\ \text{V}$, $R_{in} = 0.1\ \Omega$, $C_{nom} = 50{,}000\ \text{As}$

resistance: $A = 2\,m^2$, $C_d = 0.3$, $\rho = 1.2\,\text{kg/m}^3$, $\mu_{roll} = 0.01$, $g = 9.81\,\text{m/s}^2$

(1) A transmission consists of the planetary gear set. Calculate the total equivalent inertia at wheel. (drive: sun gear, driven: carrier, fixed: ring gear) (5 pts)

(2) When a vehicle speed is 36 km/h, calculate a vehicle acceleration speed. (8 pts)

(3) Assume the vehicle speed and motor torque are constant. After 5 minutes under this condition, calculate final SOC. (initial SOC is 50%.) (7 pts)

(4) After driving as (3), calculate the fuel efficiency (km/L). Here, ignore the motor, and consider only the engine operation. (8 pts)

(5) Convert the fuel efficiency of (4) to MPG unit. (1 mile = 1.609 km, 1 gallon = 3.785 L) (2 pts)