

Submit the compressed file as (ID)_(name).zip to [[ftp://cdl.hanyang.ac.kr](http://cdl.hanyang.ac.kr) → Undergraduate_CAE → lab → midterm] folder. It should contain the final results (graphs) of each problem using PowerPoint (ID.ppt), MATLAB file (problem#_#.m), Simulink file (problem#_#.slx)

1.[MATLAB] Solve the following initial value problem over the interval from $t=0$ to $t=2$, where $\mu=10$, $y(0)=0$ and the step size of $\Delta t=0.2$

$$\frac{dy}{dt} = \sin(t) - \mu y$$

(1) When the analytic solution of the above initial value problem is given as

$$y = \frac{\mu \sin t - \cos t + e^{-\mu t}}{\mu^2 + 1},$$

develop a function script (heun.m) that covers heun's method without corrector update. Then obtain the numerical solution and plot analytic solution and numerical solution in the format below (10 pts)

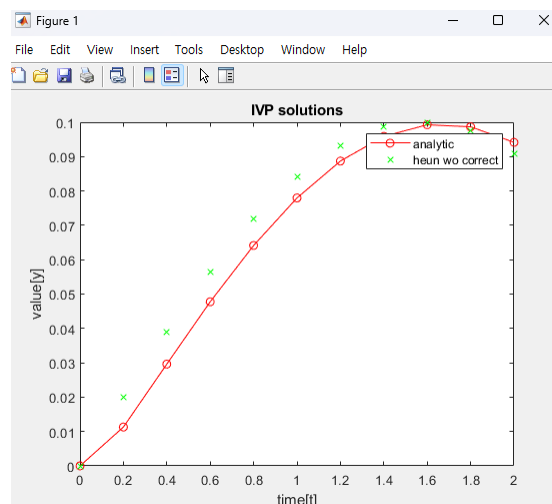
Satisfy all the post-processing styles of the plot provided below. Except font style & size

[Heun without corrector update]

for an IVP $\frac{dy}{dt} = f(t, y)$

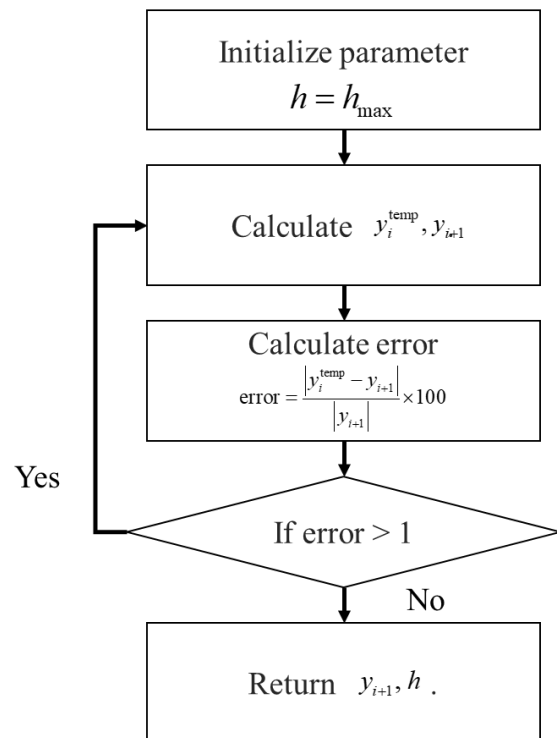
$$y_{i+1}^{\text{temp}} = y_i + f(t_i, y_i)h$$

$$y_{i+1} = y_i + \frac{1}{2} \left(f(t_i, y_i) + f(t_{i+1}, y_{i+1}^{\text{temp}}) \right) h$$



- (2) When the flowchart of the function that calculate adaptive step size and solution of next time step is given as below, develop a function script(update_h.m) and obtain numerical solution using heun_h_adaptive.m below. (20 pts)

function [h, y_heun] = update_h(dydt, yi, ti, hmax)



```

function [t, y] = heun_h_adaptive(dydt, tspan, hmax, y0)
ti = tspan(1); tf = tspan(2);
y(1) = y0;
t(1) = ti;
i = 1;
while true
    [h, y_heun] = update_h(dydt, y(i), t(i), hmax);

    if t(i) + h > tf
        break
    end
    t(i+1) = t(i) + h;
    y(i+1) = y_heun;
    i = i + 1;
end
end
  
```

2.[MATLAB] Solve the equation of motion for a simple pendulum with given initial conditions.

$$\frac{d^2\theta}{dt^2} + \frac{g}{L} \sin \theta = 0$$

initial conditions

$$\text{at } t=0, \theta = \frac{\pi}{4}, \frac{d\theta}{dt} = 0$$

(1) Given the following two scripts (main, pendulum), Develop a single function script (RK4.m) that covers the fourth-order RK method for second-order ODE and obtain the solution of θ . (20 pts)

tspan=[0 10], step size h=0.1 plot style "x:" ex: plot(x, y, 'x:')

[Fourth-order-RK]

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

$$k_1 = f(t_i, y_i)$$

$$k_2 = f\left(t_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1h\right)$$

$$k_3 = f\left(t_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2h\right)$$

$$k_4 = f(t_i + h, y_i + k_3h)$$

[main.m]

clc; clear; close all;

h=0.01;

tf=10;

g = 9.81;

L = 2;

[t, y]= RK4(@(t, y) pendulum(t, y, g, L), [0 tf], [pi/4, 0],h);

figure(1)

plot(t, y(:,1),'x:')

[pendulum.m]

function yp = pendulum(t,y,g,L)

...

3.[Simulink] Solve the following Van der Pol equation using signal-based solution. Plot numerical solution using scope block. (Simulation time: 6000s, solver : ode23tb,) (15 pts)

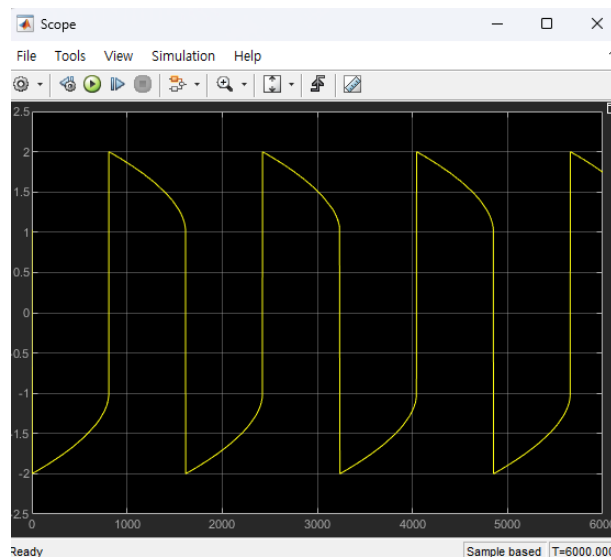
$$\frac{d^2 y}{dt^2} - \mu(1 - y^2) \frac{dy}{dt} + y = 0$$

initial condition

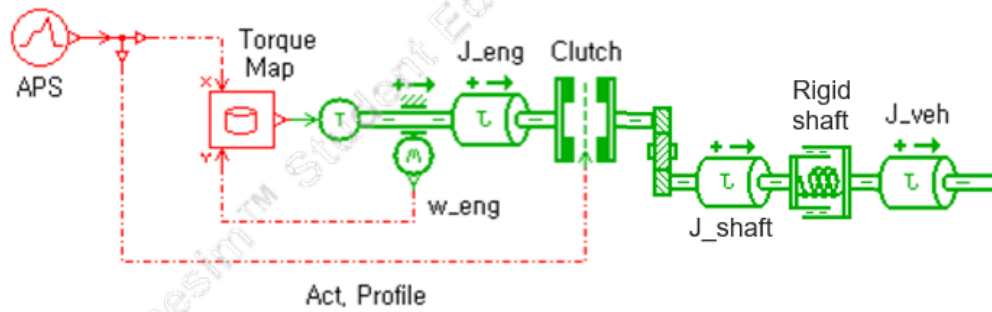
$$\text{at } t=0, y=1 \quad \frac{dy}{dt} = 1$$

parameter

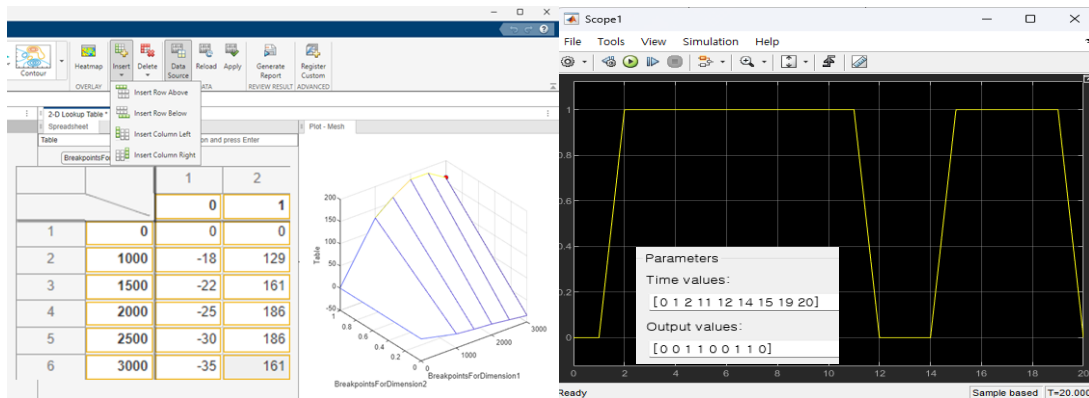
$$\mu=1,000$$



4. [Simulink] Consider the following system.



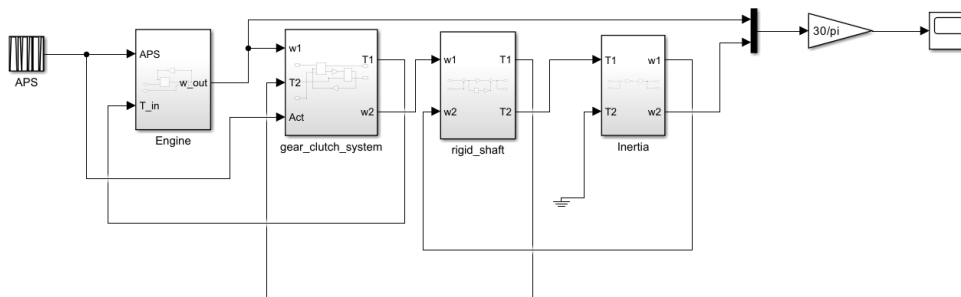
- ① $J_{eng} = 1 \text{ kgm}^2$, $w_{eng} = 0 \text{ RPM}$
- ② $J_{shaft} = 0.01 \text{ kgm}^2$
- ③ Clutch friction torque = 200 Nm
- ④ Gear Ratio = 2
- ⑤ $k_{shaft} = 1e7 \text{ N/rad}$, $c_{shaft} = 1e5 \text{ Ns/rad}$
- ⑥ $J_{eng} = 50 \text{ kgm}^2$, $w_{eng} = 200 \text{ RPM}$



[Torque map(APS, RPM)]

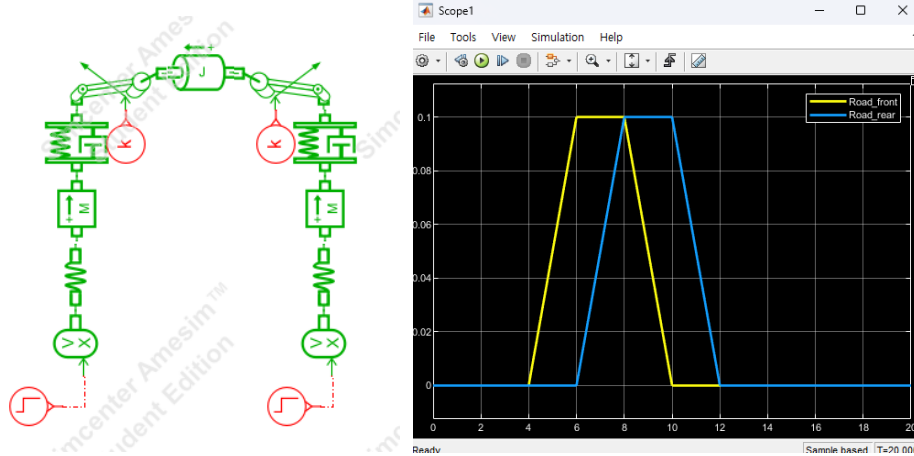
[APS]

(1) Construct a Simulink model for the system above. Show the engine and vehicle-side RPM by using scope block(simulation time : 20s, solver : auto)

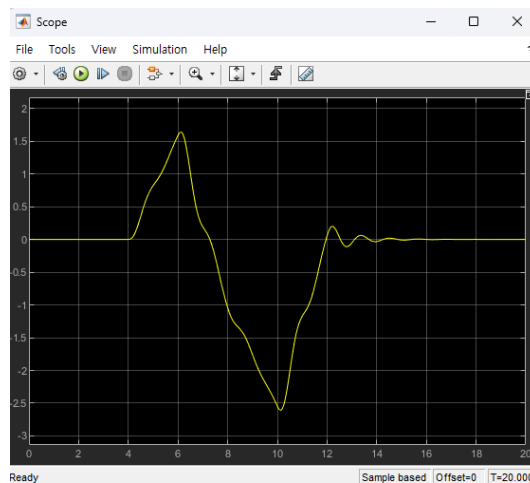


5. [Simulink] Consider quarter car model. The parameter values in the model are as follows.

[Wheel displacement]



[Angle of inertia]



$$J_p = 2000 \text{ kg m}^2$$

$$L_f = 1.2 \text{ m}, L_r = 1.5 \text{ m}$$

$$k_f = 15000 \text{ N/m}, c_f = 1000 \text{ Ns/m}$$

$$k_r = 20000 \text{ N/m}, c_r = 1500 \text{ Ns/m}$$

$$m_{\text{wheel}} = 50 \text{ kg}, k_{\text{tire}} = 200000 \text{ N/m}$$

- (1) Construct a Simulink model for this system. Then show the result of vehicle pitch angle by using scope block (Simulation time : 20s, solver :ode23tb) (20 pts)

Hint)

