Midterm Exam

10/29/2024

1. Solve the following (initial value) problem with the fourth-order RK method from $x=0$ to 0.5 with the step size of 0.5: $\frac{d^2y}{dx^2}+0.5\frac{dy}{dx}+7y=0$ where $y(0)=4$ and $y'(0)=0$ (20 pts)

   [Hint: $\phi=\frac{1}{6}(k_1+2k_2+2k_3+k_4)$ $where$ $k_3=f(x_i+\frac{1}{2}h, y_i+\frac{1}{2}k_2h)$ and $k_4=f(x_i+h, y_i+k_3h)$]

2. Solve the nondimensionalized ODE using finite difference methods that describe the temperature distribution in a circular rod with internal heat source S, $\frac{d^2T}{dr^2}+\frac{1}{r}\frac{dT}{dr}+S=0$ over the range $0\le r\le 1$, with the boundary conditions $T(r=1)=0$ and $\left.\frac{dT}{dr}\right|_{r=0}=0$. (3-point central difference for $\frac{d^2T}{dr^2}$, central difference for $\frac{dT}{dr}$): (20 pts)
   (1) Write the general finite difference equation at point *i*.
   (2) Write the end point equation at $r=0$.
   (3) Write the end point equation at $r=1$.

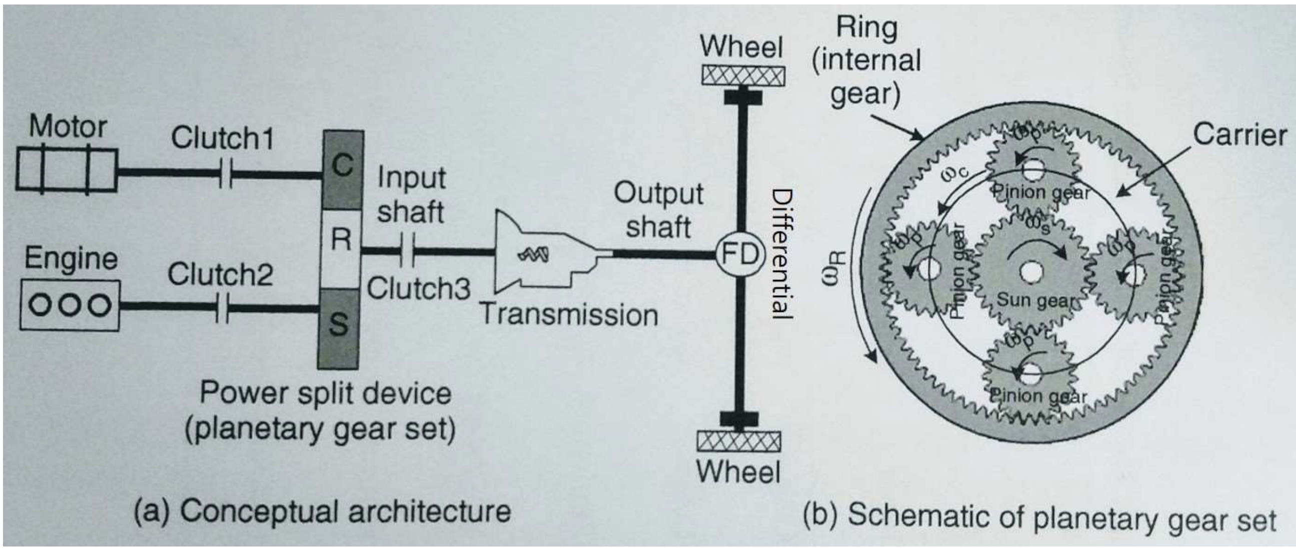
3. Consider the heat-conduction equation $k\frac{\partial^2T}{\partial x^2}=\frac{\partial T}{\partial t}$. The heat-conduction equation requires approximations for the second derivative in space and the first derivative in time. Write the general finite difference equation to solve the temperature at point *i* with time (*l*+1). (20 pts)
   (1) Using an explicit method
   (2) Using an implicit method
   (3) Explain the differences between two methods.

4. Find the general solution of the following PDE by the method of separation of variables:

   $xy\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}+yu=0$. (15 pts)

5. Describe the input(s) and the output(s) of the following vehicle component model: (1) engine (2) motor (3) clutch (4) transmission (5) battery (6) resistance (7) vehicle (equivalent inertia) (8) driver. (10 pts)

6. Consider the following parallel hybrid electric vehicle system. Be sure to check the units of parameter values.



(a) Conceptual architecture (b) Schematic of planetary gear set

Inertia/mass : $J_{engine} = 0.2\ \text{kg}\cdot\text{m}^2$, $J_{motor} = 0.05\,\text{kg}\cdot\text{m}^2$, $J_{wheel} = 1\ \text{kg}\cdot\text{m}^2$(sum of wheels), $M_{vehicle} = 1{,}500\ \text{kg}$

Powertrain : $Z_{sun} = 30$, $Z_{ring} = 60$(power-split device), $GR_{transmission} = 3$, $GR_{differential} = 4$, $R_{tire} = 0.3\ \text{m}$

Resistance: $C_d = 0.25$, $A_{front} = 1.8\,\text{m}^2$, $\rho_{air} = 1.2\,\text{kg/m}^3$, $\mu_{roll} = 0.01$, $g = 9.81\ \text{m/s}^2$

Battery: $C_{nom} = 50{,}000\ \text{As}$, $V_{battery} = 250\ \text{V}$

motion equation of planetary gear: $\omega_s + \dfrac{Z_r}{Z_s}\omega_r - \dfrac{Z_s + Z_r}{Z_s}\omega_c = 0$

(1) On EV mode, calculate the total equivalent inertia at wheel. [Hint: define the state of power-split device. Clutch 1 and 3 are engaged and a sun gear is fixed.] (6 pts)

(2) When the traction motor torque is 50 Nm and a vehicle speed is 72 km/h, calculate a vehicle acceleration speed. (8 pts)

(3) On HEV mode (All clutches are engaged.), when the engine and motor RPM are 3000 and 2000, respectively, calculate a vehicle speed. (Hint: calculate an input shaft speed.) (5 pts)

(4) This vehicle is currently driving a downhill road (EV mode). A driver is working the braking pedal to maintain the vehicle speed of 36 km/h, and the motor is charging a battery. When the sum of regenerative braking torque at wheels is 1000 Nm (negative value), calculate the motor torque and speed. How much is a SOC after driving of 20 s. (without loss, current SOC = 50 %) (6 pts)