Midterm Exam

10/31/2017

1. (각각 정의: 2점, 사례: 3점)

round-off errors (반올림오차): result when numbers having limited significant figures are used to represent exact numbers

examples: common arithmetic operations, large computations, adding a large and a small number, subtractive cancellation, smearing, inner products, Numbers ($\pi, e, \sqrt{\ }$) 중 하나 (반올림 예시는 점수X)

truncation errors(단절오차): result when approximations are used to represent exact mathematical procedures

examples: Taylor series

2. (각 4pts) (방법만 맞는경우 2pts, 답 하나만 맞는 경우 3pts)

(1) $\int \frac{dy}{\sqrt{y}} = \int (1+2x)\,dx \rightarrow 2\sqrt{y} = x + x^2 + c \xrightarrow{y(0)=1} c = 2 \rightarrow y = \frac{\left(x + x^2 + 2\right)^2}{4}$

(2) i에서 기울기로 (i+1)에서 y를 예측

Euler's method			
x	y	dy/dx	exact
0	1	1	1
0.5	1.5	2.44949	1.890625
1	2.724745		4

(3) i에서 기울기와 (i+1)에서 기울기의 평균으로 (i+1)에서 y를 예측

Heun's method withoud corrector						
x	y	dy/dx	x_end	y_end	dy/dx_enc	dy/dx_avg
0	1	1	0.5	1.5	2.44949	1.724745
0.5	1.862372	2.729375	1	3.22706	5.389206	4.059291
1	3.892018					

(4) (i+0.5*h)에서 기울기로 (i+1)에서 y를 예측

Midpoint method					
x	y	dy/dx	x_mid	y_mid	dy/dx_mic
0	1	1	0.25	1.25	1.677051
0.5	1.838525	2.711845	0.75	2.516487	3.96586
1	3.821455				

(5) 기울기 $\phi = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \leftarrow$

$k_1 : i$에서 기울기

$k_2 : i$에서 k_1기울기로 $\left(i + \frac{1}{2}\right)$에서 기울기

$k_3 : i$에서 k_2기울기로 $\left(i + \frac{1}{2}\right)$에서 기울기

$k_4 : i$에서 k_3기울기로 $(i+1)$에서 기울기

4-th order RK method

x	y	k1=dy/dx	x_mid	y_mid	2=dy/dx_m	y_3	k3	x_end	y_end	k4	phi=dy/dx
0	1	1	0.25	1.25	1.677051	1.419263	1.786992	0.5	1.893496	2.752087	1.780029
0.5	1.890014	2.749556	0.75	2.577403	4.013573	2.893408	4.252505	1	4.016267	6.012188	4.21565
1	3.99784										

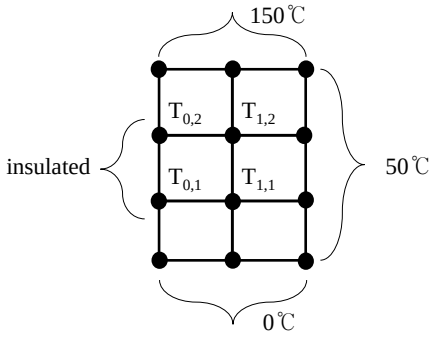
3. (5 pts)

$$z = y_1 \rightarrow \frac{dz}{dx} = \frac{dy_1}{dx} = y_2, \quad \frac{d^2z}{dx^2} = \frac{dy_2}{dx} = y_3, \quad \frac{d^3z}{dx^3} = \frac{dy_3}{dx}$$

$$z(0) = y_1(0) = 1, \quad \frac{dz}{dx}(0) = y_2(0) = 0, \quad \frac{d^2z}{dx^2}(0) = y_3(0) = -1$$

$$\begin{cases} \frac{dy_1}{dx} = y_2 \text{ where } y_1(0) = 1 \\ \frac{dy_2}{dx} = y_3 \text{ where } y_2(0) = 0 \\ \frac{dy_3}{dx} = 2y_1 + {y_2}^3 - {y_1}^2 y_3 \text{ where } y_3(0) = -1 \end{cases}$$

4. 3*4 그리드 포인트에 대한 그림 (영역내부에 3*4줄 OK)과 미지수 표기 (5점) (그리드 다른 경우 2점 감점)



단열점에 대한 식 유도 (5점), 내부점에 대한 식 유도 (5점)

$$@(0,1)\ T_{-1,1}+0+T_{1,1}+T_{0,2}-4T_{0,1}=0 \xrightarrow{\frac{\partial T}{\partial x}=\frac{T_{1,1}-T_{-1,1}}{2\Delta x}=0} 4T_{0,1}-2T_{1,1}-T_{0,2}=0$$

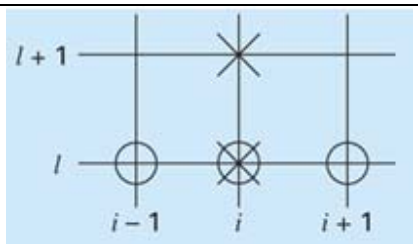
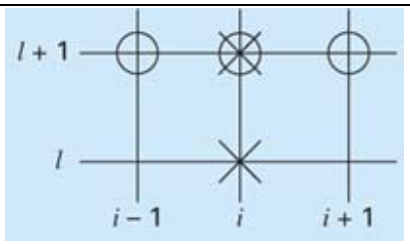
$$@(1,1)\ T_{0,1}+0+50+T_{1,2}-4T_{1,1}=0 \rightarrow -T_{0,1}+4T_{1,1}-T_{1,2}=50$$

$$@(0,2)\ T_{-1,2}+T_{0,1}+T_{1,2}+150-4T_{0,2}=0 \xrightarrow{\frac{\partial T}{\partial x}=\frac{T_{1,2}-T_{-1,2}}{2\Delta x}=0} -T_{0,1}+4T_{0,2}-2T_{1,2}=150$$

$$@(1,2)\ T_{0,2}+T_{1,1}+50+150-4T_{1,2}=0 \rightarrow -T_{1,1}-T_{0,2}+4T_{1,2}=200$$

$$\begin{bmatrix} 4 & -2 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -2 \\ 0 & -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} T_{0,1} \\ T_{1,1} \\ T_{0,2} \\ T_{1,2} \end{bmatrix} = \begin{bmatrix} 0 \\ 50 \\ 150 \\ 200 \end{bmatrix}$$ (행렬식 구성 정확히 일치하는 경우만 5점)

5.

	explicit	implicit
concept		
Space (3 pts)	$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1}^l - 2T_i^l + T_{i-1}^l}{\Delta x^2}$	$\frac{\partial^2 T}{\partial x^2} \cong \frac{T_{i+1}^{l+1} - 2T_i^{l+1} + T_{i-1}^{l+1}}{(\Delta x)^2}$
Time (2 pts)	$\frac{\partial T}{\partial t} = \frac{T_i^{l+1} - T_i^l}{\Delta t}$	$\frac{\partial T}{\partial t} = \frac{T_i^{l+1} - T_i^l}{\Delta t}$
(2 pts)	$k\frac{T_{i+1}^l - 2T_i^l + T_{i-1}^l}{(\Delta x)^2} = \frac{T_i^{l+1} - T_i^l}{\Delta t}$ $\rightarrow T_i^{l+1} = T_i^l + \lambda\left(T_{i+1}^l - 2T_i^l + T_{i-1}^l\right)$ where $\lambda = \frac{k\Delta t}{(\Delta x)^2}$	$\begin{bmatrix} 1+2\lambda & -\lambda & \cdots & 0 \\ -\lambda & 1+2\lambda & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1+2\lambda \end{bmatrix} \begin{bmatrix} T_1^{l+1} \\ T_2^{l+1} \\ \vdots \\ T_m^{l+1} \end{bmatrix} = \begin{bmatrix} T_1^l + \lambda T_0^{l+1} \\ T_i^l \\ \vdots \\ T_m^l + \lambda T_{m+1}^{l+1} \end{bmatrix}$
Stability (3 pts)	$\lambda = \frac{k\Delta t}{(\Delta x)^2} \le \frac{1}{2} \rightarrow \Delta t \le \frac{1}{2}\frac{(\Delta x)^2}{k}$	Unconditionally stable

6.

(1) $F_{fric} = \mu_{disk} F_n = 0.5 \times 2{,}000 = 1{,}000$ N, $R_{eff} = \dfrac{2(r_{out}^3 - r_{in}^3)}{3(r_{out}^2 - r_{in}^2)} = \dfrac{2(0.2^3 - 0.1^3)}{3(0.2^2 - 0.1^2)} = 0.1556$ m

$T_{out} = F_{fric} R_{eff} = 1{,}000 \times 0.1556 = 155.6$ Nm (4 pts)

(2) $\omega_{motor} = \dfrac{V_{veh}}{R_{tire}} GR_{TM} GR_{diff} = \dfrac{10}{0.3} \times 2 \times 4 = 266.67$ rad/s, $E_b = K_e \omega_{motor} = 0.4 \times 266.67 = 106.67$ V

$I = \dfrac{T_{motor}}{K_t} = \dfrac{50}{2} = 25$ A, $V = IR + E_b = 25 \times 10 + 106.67 = 356.67$ V (6 pts)

(3) $T_{wheel} = (T_{eng} + T_{motor}) GR_{TM} GR_{diff} = (30 + 50) \times 2 \times 4 = 640$ Nm

$J_{eq} = (J_{eng} + J_{motor}) GR_{TM}^2 GR_{diff}^2 + J_{wheel} + m_{body} R_{tire}^2 = (0.2 + 0.05) \times 8^2 + 1 + 1{,}500 \times 0.3^2 = 152$ kgm^2

$F_{drag} = \dfrac{1}{2} C_d A_{front} \rho_{air} V_{vehicle}^2 + \mu_{roll} M_{vehicle} g = \dfrac{1}{2} \times 0.25 \times 1.8 \times 1.2 \times 10^2 + 0.01 \times 1500 \times 9.81 = 174.15$ N

$T_{drag} = F_{drag} R_{tire} = 174.15 \times 0.3 = 52.25$ Nm, $a_{vehicle} = \dfrac{T_{wheel} - T_{drag}}{J_{eq}} R_{tire} = \dfrac{640 - 52.25}{152} \times 0.3 = 1.16$ m/s^2

(9 pts)

(4) $W_{motor} = T_{motor} \omega_{motor} = 50 \times 266.67 = 13{,}333$ W, $I_{battery} = \dfrac{W_{motor}}{V_{battery}} = \dfrac{13{,}333}{250} = 53.33$ A (6 pts)

$\dfrac{dSOC}{dt} = I_{battery} \dfrac{100}{C_{nom}} = 53.33 \dfrac{100}{50{,}000} = 0.1067$ %/s,

$t_{driving} = SOC_{ini} \dfrac{1}{dSOC/dt} = 50 \dfrac{1}{0.1067} = 468.75$ s, $s_{dist} = V_{vehicle} t_{driving} = 4687.5$ m