10/30/2018

1. (각 5점)

$$\frac{dy}{dx} = yx^2 - 1.1y,\ y(0) = 1$$

(1) explicit Euler: $y'(0) = -1.1 \rightarrow y(1) = y(0) + (\Delta x)\, y'(0) = 1 - (1)1.1 = -0.1$

(2) implicit Euler

$$y(1) = y(0) + (\Delta x)\, y'(1) = 1 + (1)\left[y(1)(1)^2 - 1.1y(1) \right] = 1 - 0.1y(1) \rightarrow 1.1y(1) = 1 \rightarrow y(1) = \frac{10}{11} = 0.909$$

(3) Heun's w/o corrector: with the prediction of explicit Euler: $y(1) = -0.1$

$$y'(1) = y(1)(1)^2 - 1.1y(1) = -0.1 + 0.11 = 0.01$$

$$\left[y'(0) \right]_{new} = \frac{1}{2}\left[y'(0) + y'(1) \right] = \frac{-1.1 + 0.01}{2} = -0.545 \rightarrow y(1) = y(0) + (\Delta x)\, y'(0) = 1 - 0.545 = 0.455$$

2.

$$f(x,u) = \frac{du}{dx} = -2u + x + 4,\ u(0) = 1,\ \Delta x = 2$$

$k_1 = f(x,u) = f(0,1) = -2(1) + 0 + 4 = 2$ (2 pts)

$k_2 = f(x + \Delta x/2, u + k_1\, \Delta x/2) = f(1,3) = -2(3) + 1 + 4 = -1$ (2 pts)

(1) 3rd order Runge – Kutta method

$k_3 = f(x + \Delta x, u - k_1\Delta x + 2k_2\Delta x) = f\left(2, 1 - 2(2) + 2(-1)2\right) = f(2,-7) = -2(-7) + 2 + 4 = 20$ (2 pts)

$u(2) = u(0) + \frac{1}{6}(k_1 + 4k_2 + k_3)\Delta x = 1 + \frac{1}{6}\left(2 + 4(-1) + 20\right)(2) = 7$ (2 pts)

(2) 4th order Runge – Kutta method

$k_3 = f(x + \Delta x/2, u + k_2\, \Delta x/2) = f(1,0) = -2(0) + 1 + 4 = 5$ (2 pts)

$k_4 = f(x + \Delta x, u + k_3\Delta x) = f\left(2, 1 + 5(2)\right) = f(2,11) = -2(11) + 2 + 4 = -16$ (2 pts)

$u(2) = u(0) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\Delta x = 1 + \frac{1}{6}\left(2 + 2(-1) + 2(5) - 16\right)(2) = -1$ (2 pts)

(3) exact solution

$$u(x) = ae^{-2x} + bx + c \rightarrow -2ae^{-2x} + b = -2\left(ae^{-2x} + bx + c\right) + x + 4 \leftrightarrow \begin{cases} -2b + 1 = 0 \rightarrow b = 1/2 \\ b = -2c + 4 \rightarrow c = 7/4 \end{cases}$$

$$u(x) = ae^{-2x} + \frac{1}{2}x + \frac{7}{4} \xrightarrow{u(0)=1} 1 = a + \frac{7}{4} \rightarrow a = -\frac{3}{4}$$

$u(x) = -\frac{3}{4}e^{-2x} + \frac{1}{2}x + \frac{7}{4} \rightarrow u(2) = -\frac{3}{4}e^{-4} + \frac{11}{4}$ (4 pts)

exact @ u=2	3rd order R-K	4^{th} order R-K
2.736263271	7	-1
Error (1 pt each)	156%	-137%

3. start with Taylor's series

$$(1): f(x_{i-1}) = f(x_i) - f'(x_i)(2h) + \frac{1}{2} f''(x_i)(2h)^2 \quad \text{(3 pts)}$$

$$(2): f(x_{i+1}) = f(x_i) + f'(x_i)(h) + \frac{1}{2} f''(x_i)(h)^2 \quad \text{(3 pts)}$$

$$(1) + 2\times(2):\ f(x_{i-1}) + 2f(x_{i+1}) = 3f(x_i) + f''(x_i)(3h^2) + O(h^3)$$

$$\rightarrow f''(x_i) = \frac{1}{3h^2}\left[f(x_{i-1}) - 3f(x_i) + 2f(x_{i+1})\right] + O(h) \quad \text{(4 pts)}$$

4.

$$\frac{d^2u}{dx^2} = 1 \rightarrow \frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta x)^2} = 1 \rightarrow u_{i+1} - 2u_i + u_{i-1} = 0.04,\ i = 0,1,2,3,4$$

$$u'(0) = 0.5 \rightarrow \frac{u_1 - u_{-1}}{2(\Delta x)} = 0.5 \rightarrow u_{-1} = u_1 - 0.2 \text{(backward difference:} -1 \text{ pt)},\ u(1) = 2 \rightarrow u_5 = 2$$

$$\left.\begin{aligned}
&i = 0\text{(5 pts)}: u_1 - 2u_0 + u_{-1} = 0.04 \rightarrow 2u_1 - 2u_0 = 0.24 \\
&\frac{u_1 - u_0}{(\Delta x)} = 0.5\text{로 작성한 경우, } -3\text{ pts} \\
&i = 1\text{(2 pts)}: u_2 - 2u_1 + u_0 = 0.04 \\
&i = 2\text{(2 pts)}: u_3 - 2u_2 + u_1 = 0.04 \\
&i = 3\text{(2 pts)}: u_4 - 2u_3 + u_2 = 0.04 \\
&i = 4\text{(4 pts)}: u_5 - 2u_4 + u_3 = 0.04 \rightarrow -2u_4 + u_3 = -1.96
\end{aligned}\right\} \rightarrow
\begin{bmatrix} -2 & 2 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}
\begin{Bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} =
\begin{Bmatrix} 0.24 \\ 0.04 \\ 0.04 \\ 0.04 \\ -1.96 \end{Bmatrix}$$

5.

$$xy\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} + yu = 0 \leftarrow u(x,y) = G(x)H(y) \quad \text{(5 pts: 변수분리 아이디어)}$$

$$xyH\frac{dG}{dx} - G\frac{dH}{dy} + yGH = 0 \rightarrow xyH\frac{dG}{dx} + yGH = G\frac{dH}{dy}$$

$$y\left(xH\frac{dG}{dx} + GH\right) = G\frac{dH}{dy} \rightarrow xH\frac{dG}{dx} + GH = \frac{G}{y}\frac{dH}{dy}$$

$$\frac{x}{G}\frac{dG}{dx} + 1 = \frac{1}{yH}\frac{dH}{dy} \text{(3 pts)} \rightarrow \begin{cases} \dfrac{x}{G}\dfrac{dG}{dx} + 1 = c \rightarrow \dfrac{1}{G}dG = \dfrac{c-1}{x}dx \rightarrow \ln G = (c-1)\ln x \rightarrow G = k_1 x^{(c-1)} \quad \text{(3 pts)} \\ \dfrac{1}{yH}\dfrac{dH}{dy} = c \rightarrow \dfrac{dH}{H} = cydy \rightarrow \ln H = \dfrac{cy^2}{2} + d \Longrightarrow H = k_2 e^{\frac{c}{2}y^2} \quad \text{(3 pts)} \end{cases}$$

$$u(x,y) = k_1 x^{(c-1)} k_2 e^{\frac{c}{2}y^2} = kx^{(c-1)} e^{\frac{c}{2}y^2} \quad \text{(1 pts)}$$

6.

(1) $GR_{PSD} = \dfrac{Z_{ring}}{Z_{sun} + Z_{ring}} = \dfrac{60}{90} = \dfrac{2}{3}$ (2pts), $J_{vehicle} = mR_{tire}^2 = 1500 \times 0.3^2 = 135\ \text{kg}\bullet\text{m}^2$ (2pts)

$$J_{eq} = J_{motor} \times (GR_{PSD} \times GR_{TM} \times GR_{diff})^2 + J_{wheel} + J_{vehicle} = 0.05 \times \left(\frac{2}{3} \times 3 \times 4\right)^2 + 1 + 135 = 139.2\,\text{kg}\bullet\text{m}^2$$ (2pts)

(2) $T_{wheel} = T_{motor} \times (GR_{PSD} + GR_{TM} + GR_{diff}) = 50 \times \left(\dfrac{2}{3} \times 3 \times 4\right) = 400\,\text{Nm}$ (2pts)

$$F_{drag} = \frac{1}{2} C_d A_{front} \rho_{air} V_{vehicle}^2 + \mu_{roll} M_{vehicle} g = \frac{1}{2} \times 0.25 \times 1.8 \times 1.2 \times 20^2 + 0.01 \times 1500 \times 9.81 = 255.15\ \text{N}$$ (2pts)

$$T_{drag} = F_{drag} R_{tire} = 255.15 \times 0.3 = 76.55\ \text{Nm}$$,

$$a_{vehicle} = \frac{T_{wheel} - T_{drag}}{J_{eq}} R_{tire} = \frac{400 - 76.55}{139.2} \times 0.3 = 0.697\ \text{m/s}^2$$ (4pts)

(3) $\omega_{ring} = \dfrac{Z_{sun}}{Z_{ring}} (\omega_{carrier} - \omega_{sun}) + \omega_{carrier} = \dfrac{30}{60}(2000 - 3000) + 2000 = 1500\ \text{RPM}$ (2pts)

$$\omega_{wheel} = \frac{\omega_{ring}}{GR_{TM} \times GR_{diff}} = \frac{1500}{3 \times 4} = 125\ \text{RPM} = 13.09\,\text{rad/s}$$ (2pts)

$$V_{vehicle} = R_{tire} \omega_{wheel} = 0.3 \times 13.09 = 3.93\ \text{m/s}$$ (1pt)

(4) $\omega_{motor} = \dfrac{V_{vehicle}}{R_{tire}} \times GR_{diff} \times GR_{TM} \times GR_{PSD} = 266.67\,\text{rad/s}$ (1pt)

$$T_{motor} = T_{regen} \times \frac{1}{GR_{diff} \times GR_{TM} \times GR_{PSD}} = -125\,\text{Nm}$$ (1pt)

$$I_{battery} = \frac{T_{regen} \omega_{wheel}}{V_{battery}} = \frac{-1000 \times 10 / 0.3}{250} = -133.33\ \text{A}$$ (1pt)

$$\frac{dSOC}{dt} = -I_{battery} \frac{100}{C_{nom}} = 133.33 \frac{100}{50{,}000} = 0.267\ \%/\text{s}$$ (2pts)

$$SOC_{final} = SOC_{initial} + \frac{dSOC}{dt} \Delta t = 50 + 0.267 \times 20 = 55.34\ \%$$ (1pt)