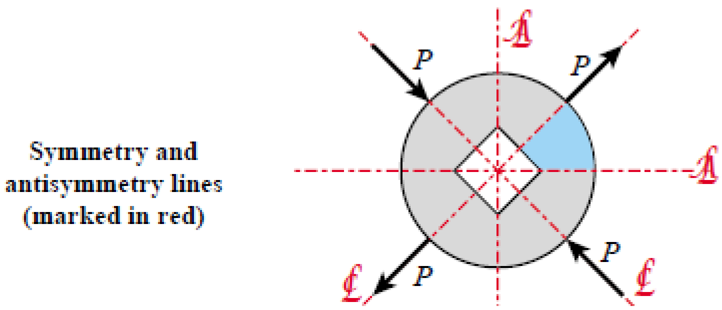
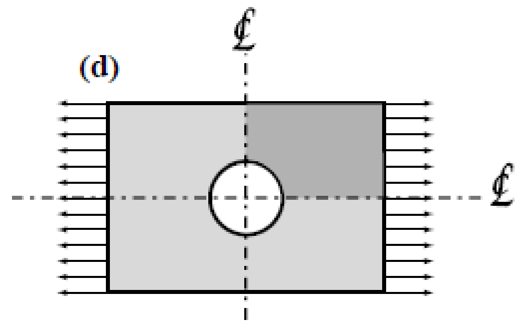
1. (2 pts each) (1) displacement (2) heat flux (3) pressure (4) charge density (electric intensity) (5) magnetic potential

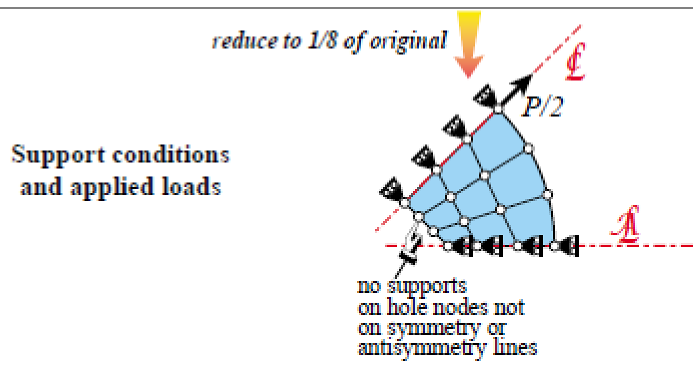
2. 요소절점에 경계조건과 하중조건이 작용되어야 함



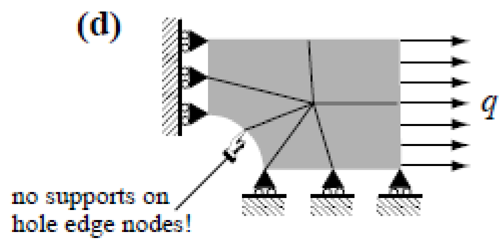
Symmetry Line 2개, Anti-symmetry Line 2개 (각1점)



Symmetry Line 2개 (각2점)



요소망 1점, 하중조건 1점, roller조건 각 2점



요소망 1점, 하중 1점, roller대칭/비대칭 각 2점

3.

$$\mathbf{N} = \begin{bmatrix} 1-\frac{x}{l} & \frac{x}{l} \end{bmatrix} = \begin{bmatrix} 1-\zeta & \zeta \end{bmatrix} \text{ (5 pts)}$$

$$\mathbf{K}^e = \int_0^1 EA\mathbf{B}^T\mathbf{B}l\,d\zeta = \int_0^1 \frac{EA}{l^2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} l\,d\zeta = \frac{E}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\int_0^1 A(\zeta)\,d\zeta \text{ (5 pts)}$$

$$= \frac{E}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\int_0^1 \left[A_i(1-\zeta)+A_j\zeta\right]d\zeta = \frac{E(A_i+A_j)}{2l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \leftrightarrow \mathbf{K}^e = \frac{E\bar{A}}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \text{ (5 pts)}$$

(형상함수 설명없이 유도 시 최대 7점)

4.

(1) master-slave method: $u_1(\text{master}) = u_3$ (5 pts)

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \xrightarrow{\mathbf{u}=\mathbf{T}\hat{\mathbf{u}}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 4 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \rightarrow u_1 = u_2 = \frac{3}{2} = u_3$$

(2) penalty function method (5 pts)

$$w[1 \quad -1] \begin{bmatrix} u_1 \\ u_3 \end{bmatrix} = 0 \xrightarrow[{[1 \quad -1]^T}]{\text{premultiply}} w \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \xrightarrow{\text{add}} \begin{bmatrix} 2+w & -1 & 0-w \\ -1 & 2 & -1 \\ 0-w & -1 & 2+w \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

(3) Lagrange multiplier method (5 pts)

$$u_1 = u_3 \xrightarrow{\text{add}} \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1-\lambda \\ 0 \\ 2+\lambda \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -1 & 0 & 1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \lambda \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

5. (10 pts)

$$\left. \begin{aligned} N_1^e &= \frac{1}{4}(1-\xi)(1-\eta) \\ N_2^e &= \frac{1}{4}(1+\xi)(1-\eta) \\ N_3^e &= \frac{1}{4}(1+\xi)(1+\eta) \\ N_4^e &= \frac{1}{4}(1-\xi)(1+\eta) \end{aligned} \right\} \xrightarrow{\xi=0.8,\ \eta=0.9} \left\{ \begin{aligned} N_1^e &= \frac{1}{4}(1-0.8)(1-0.9) = 0.005 \\ N_2^e &= \frac{1}{4}(1+0.8)(1-0.9) = 0.045 \\ N_3^e &= \frac{1}{4}(1+0.8)(1+0.9) = 0.855 \\ N_4^e &= \frac{1}{4}(1-0.8)(1+0.9) = 0.095 \end{aligned} \right.$$

$$x = \sum_{i=1}^{4} N_i^e x_i = 0.005(1.0) + 0.045(3.0) + 0.855(3.5) + 0.095(1.5) = 3.275$$

$$y = \sum_{i=1}^{4} N_i^e y_i = 0.005(1.0) + 0.045(1.5) + 0.855(4.0) + 0.095(2.5) = 3.73$$

6. (5 pts each)

(1) $N_1 = c_1(\eta-1)(\xi-1)\xi \to N_1(-1,-1) = -4c_1 = 1 \to c_1 = -\frac{1}{4} \to N_1 = -\frac{1}{4}(\eta-1)(\xi-1)\xi$

$$(2)\ \text{compatibility}\begin{cases}\text{edge1-4: } N_1(\xi=-1) = -\frac{1}{2}(\eta-1) \to \text{linear} \xleftrightarrow{OK} \text{two points} \\ \text{edge1-5-2: } N_1(\eta=-1) = -\frac{1}{2}(\xi-1)\xi \to \text{quadratic} \xleftrightarrow{OK} \text{three points}\end{cases}$$

(3) completeness: $\sum_{i=1}^{6} N_i = 1$

$$(4)\ \begin{cases}\text{edge1-4: } N_1 \text{ varies linearly over 6-node element side} \\ \xleftrightarrow{OK} \text{shape functions of 4-node bilinear quadrilateral vary linearly over any side} \\ \text{edge1-5-2: } N_1 \text{ varies quadratically over 6-node element side} \\ \xleftrightarrow{OK} \text{shape functions of 9-node bilinear quadrilateral vary quadratically over any side}\end{cases}$$

(5) $r = \min(n_F - n_R, 3n_G) = \min(12-3, 3n_G) \to n_G \geq 3$

(6) $n_G \geq 3 \to p = 2$