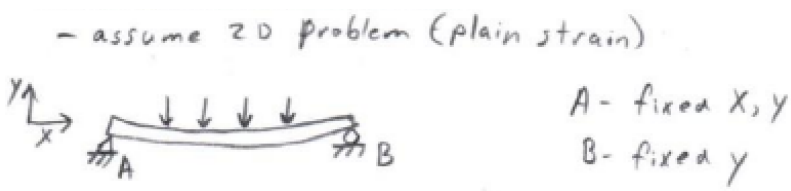
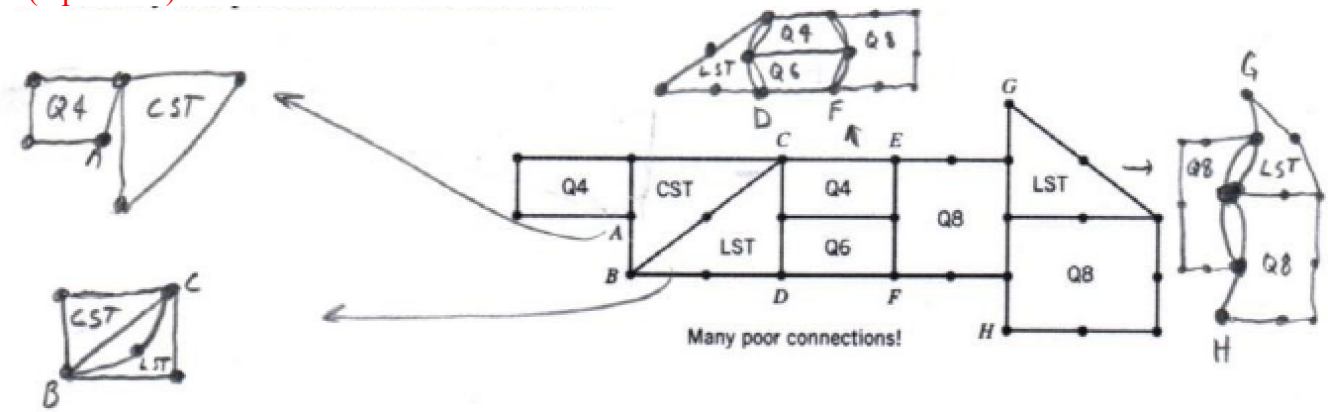
1. (5 pts each)

(1)



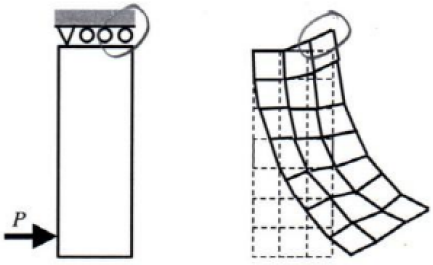
(2)

2. (5 pts each)

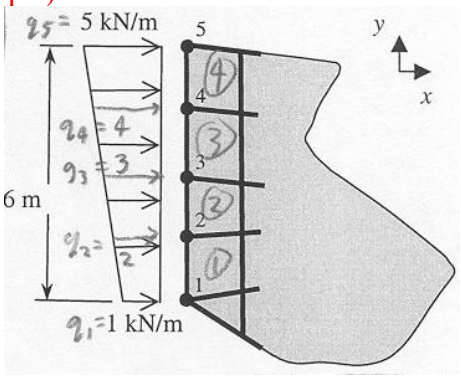


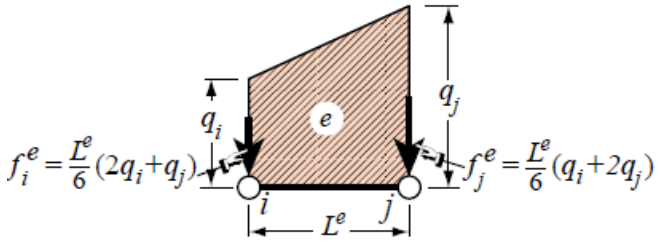
- Node A is not really a node from the CST. It could open up a gap as shown.
- Should not connect 2 node edge BC with 3 node edge. A gap could open up as shown.
- 2 node edges should not be connected to 3 node edge CD and EF.
- GH nodes belong to different parabolas.

3. (10 pts) No, the two roller supports on the right have not been entered correctly.



4. (10 pts)





$$\begin{bmatrix} f_i^e \\ f_j^e \end{bmatrix} = \frac{L^e}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} q_i \\ q_j \end{bmatrix}$$

$$\left.\begin{aligned} for\ element\ 1: \begin{bmatrix} F_{1x} \\ F_{2x} \end{bmatrix} &= \frac{6/4}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1.25 \end{bmatrix} kN \\ for\ element\ 2: \begin{bmatrix} F_{2x} \\ F_{3x} \end{bmatrix} &= \frac{6/4}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1.75 \\ 2 \end{bmatrix} kN \\ for\ element\ 3: \begin{bmatrix} F_{3x} \\ F_{4x} \end{bmatrix} &= \frac{6/4}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 2.75 \end{bmatrix} kN \\ for\ element\ 4: \begin{bmatrix} F_{4x} \\ F_{5x} \end{bmatrix} &= \frac{6/4}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 3.25 \\ 3.5 \end{bmatrix} kN \end{aligned}\right\} \rightarrow \begin{bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \\ F_{5x} \end{bmatrix} = \begin{bmatrix} 1 \\ 1.25+1.75 \\ 2+2.5 \\ 2.75+3.25 \\ 3.5 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4.5 \\ 6 \\ 3.5 \end{bmatrix} kN$$

5.

(1) (10+5 pts)

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \varepsilon = Bd = \frac{1}{2A}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix}$$

$$2A = x_{21}y_{31} - x_{31}y_{21} = (0.03)(0.03) - (0.02)(0.01) = 0.0007m^2$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \frac{1}{0.0007}\begin{bmatrix} -0.02 & 0 & 0.03 & 0 & -0.01 & 0 \\ 0 & -0.01 & 0 & -0.02 & 0 & 0.03 \\ -0.01 & -0.02 & -0.02 & 0.03 & 0.03 & -0.01 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 1 \\ -1 \end{bmatrix}10^{-6} = \begin{bmatrix} 71.4 \\ -42.9 \\ 0 \end{bmatrix}10^{-6}$$

(2) (5 pts)

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \sigma = E\varepsilon = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{(1-\nu)}{2} \end{bmatrix}\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \frac{200\times10^9}{1-0.27^2}\begin{bmatrix} 1 & 0.27 & 0 \\ 0.27 & 1 & 0 \\ 0 & 0 & \frac{(1-0.27)}{2} \end{bmatrix}\begin{bmatrix} 71.4 \\ -42.9 \\ 0 \end{bmatrix}10^{-6}$$

$$= \begin{bmatrix} 12.9 \\ -5.1 \\ 0 \end{bmatrix}10^6 Pa$$

6.

(1) (3 pts * 3) $N_1 = \frac{(a-x)(b-y)}{4ab}, N_2 = \frac{(a+x)(b-y)}{4ab}, N_3 = \frac{(a+x)(b+y)}{4ab}, N_4 = \frac{(a-x)(b+y)}{4ab}$

(2) (5 pts)

$$N_1 = \frac{(0.01-0)(0.01-0)}{4(0.01)(0.01)} = 0.25, N_2 = \frac{(0.01+0)(0.01-0)}{4(0.01)(0.01)} = 0.25,$$
$$N_3 = \frac{(0.01+0)(0.01-0)}{4(0.01)(0.01)} = 0.25, N_4 = \frac{(0.01-0)(0.01+0)}{4(0.01)(0.01)} = 0.25$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = u = Nd = \begin{bmatrix} 0.25 & 0 & 0.25 & 0 & 0.25 & 0 & 0.25 & 0 \\ 0 & 0.25 & 0 & 0.25 & 0 & 0.25 & 0 & 0.25 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} 10^{-6} = \begin{bmatrix} 0.75 \\ -0.25 \end{bmatrix} 10^{-6} m$$

(3) (5+5 pts)

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \varepsilon = Bd = \partial Nd = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} Nd =$$

$$= \frac{1}{4ab} \begin{bmatrix} -(b-y) & 0 & (b-y) & 0 & (b+y) & 0 & -(b-y) & 0 \\ 0 & -(a-x) & 0 & -(a+x) & 0 & (a+x) & 0 & (a-x) \\ -(a-x) & -(b-y) & -(a+x) & (b-y) & (a+x) & (b+y) & (a-x) & -(b-y) \end{bmatrix} d$$

$$= \frac{1}{4(0.01)(0.01)} \begin{bmatrix} -0.01 & 0 & 0.01 & 0 & 0.01 & 0 & -0.01 & 0 \\ 0 & -0.01 & 0 & -0.01 & 0 & 0.01 & 0 & 0.01 \\ -0.01 & -0.01 & -0.01 & 0.01 & 0.01 & 0.01 & 0.01 & -0.01 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} 10^{-6} = \begin{bmatrix} 75 \\ -25 \\ -50 \end{bmatrix} 10^{-6}$$

(4) (4+2 pts)

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \sigma = E\varepsilon = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{(1-2\nu)}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$= \frac{200 \times 10^9}{(1+0.27)(1-2\times 0.27)} \begin{bmatrix} 1-0.27 & 0.27 & 0 \\ 0.27 & 1-0.27 & 0 \\ 0 & 0 & \frac{(1-2\times 0.27)}{2} \end{bmatrix} \begin{bmatrix} 75 \\ -25 \\ -50 \end{bmatrix} 10^{-6}$$

$$= \begin{bmatrix} 16.43 \\ 0.685 \\ -3.937 \end{bmatrix} 10^6 Pa$$