1. Develop a single function script that covers various methods up to the 4th order Runge-Kutta method by taking parameters a_{mn}, b_m, and c_m as in the equation below. (30 pts)

$$
\begin{aligned}
y_{i+1} &= y_i + h\sum_{m=1}^{N} b_m k_m \\
k_1 &= f\left(t_i, y_i\right) \\
k_2 &= f\left(t_i + c_2 h, y_i + h\left(a_{21}k_1\right)\right) \\
k_3 &= f\left(t_i + c_3 h, y_i + h\left(a_{31}k_1 + a_{32}k_2\right)\right) \\
&\vdots \\
k_m &= f\left(t_i + c_m h, y_i + h\sum_{n=1}^{m-1} a_{mn}k_n\right) \\
&\vdots
\end{aligned}
$$

Hint: for an IVP $\dfrac{dy}{dt} = f\left(t, y\right)$

[Forward Euler]

$$y_{i+1} = y_i + f\left(t_i, y_i\right)h$$

[Heun w/o update]

$$
\begin{aligned}
y_{i+1}^{\text{temp}} &= y_i + f\left(t_i, y_i\right)h \\
y_{i+1} &= y_i + \frac{1}{2}\left(f\left(t_i, y_i\right) + f\left(t_{i+1}, y_{i+1}^{\text{temp}}\right)\right)h
\end{aligned}
$$

[Midpoint]

$$
\begin{aligned}
y_{i+1/2} &= y_i + f\left(t_i, y_i\right)\frac{h}{2} \\
y_{i+1} &= y_i + f\left(t_{i+1/2}, y_{i+1/2}\right)h
\end{aligned}
$$

[Ralston]

$a_{21} = \dfrac{2}{3}$, $b_1 = b_2 = \dfrac{1}{2}$, $c_2 = \dfrac{2}{3}$, and the others are zeros.

```
function [t,y] = myode(dydt, tspan, y0, h, a, b, c)
t0 = tspan(1);
tn = tspan(2);
t = (t0:h:tn)';
n = length(t);
y = y0 * ones(n,1);
for i = 1:n-1
    k = zeros(1,4);
    for j = 1:4
        k(j) = dydt(t(i) + c(j)*h, y(i) + dot(a(j,:),k)*h);
    end
    y(i+1) = y(i) + dot(b,k)*h;
end
```

Alternatively,

```
function [t,y] = myode(dydt, tspan, y0, h, a, b, c)
t0 = tspan(1);
tn = tspan(2);
t = (t0:h:tn)';
n = length(t);
y = y0 * ones(n,1);
for i = 1:n-1
    k1 = dydt(t(i), y(i));
    k2 = dydt(t(i) + c(2)*h, y(i) + a(2,1)*k1*h);
    k3 = dydt(t(i) + c(3)*h, y(i) + (a(3,1)*k1 + a(3,2)*k2)*h);
    k4 = dydt(t(i) + c(4)*h, y(i) + (a(4,1)*k1 + a(4,2)*k2 + a(4,3)*k3)*h);
    y(i+1) = y(i) + (b(1)*k1 + b(2)*k2 + b(3)*k3 + b(4)*k4)*h;
end
```

안풀면 0점, 제대로 작동하지 않으면 10점, 작동은 하지만 해가 다르게 나오면 20점

2. Solve the following initial value problem over the interval from $t = 0$ to 10 with the **Euler's method**, **Heun's method** (w/o update), **Midpoint method**, **Ralston's method** using the MATLAB function developed in prob. 1 where $y(0) = 0$:

$$\frac{dy}{dt} = \frac{1+\cos t}{1+2y}$$

(1) Compare the four different results by using graphical method. (10 pts)

```
dydt =@(t,y) (1 + cos(t))/(1 + 2*y);
tspan = [0 10];
y0 = 0;
h = 1;
[t,y] = myode(dydt, tspan, y0, h, a, b, c);
```

[Euler]

```
a = [0 0 0 0; 0 0 0 0; 0 0 0 0; 0 0 0 0];
b = [1 0 0 0];
c = [0 0 0 0];
```

[Midpoint]

```
a = [0 0 0 0;1/2 0 0 0;0 0 0 0;0 0 0 0];
b = [0 1 0 0];
c = [0 1/2 0 0];
```

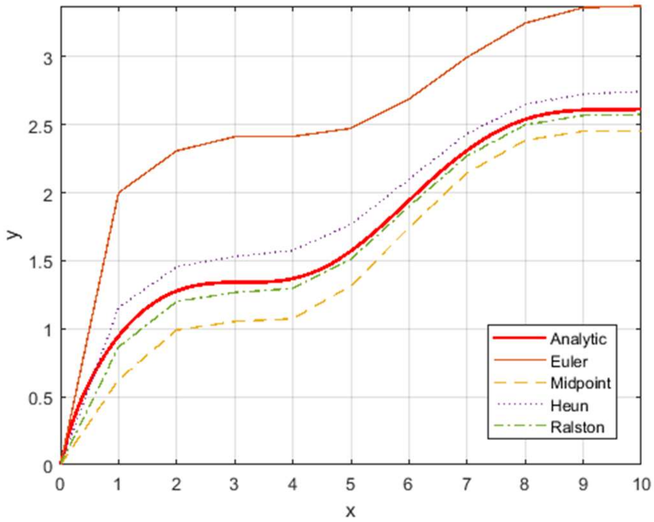
[Heun]

```
a = [0 0 0 0; 1 0 0 0; 0 0 0 0; 0 0 0 0];
b = [1/2 1/2 0 0];
c = [0 1 0 0];
```

[Ralston]

```
a = [0 0 0 0; 2/3 0 0 0; 0 0 0 0; 0 0 0 0];
```

```
b = [1/4 3/4 0 0];
c = [0 2/3 0 0];
```



그래프 안그리면 0점, 그렸는데 틀렸으면 5점, 아쉬우면 8점

(2) When the analytic solution of the above initial value problem is given as

$$y = \frac{1}{2}\sqrt{4t + 4\sin t + 1} - \frac{1}{2}$$

suggest the reasonable value of step size h, for each method, which the NRMSE is less than 2%. (10 pts)

$$\mathrm{NRMSE}(y) = \frac{\mathrm{RMSE}(y)}{y_{\max} - y_{\min}}$$

$$\mathrm{RMSE}(y) = \sqrt{\frac{\sum_{i=1}^{N}\left(y_i - \hat{y}_i\right)^2}{N}}$$

where y is the analytic solution and $\hat{y}$ is the numerical one.

Approximately,

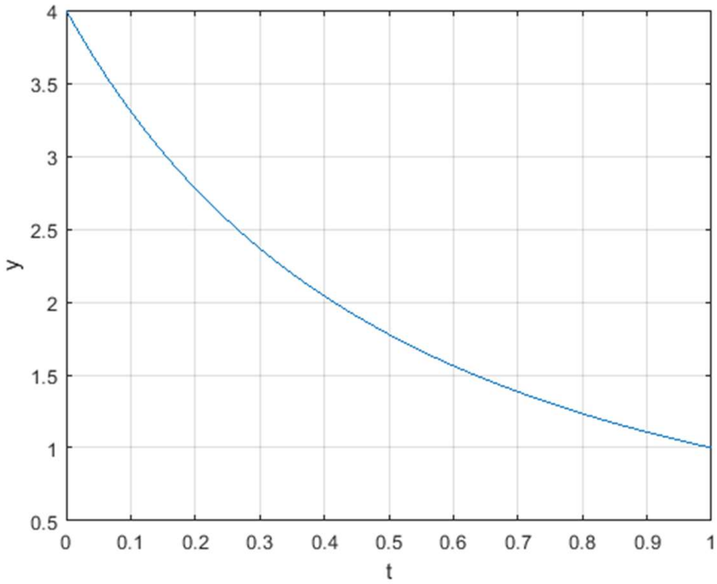
```
h_Euler = 0.11504;

h_Midpoint = 0.41436;

h_Heun = 0.625;

h_Ralston = 0.83333;
```

위 값은 fzero 함수 이용하여 NRMSE가 근사적으로 2% 나오도록 하는 step size를 구한 것이다. 이처럼 해를 구하지 않고 h를 줄여가면서 2% 이내로 들어오면 멈추는 식으로 대략 구하면 부분점수 8점 부여한다.

3. Solve the following boundary value problem using shooting method (10 pts)

$$\frac{d^2y}{dt^2} = \frac{3}{2}y^2, \quad y(0)=4, \quad y(1)=1$$

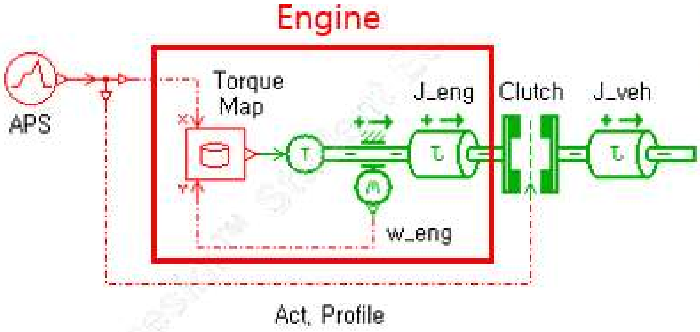
```
function prob3()
close all; clear; clc

dydt =@(t,y) [y(2);3*y(1)^2/2];

tspan = [0 1];
y1 = 4;
y2 = 1;
z1 = 0;
za = fzero(@res, z1, [], dydt, tspan, y1, y2);
[t,y] = ode45(dydt, tspan, [y1, za]);
plot(t, y(:,1))
y(end,1)

function r = res(za, dydt, tspan, y1, y2)
[~,y] = ode45(dydt, tspan, [y1,za]);
r = y(end,1) - y2;
```

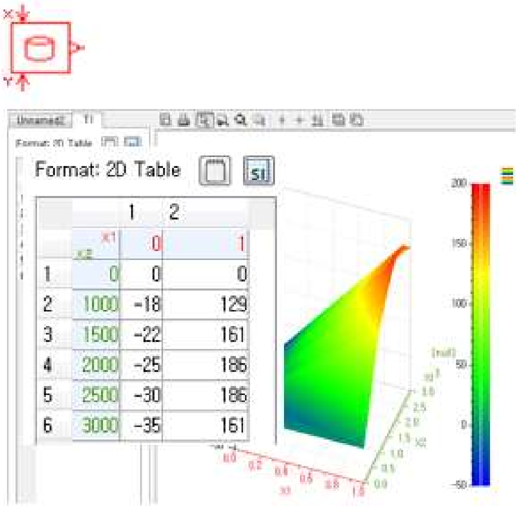


za = -8;

방법 맞는데 답 틀리면 8점, 하긴 했는데 틀리면 5점, 매우 이상한 결과 1점

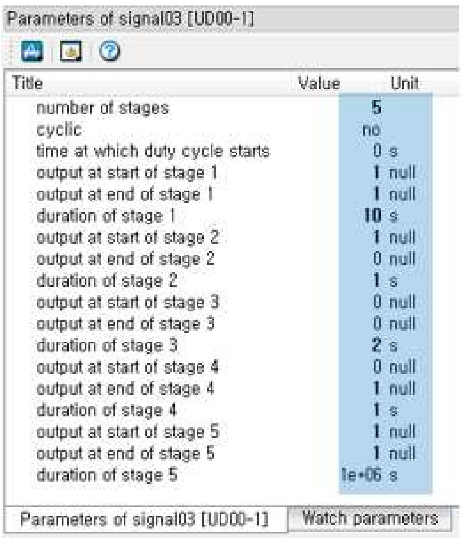
4. [AMESim/Simulink] Consider the following engine model by AMESim.





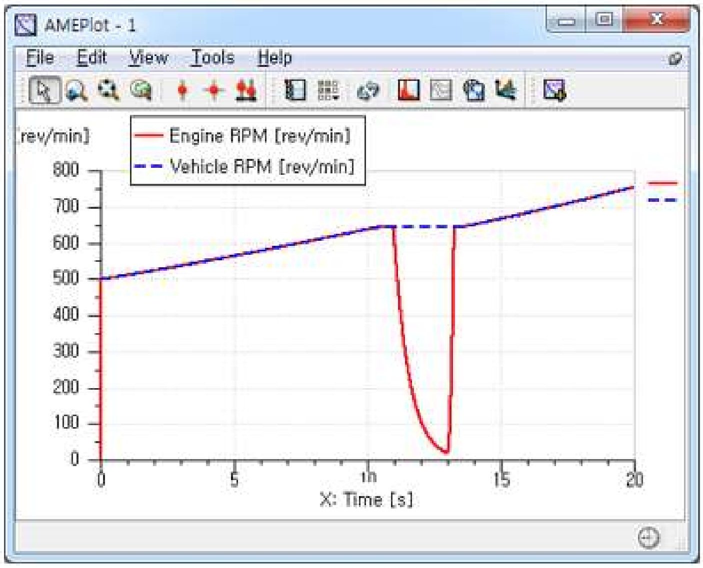
	x1	0	1
	x2		
1	0	0	0
2	1000	-18	129
3	1500	-22	161
4	2000	-25	186
5	2500	-30	186
6	3000	-35	161

① J_eng = 0.1 kgm^2, w_eng = 0 RPM
② J_veh = 50 kgm^2, w_veh = 500 RPM
③ clutch friction torque = 200 Nm


Parameters of signal03 [UD00-1]

Title	Value	Unit
number of stages	5	
cyclic	no	
time at which duty cycle starts	0	s
output at start of stage 1	1	null
output at end of stage 1	1	null
duration of stage 1	10	s
output at start of stage 2	1	null
output at end of stage 2	0	null
duration of stage 2	1	s
output at start of stage 3	0	null
output at end of stage 3	0	null
duration of stage 3	2	s
output at start of stage 4	0	null
output at end of stage 4	1	null
duration of stage 4	1	s
output at start of stage 5	1	null
output at end of stage 5	1	null
duration of stage 5	1e+06	s

Parameters of signal03 [UD00-1] Watch parameters



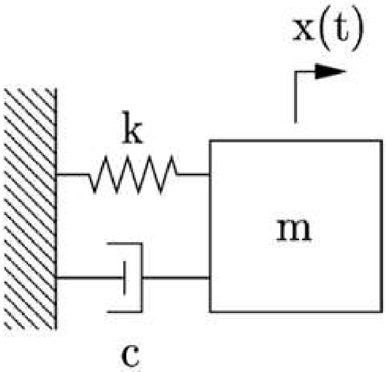
(1) Construct this AMESim model. Compare the engine- and vehicle-side RPM. (Simulation time: 20 s, Print interval: 0.001 s) (10 pts)

제대로된 모델은 10점 했는데 틀린 경우 5점

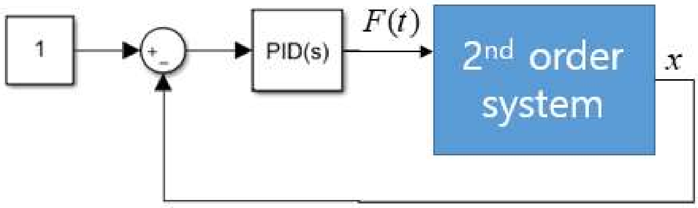
(2) Construct a Simulink model for this system. Show the engine- and vehicle-side RPM by using scope block. (15 pts)

모델의 정확도 및 완성도에 따라 15, 10, 5점

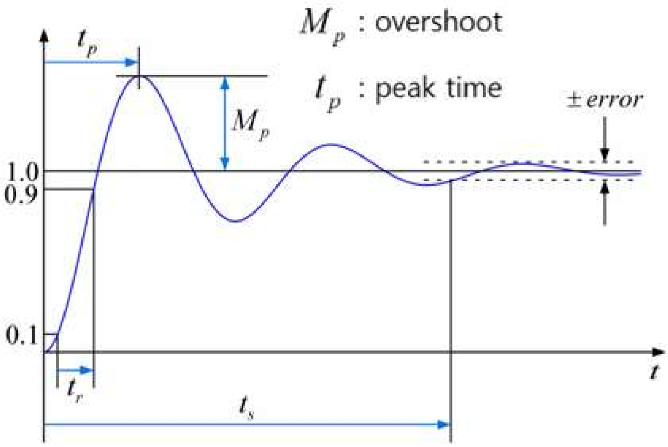
5. [Simulink] Consider the following PID control system.



$$m\frac{d^2x}{dt}+c\frac{dx}{dt}+kx=F(t) \qquad m,c,k=1$$



No.	Kp	Ki	Kd
①	1	0	0
②	5	0	0
③	5	0	4
④	5	5	4
⑤	5	5	5



Construct this Simulink model. Confirm the output displacement values for each PID parameter case (①~⑤). Choose the proper cases for the following questions. (Simulation time: 10 s) (15 pts)

(1) No overshoot : ①,③,⑤

(2) Peak time less than 1 second : ③,④,⑤

(3) Error less than 0.1 : ④,⑤

(4) Best controlled case : ⑤

맞춘 개수에 비례하여 채점