Midterm Exam

10/29/2024

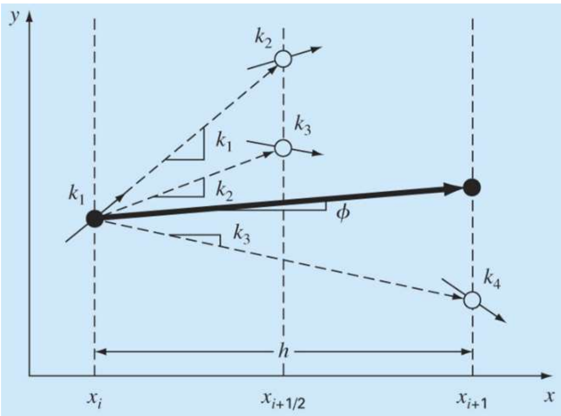
1.

$$\frac{d^2y}{dx^2}+0.5\frac{dy}{dx}+7y=0 \rightarrow \begin{cases} \dfrac{dy}{dx}=z, y(0)=4 \\ \dfrac{dz}{dx}=-0.5z-7y, y'(0)=0 \rightarrow z(0)=0 \end{cases} \color{red}{(5\text{ pts})}$$

$$\Delta x = 0.5 = h$$

$$\color{red}{(10\text{ pts})}\begin{cases} k_{1,y}=\color{blue}{0} \rightarrow y(0)+k_{1,1}\dfrac{h}{2}=4+(0)\dfrac{0.5}{2}=4 \\ k_{1,z}=-0.5(0)-7(4)=\color{blue}{-28} \rightarrow z(0)+k_{1,2}\dfrac{h}{2}=0+(-28)\dfrac{0.5}{2}=-7 \\ k_{2,y}=\color{blue}{-7} \rightarrow y(0)+k_{2,y}\dfrac{h}{2}=4+(-7)\dfrac{0.5}{2}=2.25 \\ k_{2,z}=-0.5(-7)-7(4)=\color{blue}{-24.5} \rightarrow z(0)+k_{2,z}\dfrac{h}{2}=0+(-24.5)\dfrac{0.5}{2}=-6.125 \\ k_{3,y}=\color{blue}{-6.125} \rightarrow y(0)+k_{3,y}h=4+(-6.125)(0.5)=0.9375 \\ k_{3,z}=-0.5(-6.125)-7(2.25)=\color{blue}{-12.6875} \rightarrow z(0)+k_{3,z}h=0+(-12.6875)(0.5)=-6.3438 \\ k_{4,1}=\color{blue}{-6.3438} \\ k_{4,2}=-0.5(-6.3438)-7(0.9375)=\color{blue}{-3.3906} \end{cases}$$

$$\left.\begin{aligned} y&=y(0)+\frac{1}{6}\left(k_{1,y}+2k_{2,y}+2k_{3,y}+k_{4,y}\right)h=4+\frac{1}{6}\left[0+2(-7-6.125)-6.3438\right](0.5)=\color{red}{1.2839} \\ z&=z(0)+\frac{1}{6}\left(k_{1,z}+2k_{2,z}+2k_{3,z}+k_{4,z}\right)h=0+\frac{1}{6}\left[-28+2(-24.5-12.6875)-3.3906\right](0.5)=\color{red}{-8.8138} \end{aligned}\right\}\color{red}{(5\text{ pts})}$$



2. r도 이산화하여 i로 표현하지 않으면 -5점, r=0과 r=1도 이산화로 표시해야 함

(1) divide the radial coordinate into n finite points: $\Delta r = \dfrac{1}{n-1}$

$$\frac{d^2T}{dr^2} = \frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta r)^2}, \frac{dT}{dr} = \frac{T_{i+1} - T_{i-1}}{2(\Delta r)}, r = \Delta r(i-1)$$

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + S = 0 \rightarrow \frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta r)^2} + \frac{1}{\Delta r(i-1)}\frac{T_{i+1} - T_{i-1}}{2(\Delta r)} + S = 0 \ (10 \text{ pts})$$

$$-\left[1 - \frac{1}{2(i-1)}\right]T_{i-1} + 2T_i - \left[1 + \frac{1}{2(i-1)}\right]T_{i+1} = (\Delta r)^2 S$$

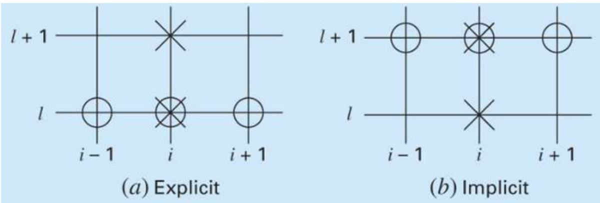
(2) $@i=1$: $\left.\dfrac{dT}{dr}\right|_{r=0} = 0 \rightarrow \dfrac{T_2 - T_0}{2(\Delta r)} = 0 \rightarrow T_0 = T_2$ (5 pts)

$$-\left[1 - \frac{1}{2(i-1)}\right]T_0 + 2T_1 - \left[1 + \frac{1}{2(i-1)}\right]T_2 = (\Delta r)^2 S \rightarrow 2T_1 - 2T_2 = (\Delta r)^2 S$$

(3) $@i = n-1$: $T(r=1) = 0 \rightarrow T_n = 0$ (5 pts)

$$-\left[1 - \frac{1}{2(i-1)}\right]T_{n-2} + 2T_{n-1} - \left[1 + \frac{1}{2(i-1)}\right]T_n = (\Delta r)^2 S \rightarrow -\left[1 - \frac{1}{2(i-1)}\right]T_{n-2} + 2T_{n-1} = (\Delta r)^2 S$$

3.



(1) explicit: $\dfrac{\partial^2 T}{\partial x^2} = \dfrac{T_{i+1}^l - 2T_i^l + T_{i-1}^l}{\Delta x^2}$, $\dfrac{\partial T}{\partial t} = \dfrac{T_i^{l+1} - T_i^l}{\Delta t}$

$$k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \rightarrow k\frac{T_{i+1}^l - 2T_i^l + T_{i-1}^l}{(\Delta x)^2} = \frac{T_i^{l+1} - T_i^l}{\Delta t} \ (8 \text{ pts}) \rightarrow T_i^{l+1} = T_i^l + \lambda\left(T_{i+1}^l - 2T_i^l + T_{i-1}^l\right) \text{ where } \lambda = \frac{k\Delta t}{(\Delta x)^2}$$

(2) implicit: $\dfrac{\partial^2 T}{\partial x^2} = \dfrac{T_{i+1}^{l+1} - 2T_i^{l+1} + T_{i-1}^{l+1}}{(\Delta x)^2}$

$$k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \rightarrow k\frac{T_{i+1}^{l+1} - 2T_i^{l+1} + T_{i-1}^{l+1}}{(\Delta x)^2} = \frac{T_i^{l+1} - T_i^l}{\Delta t} \ (8 \text{ pts}) \rightarrow -\lambda T_{i-1}^{l+1} + (1+2\lambda)T_i^{l+1} - \lambda T_{i+1}^{l+1} = T_i^l$$

(3) 수렴성 (λ조건 vs. 무조건) (4 pts)

4.

$$xy\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}+yu=0 \leftarrow u(x,y)=G(x)H(y)$$ (5 pts: 변수분리 아이디어)

$$xyH\frac{dG}{dx}-G\frac{dH}{dy}+yGH=0 \rightarrow xyH\frac{dG}{dx}+yGH=G\frac{dH}{dy}$$

$$y\left(xH\frac{dG}{dx}+GH\right)=G\frac{dH}{dy} \rightarrow xH\frac{dG}{dx}+GH=\frac{G}{y}\frac{dH}{dy}$$

$$\frac{x}{G}\frac{dG}{dx}+1=\frac{1}{yH}\frac{dH}{dy} \text{(3 pts)} \rightarrow \begin{cases} \frac{x}{G}\frac{dG}{dx}+1=c \rightarrow \frac{1}{G}dG=\frac{c-1}{x}dx \rightarrow \ln G=(c-1)\ln x \rightarrow G=k_1x^{(c-1)} \text{ (3 pts)} \\ \frac{1}{yH}\frac{dH}{dy}=c \rightarrow \frac{dH}{H}=cydy \rightarrow \ln H=\frac{cy^2}{2}+d \Longrightarrow H=k_2e^{\frac{c}{2}y^2} \text{ (3 pts)} \end{cases}$$

$$u(x,y)=k_1x^{(c-1)}k_2e^{\frac{c}{2}y^2}=kx^{(c-1)}e^{\frac{c}{2}y^2}$$ (1 pts)

5. 각1점, 8개 모두 작성하면 +2점

	Component	Input	Output
(1)	Engine	APS, RPM	Torque
(2)	Motor	APS, RPM	Torque
(3)	Clutch	Force	Torque
(4)	Transmission	Torque, RPM	Torque, RPM
(5)	Battery	Power(motor)	SOC(current)
(6)	Resistance	Vehicle speed, Gradient	Force(drag)
(7)	Vehicle (equivalent inertia)	Torque	Vehicle speed
(8)	driver	Vehicle speed	APS

6.

(1) $\omega_s=0 \rightarrow \frac{\omega_c}{\omega_r}=\frac{Z_r}{Z_s+Z_r}=GR_{PSD}=\frac{2}{3}$ (2pts), $J_{vehicle}=mR_{tire}^2=1500\times0.3^2=135\ \text{kg}\cdot\text{m}^2$ (2pts)

$$J_{eq}=J_{motor}\times(GR_{PSD}\times GR_{TM}\times GR_{diff})^2+J_{wheel}+J_{vehicle}=0.05\times\left(\frac{2}{3}\times3\times4\right)^2+1+135=139.2\,\text{kg}\cdot\text{m}^2$$ (2pts)

(2) $T_{wheel}=T_{motor}\times(GR_{PSD}+GR_{TM}+GR_{diff})=50\times\left(\frac{2}{3}\times3\times4\right)=400\ \text{Nm}$ (2pts)

$$F_{drag}=\frac{1}{2}C_dA_{front}\rho_{air}V_{vehicle}^2+\mu_{roll}M_{vehicle}g=\frac{1}{2}\times0.25\times1.8\times1.2\times20^2+0.01\times1500\times9.81=255.15\ \text{N}$$ (2pts)

$$T_{drag}=F_{drag}R_{tire}=255.15\times0.3=76.55\ \text{Nm}$$,

$$a_{vehicle}=\frac{T_{wheel}-T_{drag}}{J_{eq}}R_{tire}=\frac{400-76.55}{139.2}\times0.3=0.697\ \text{m/s}^2$$ (4pts)

(3) $\omega_s:(\text{engine rpm}),\ \omega_c:(\text{motor}),\ \omega_s+2\omega_r-3\omega_c=0\rightarrow\omega_r=\dfrac{3(2000)-3000}{2}=1500\ \text{RPM}$ (2pts)

$$\omega_{wheel}=\frac{\omega_{ring}}{GR_{TM}\times GR_{diff}}=\frac{1500}{3\times4}=125\ \text{RPM}=13.09\,\text{rad/s}\quad \text{(2pts)}$$

$$V_{vehicle}=R_{tire}\omega_{wheel}=0.3\times13.09=3.93\ \text{m/s}\quad \text{(1pt)}$$

(4) $\omega_{motor}=\dfrac{V_{vehicle}}{R_{tire}}\times GR_{diff}\times GR_{TM}\times GR_{PSD}=266.67\,\text{rad/s}$ (1pt)

$$T_{motor}=T_{regen}\times\frac{1}{GR_{diff}\times GR_{TM}\times GR_{PSD}}=-125\,\text{Nm}\quad \text{(1pt)}$$

$$I_{battery}=\frac{T_{regen}\omega_{wheel}}{V_{battery}}=\frac{-1000\times10/0.3}{250}=-133.33\ \text{A}\quad \text{(1pt)}$$

$$\frac{dSOC}{dt}=-I_{battery}\frac{100}{C_{nom}}=133.33\frac{100}{50,000}=0.267\ \text{\%/s}\quad \text{(2pts)}$$

$$SOC_{final}=SOC_{initial}+\frac{dSOC}{dt}\Delta t=50+0.267\times20=55.34\ \%\quad \text{(1pt)}$$