Final Exam

06/14/2011

1. (30 pts) Describe the process to solve $u' = -10u$ by the second order methods and check the stability.

(1) Runge-Kutta

(2) Adams-Bashforth

2. (30 pts) The vibrations of a string in a piano do not precisely obey the wave equation, because the nonzero diameter of the wire introduces some resistance to bending, and this adds to the restoring force contribution from the string tension. The resulting dispersion yields inharmonicities for overtones, i.e., higher normal modes have frequencies that are noninteger multiples of the fundamental. An exact treatment would use a nonlinear formula like $u_{tt} = Ku_{xx} + \kappa u_{xx}\left(1+u_x^2\right)^{-3/2}$, but we will consider the traditional model for this phenomenon, given by the equation $u_{tt} = c^2 u_{xx} - d^2 u_{xxxx}$. κ and d are determined by the Young's modulus of the string material and the area moment of inertia of the string cross section.

(1) Compute the dispersion relation, i.e., find how the speed of wave propagation depends on the wavenumber k of the initial conditions $u = (x,0) = e^{ikx}$.

(2) Let $r = \frac{c\Delta t}{\Delta x}$ and $R = \frac{d\Delta t}{(\Delta x)^2}$. Evaluate a leapfrog method for accuracy and stability.

3. (40 pts) Describe the process to solve the linear systems by

(1) Elimination with reordering

(2) Jacobi iteration

(3) A simple multigrid method (V-cycle)

(4) Conjugate gradients