Final Exam

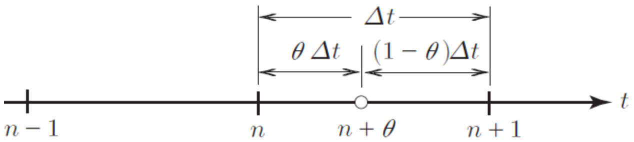
06/20/2016

1. (30 pts) Consider the unsteady heat conduction equation $\mathbf{M}\dot{\mathbf{T}}^{n+\theta} + \mathbf{K}\mathbf{T}^{n+\theta} = \mathbf{F}$ discretized in space at time step $n+\theta\left(0 \le \theta \le 1\right)$

(1) Derive $\dot{\mathbf{T}}^{n+\theta}$ and $\mathbf{T}^{n+\theta}$ using Taylor series expansion.

(2) Derive the equation to obtain the temperature at time step $n+1$.

(3) Classify the forward difference method, backward difference method and Crank-Nicolson method based on the value of θ and compare the accuracy.



2. (30 pts) Consider the wave equation $\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$ and discretized equation in space at node j using lumped mass can be written as $u_j^{n+1} = \frac{1}{6}\left(u_{j-1}^n + 4u_j^n + u_{j+1}^n\right) - \frac{1}{2}\frac{c\Delta t}{h}\left(-u_{j-1}^n + u_{j+1}^n\right)$.

(1) Rearrange the discretized equation to show the artificial diffusion.

(2) Describe how to stabilize it using finite difference method.

(3) Describe how to stabilize it using finite element method.

3. (40 pts) Consider the dynamic equation discretized in space $m\ddot{x}+c\dot{x}+kx=f$.

(1) Write the velocity using central difference method and acceleration at time step i .

(2) Derive the equation to obtain the displacement at time step $i+1$.

(3) At time $t=0$, x^0 and $\dot{x}^0$ are given. What is x^{-1} ?

(4) Write the velocity and displacement at time step $i+1$ using (1) averaging and (2) linear acceleration method. (Use the following figure and do not employ Taylor series expansion.)

(5) Propose the generalized velocity and displacement based on the result of linear acceleration method. (One example is Newmark β method.)