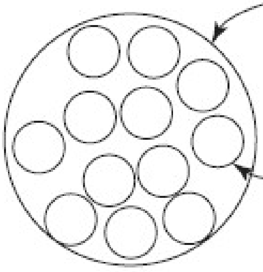
1. Optimization problem formulation (Please do not solve!)

(1) A company is redesigning its parallel flow heat exchanger of length l to increase its heat transfer. An end view of the unit is shown in the figure. There are certain limitations on the design problem. The smallest available conducting tube has a radius of 0.5 cm and all tubes must be of the same size. Further, the total cross-sectional area of all the tubes cannot exceed 2000cm^2 to ensure adequate space inside the outer shell. Formulate the problem to determine the number of tubes and the radius of each tube to maximize the surface area of the tubes in the exchanger. (10 pts)



(2) A manufacturer sells products A and B. Profit from A is \$10/kg and from B \$8/kg. Available raw materials for the products are: 100 kg of C and 80 kg of D. To produce 1 kg of A, 0.4 kg of C and 0.6 kg of D are needed. To produce 1 kg of B, 0.5 kg of C and 0.5 kg of D are needed. The markets for the products are 70 kg for A and 110 kg for B. How much of A and B should be produced to maximize profit? Formulate the design optimization problem. (10 pts)

(3) A company has m manufacturing facilities. The facility at the i-th location has capacity to produce b_i units of an item. The product should be shipped to n distribution centers. The distribution center at the j-th location requires at least a_j units of the item to satisfy demand. The cost of shipping an item from the i-th plant to the j-th distribution center is c_{ij}. Formulate a minimum cost transportation system to meet each distribution center's demand without exceeding the capacity of any manufacturing facility. (10 pts)

2. Find the extreme points of the function $f(x_1, x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6$ and justify the nature of these extreme points. (10 pts)

3. A problem of the form "$\min f(\tilde{x})$ subject to $h = 0$" is scaled, becoming the form of "$\min k_1 f(\tilde{x})$ subject to $k_2 h = 0$". What is the relation between the Lagrange multipliers of h for the unscaled and scaled problem? (10 pts)

4. Verify whether $\boldsymbol{x}^* = (0, 0)$ is optimal. (20 pts)

$$\text{Minimize } f = (x_1 - 1)^2 + x_2^{\,2}$$
$$\text{subject to } -x_1 + x_2^{\,2} \geq 0$$

5. A cylinder tank that is closed at both ends is to be fabricated to have a volume of 250π m^3. The fabrication cost is found to be proportional to the surface area of the sheet metal needed for fabrication of the tank and is \$400/m^2. The tank is to be housed in a shed with a sloping roof which limits the height of the tank by the relation $H \leq 8D$, where H is the height and D is the diameter of the tank.

(1) Formulate the minimum cost design problem. (5 pts)

(2) Check the convexity of the problem. (5 pts)

(3) Write KKT necessary conditions. (5 pts)

(4) Solve KKT necessary conditions for local minimum points. Check sufficient conditions and verify the conditions graphically. (10 pts)

(5) What will be the change in cost if the volume requirement is changed to 255π m^3 in place of 250π m^3? (5 pts)