Final Exam

1. (25 pts) The objective is to design a minimum-mass *tubular* column of length *l* supporting a load *P* without buckling or overstressing. The column is fixed at the base and free at the top, as shown in the figure. This type of structure is called a cantilever column. Many formulations for the design problem are possible depending on how the design variables are defined. Formulate two design optimization problems, but do not solve. (The buckling load (also called the critical load) for a cantilever column is given as $P_{cr} = \pi^2 E I / 4l^2$)



2. (20 pts) Describe a procedure for preliminary structure sizing for front impact.

3. (20 pts) In the standard rear impact test, the stationary target vehicle is impacted by a moving barrier.

(1) What is the criterion for this test?

(2) If we could replace this impact with one between a moving vehicle and fixed barrier, we could apply the structure-sizing procedure developed for the front barrier case. Derive the equivalent impact velocity which would result in the same work of deformation to be done by the structure.



4. (10 pts) Derive a general equation for the energy conservation in case of an impact between two vehicles, illustrated in the figure where

 m_i : mass of vehicle *i*

- v_i : velocity of vehicle *i* before impact
- v'_i : velocity of vehicle *i* after impact
- I_i : mass moment of inertia of vehicle *i*
- ω_i : angular velocity of vehicle *i* after impact
- W_i : energy of deformation of vehicle *i* by plastic deformation



5. (25 pts) With the source-path-receiver model below, we have engine unbalance forces applied to the powertrain. The path into the body is through the mounted powertrain acting as an isolator. For body design, we are interested in the force transmitted through the engine mounts into the body structure. The mounted powertrain is a SDOF system. *F* is the magnitude of the unbalance force and $F_{\rm T}$ is the force applied to the body structure through the engine mounts. Derive the isolation frequency.

