1. [Design Optimization Problem Formulation]

$$\underset{R,t}{\text{minimize}}\ \text{mass} = \rho(lA) = 2\rho l\pi Rt$$

(1)
$$\text{subject to} \begin{cases} \sigma = \dfrac{P}{A} = \dfrac{P}{2\pi Rt} \le \sigma_a \\ P \le \dfrac{\pi^2 EI}{4l^2} = \dfrac{\pi^3 ER^3 t}{4l^2} \end{cases}$$

$$R_{\min} \le R \le R_{\max};\quad t_{\min} \le t \le t_{\max}$$

$$\left[\text{assuming thin wall } (R >> t) \rightarrow A = 2\pi Rt;\ I = \pi R^3 t\right]$$

$$\underset{R,t}{\text{minimize}}\ \text{mass} = \rho(lA) = \pi\rho l\left(R_0^2 - R_i^2\right)$$

$$\text{subject to} \begin{cases} \sigma = \dfrac{P}{A} = \dfrac{P}{\pi\left(R_0^2 - R_i^2\right)} \le \sigma_a \\ P \le \dfrac{\pi^2 EI}{4l^2} = \dfrac{\pi^3 E}{16l^2}\left(R_0^4 - R_i^4\right) \end{cases}$$

(2)
$$(R_0)_{\min} \le R_0 \le (R_0)_{\max};\quad (R_i)_{\min} \le R_i \le (R_i)_{\max}$$

$$R_0 > R_i;\quad \frac{R}{t} = \frac{R_0 + R_i}{2(R_0 - R_i)} \ge k \quad (\text{thin-walled: } R >> t, k \ge 20)$$

$$\left[A = \pi\left(R_0^2 - R_i^2\right);\ I = \frac{\pi}{4}\left(R_0^4 - R_i^4\right)\right]$$

각 정식화별 목적함수, 구속조건2개(응력, 좌굴), 설계변수 side constraint: 4*3점

2. 붉은색 키워드 각 4점

- ✓ Select the maximum allowable cabin decelerations based on occupant injury (amax)
- ✓ Determine a consistent structural efficiency and crush space (η, Δ)
- ✓ Compute the average and maximum allowable crush forces which the vehicle must generate during impact (amax→Fmax,η→Favg)
- ✓ Allocate these total forces to the structural elements within the vehicle front end
- ✓ Size the crushable midrail using the average required crush force requirement
- ✓ If the peak crush load Pmax exceeds the maximum load requirement, then consider crush interior designs
- ✓ The cabin reaction structure capacity must exceed the maximum midrail crush load. Use limit analysis to determine the required plastic moments for the hinges
- ✓ Size the reaction structure sections to generate the hinge moments

3.

(1) Minimize fuel system leakage (fuel tank integrity) (5 pts)

$$M_1(0) + M_2 V_0 = (M_1 + M_2) V_F \rightarrow V_F = \frac{M_2}{M_1 + M_2} V_0 \quad (5\text{ pts})$$

$$\begin{cases} M_1 : \text{ struck vehicle mass} \\ M_2 : \text{ moving barrier mass} \\ V_0 : \text{ initial moving barrier speed} \\ V_F : \text{ final speed of vehicle and barrier} \end{cases}$$

(2) (work of deformation)

= (change of kinetic energy before and after the impact)

$$W = \frac{1}{2} M_2 {V_0}^2 - \frac{1}{2}(M_1 + M_2){V_F}^2 = \frac{1}{2}\left(\frac{M_1 M_2}{M_1 + M_2}\right){V_0}^2 \quad (5\text{ pts})$$

$$\frac{1}{2} M_1 {V_{EQ}}^2 = W \rightarrow V_{EQ} = V_0 \sqrt{\frac{M_2}{M_1 + M_2}} \quad (5\text{ pts})$$

4. $$\underbrace{\frac{1}{2} m_1 {v_1}^2 + \frac{1}{2} m_2 {v_2}^2}_{\text{initial kinetic energy}} = \underbrace{\frac{1}{2} m_1 {v_1'}^2 + \frac{1}{2} m_2 {v_2'}^2}_{\text{translational kinetic energy after impact}} + \underbrace{\frac{1}{2} I_1 {\omega_1}^2 + \frac{1}{2} I_2 {\omega_2}^2}_{\text{rotational kinetic energy after impact}} + \underbrace{W_1 + W_2}_{\text{energy of deformation}} \;) \; (10\text{ pts})$$

$$f(t) = F \sin(\omega t) \rightarrow x(t) = X \sin(\omega t)$$

$$f(t) - kx(t) = m\frac{d^2 x}{dt^2} \rightarrow F \sin(\omega t) = kX \sin(\omega t) - mX\omega^2 \sin(\omega t)$$

5. $$F = kX - m\omega^2 X \rightarrow \frac{X}{F} = \frac{1}{k - m\omega^2} = \frac{1/k}{1 - \left(\frac{m}{k}\right)\omega^2} = \frac{1/k}{1 - \left(\frac{\omega}{\omega_n}\right)^2} = P(\omega) \quad (10\text{ pts})$$

$$F_T = kX \quad (5\text{ pts})$$

$$\left|\frac{F_T}{F}\right| = |T(\omega)| = \left|\frac{1}{1 - (\omega/\omega_n)^2}\right| < 1 \rightarrow \omega > \omega_n\sqrt{2} \quad (10\text{ pts})$$